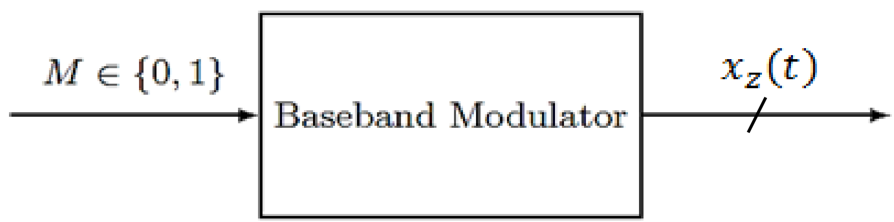
Digital Communication Part II
Optimal Single-Bit Transmission

(Chapter 13)

- Baseband modulator



$x_{z,0}(t)$ if $M = 0$

Energy $E_0 = \int |x_{z,0}(t)|^2 \, dt$

$x_{z,1}(t)$ if $M = 1$

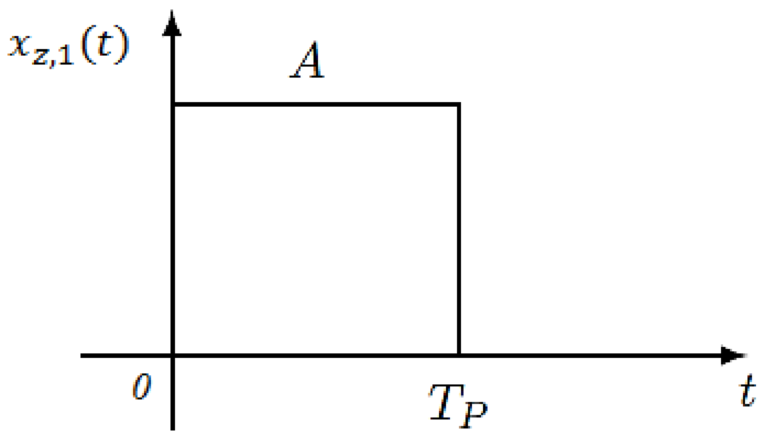
Energy $E_1 = \int |x_{z,1}(t)|^2 \, dt$

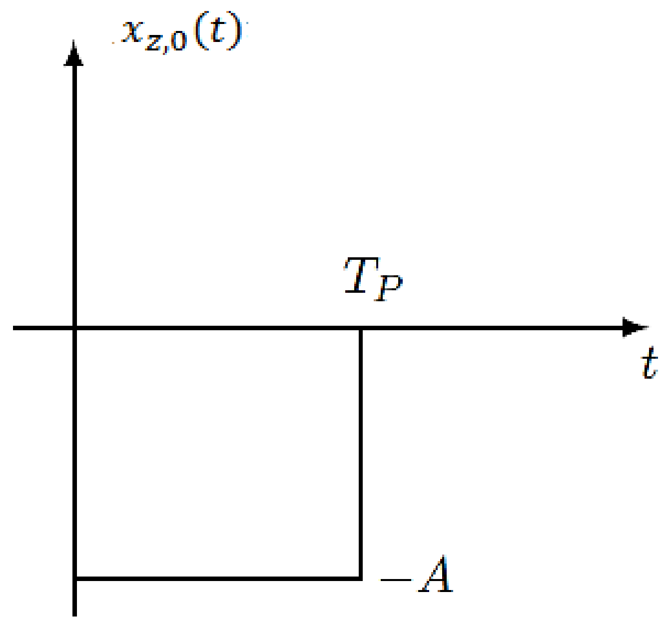
$$\pi_1 = \Pr(M = 1)$$
$$\pi_0 = \Pr(M = 0)$$
$$(\pi_0 + \pi_1 = 1)$$

Optimal Single-Bit Transmission

- Average energy per bit: $E_b = \pi_0 E_0 + \pi_1 E_1$
- If $x_z(t)$ is of duration T_P,
  - Average power $= P_s = \frac{E_s}{T_P}$
  - Bit rate $= R_b = \frac{1}{T_P}$

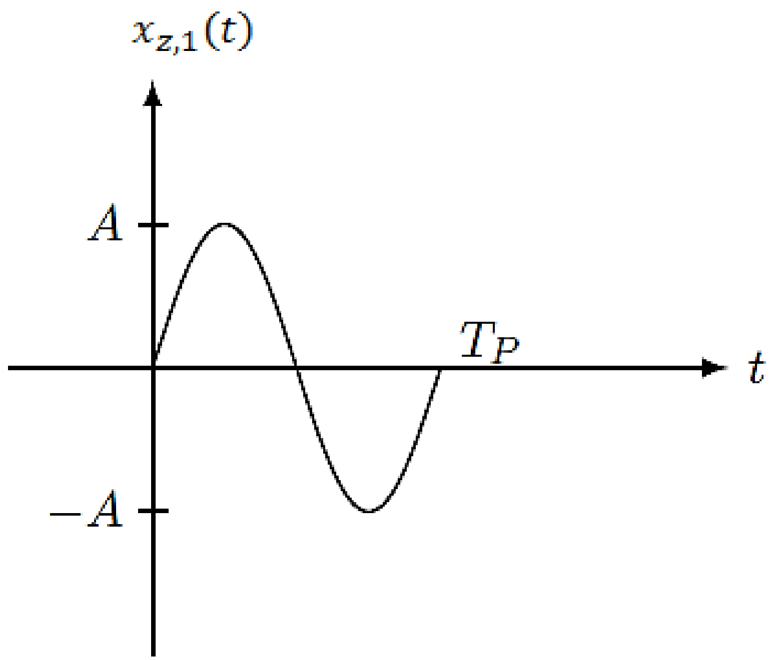
Example a)

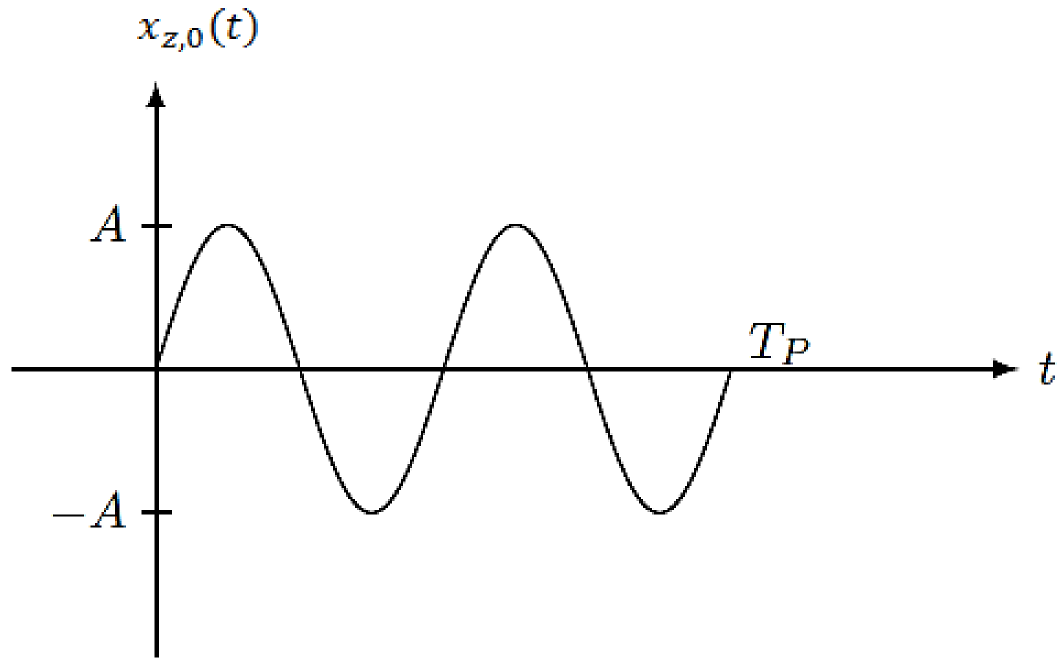




$$E_0 = E_1 = A^2 T_P \qquad \Longrightarrow \qquad E_b = \pi_0 E_0 + \pi_1 E_1 = A^2 T_P$$

Example b)





$$E_0 = E_1 = \frac{A^2 T_P}{2} \quad \Longrightarrow \quad E_b = \pi_0 E_0 + \pi_1 E_1 = \frac{A^2 T_P}{2}$$

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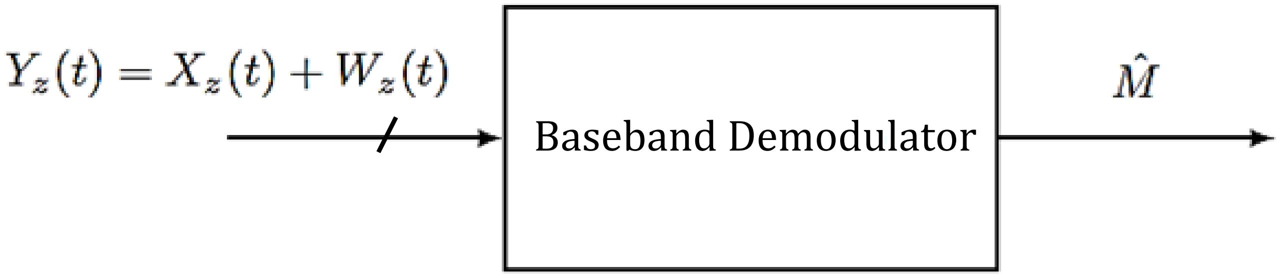
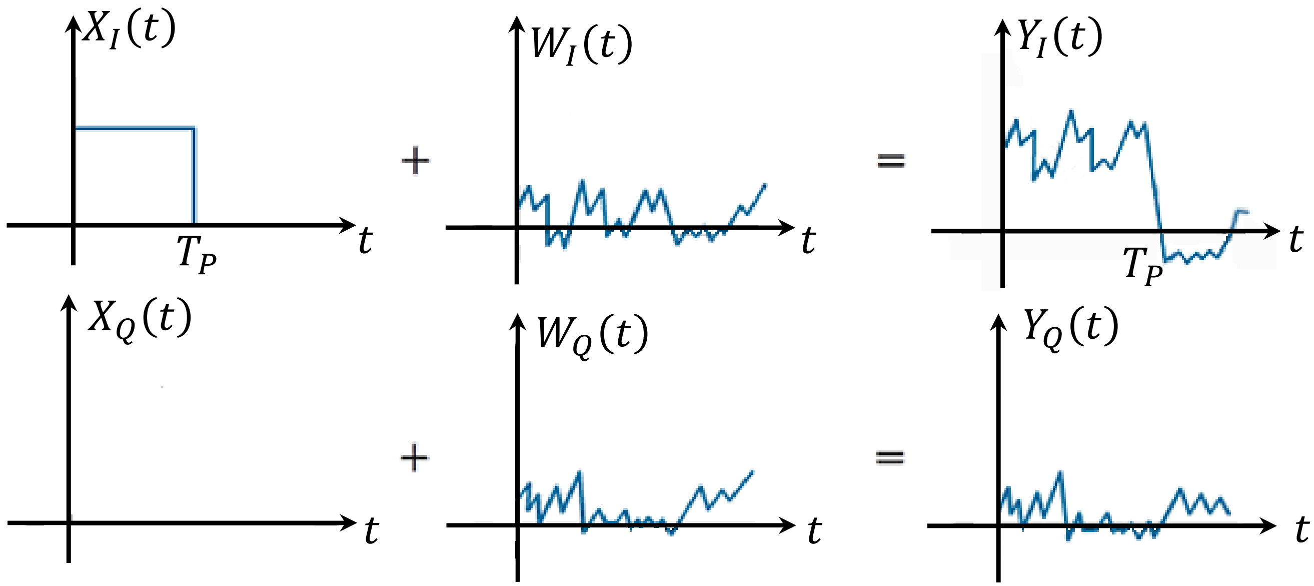
Baseband Demodulator



Ex.:

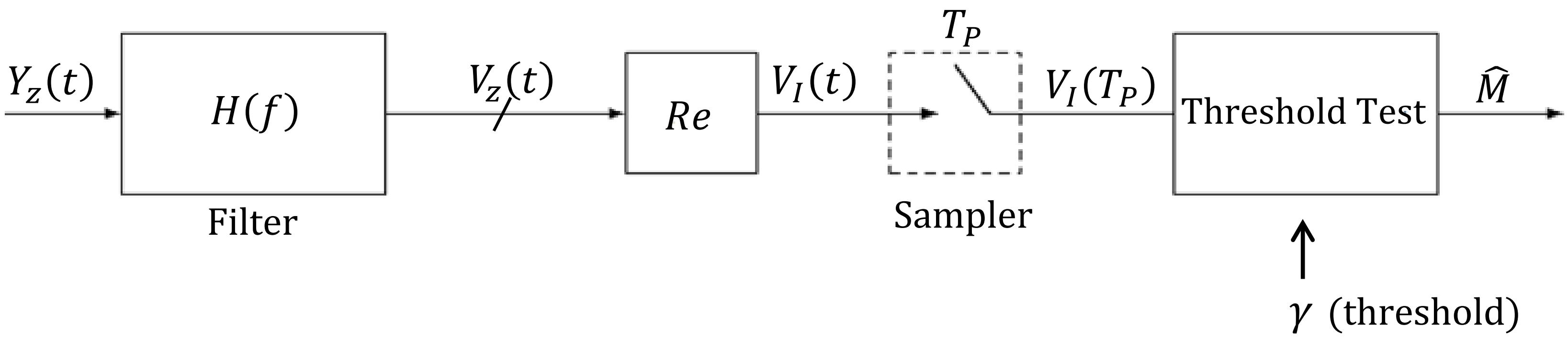


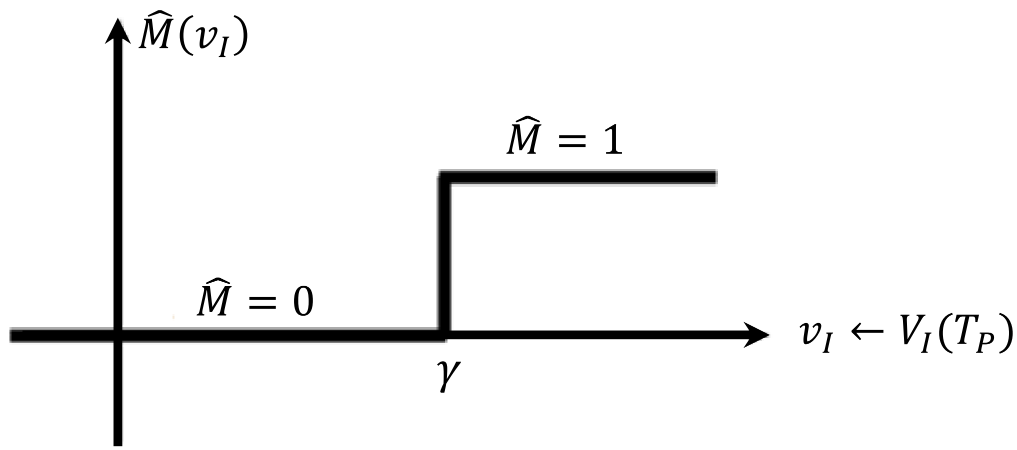
$X_z(t) \quad + \quad W_z(t) \quad = \quad Y_z(t)$

Design Problem

- Design domain
  1. Baseband modulator: Waveforms $x_{z,0}(t)$ and $x_{z,1}(t)$
  2. Baseband demodulator: to be discussed
- Design goal: Minimize the BEP
- Design constraints:
  – Duration T_P
  – Bandwidth B_T (or spectral efficiency $\eta_B = \frac{1/T_P}{B_T}$)
  – Average Power P_s (or $SNR = \frac{P_s}{N_0 B_T}$)

Structure of the Optimal Baseband Demodulator



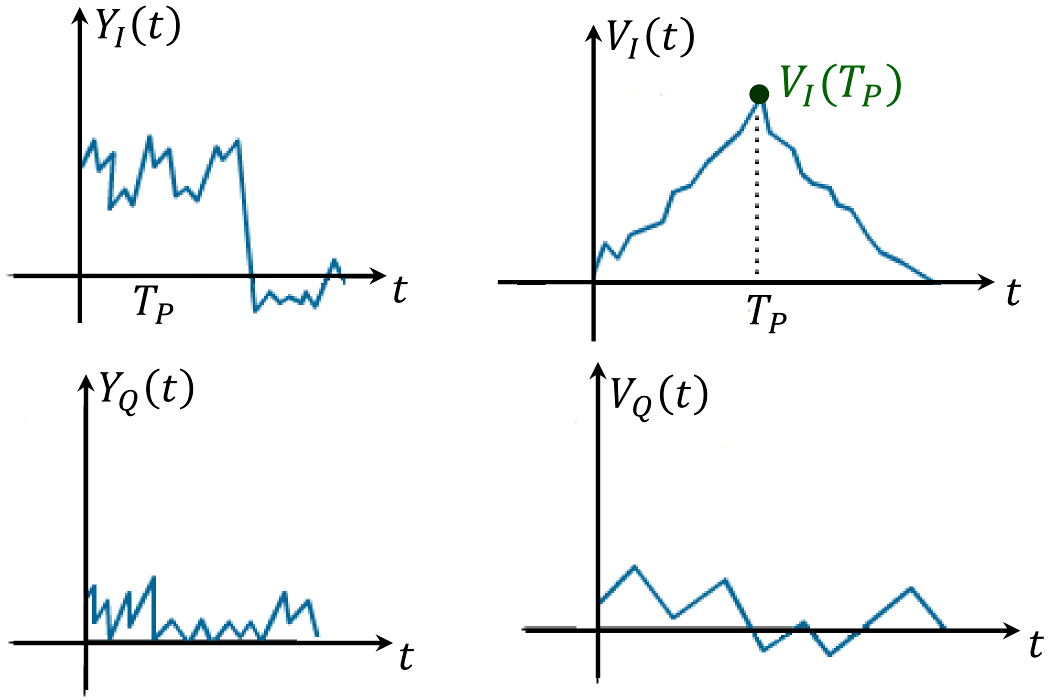


Remark: The threshold test can be expressed as

$$V_I(T_P) \underset{\hat{M}=0}{\overset{\hat{M}=1}{\gtrless}} \gamma$$

Structure of the Optimal Baseband Demodulator

Ex.:



Remark: $V_I(T_P)$ is a so called sufficient statistics for the estimate of M from $V_z(t)$

Design Problem

- Design domain
  1. Baseband modulator: Waveforms $x_{z,0}(t)$ and $x_{z,1}(t)$
  2. Baseband demodulator:
     1. Filter $H(f)$
     2. Threshold γ
- Design goal: Minimize the BEP
- Design constraints:
  - Duration T_P
  - Bandwidth B_T (or spectral efficiency $\eta_B = \frac{1/T_P}{B_T}$)
  - Average Power P_s (or $SNR = \frac{P_S}{N_0 B_T}$)

Three Steps to Solve the Design Problem

1. Optimum threshold γ

   Given $\{x_{z,0}(t), x_{z,1}(t)\}$ and $H(f)$, design γ

   $\longrightarrow$ maximum a posteriori bit demodulation (MAPBD)

2. Optimum filter $H(f)$:

   Given $\{x_{z,0}(t), x_{z,1}(t)\}$ and the optimal γ, design $H(f)$

   $\longrightarrow$ matched filter

3. Optimum waveforms $\{x_{z,0}(t), x_{z,1}(t)\}$

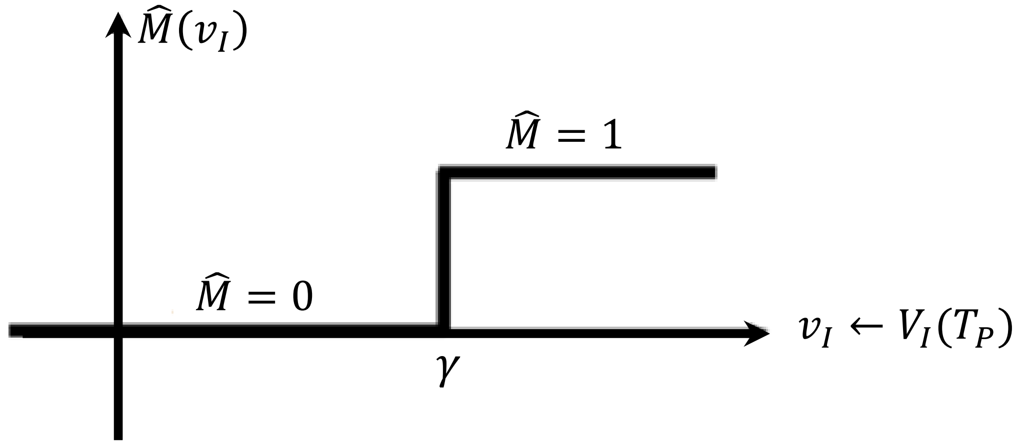
   Given γ and $H(f)$, design $\{x_{z,0}(t), x_{z,1}(t)\}$

Optimum Threshold γ

- Given $\{x_{z,0}(t), x_{z,1}(t)\}$ and $H(f)$, we wish to find γ that minimizes the BEP

$$P_B(E) = \Pr(\widehat{M} \neq M)$$

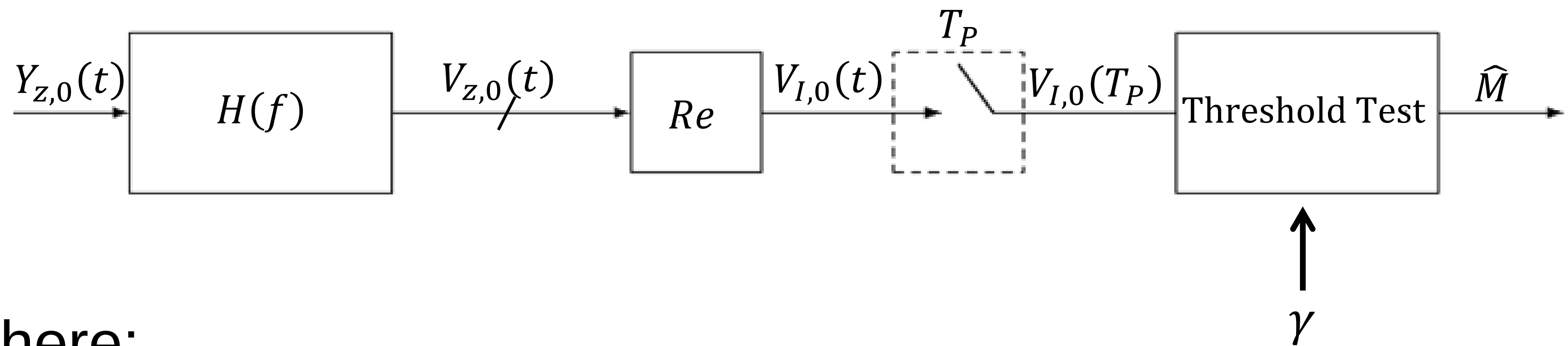
- Recall the rule



- That is, if $v_I > \gamma$ we have $\widehat{M} = 1$ and, otherwise, if $v_I < \gamma$ we have $\widehat{M} = 0$

Optimum Threshold γ

- If $M = 0$, the baseband signals at the receiver side are as follows:



where:

- $Y_{z,0}(t) = x_{z,0}(t) + W_{z,0}(t)$
- $V_{z,0}(t) = Y_{z,0}(t) * h(t)$

impulse response of the filter

convolution

$$= \underbrace{x_{z,0}(t) * h(t)}_{\triangleq m_0(t)} + \underbrace{W_{z,0}(t) * h(t)}_{\triangleq N_z(t)}$$

Optimum Threshold γ

- The noise $N_z(t) = N_I(t) + jN_Q(t)$ is such that:
  - $N_I(t)$ and $N_Q(t)$ are independent
  - $S_{N_I}(f) = S_{N_Q}(f) = |H(f)|^2 \frac{N_0}{2}$

    $\Rightarrow N_I(t) \sim N\left(0, \sigma_{N_I}^2\right)$ with $\sigma_{N_I}^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)|^2 \, df$

    and $N_Q(t) \sim N\left(0, \sigma_{N_Q}^2\right)$ with $\sigma_{N_Q}^2 = \sigma_{N_I}^2$

- $V_{I,0} = Re\{m_0(T_P)\} + N_I(T_P)$ where $Re\{m_0(T_P)\} \triangleq m_0$ and $N_I(T_P) \sim N\left(0, \sigma_{N_I}^2\right)$

Optimum Threshold γ

- To summarize, if $M = 0$, then the sufficient statistics $V_I(T_p)$ at the input of the threshold test is:

$$V_{I,0}(T_p) = m_0 + N_I$$

where $m_0 = Re\{x_{z,0}(t) * h(t)|_{t=T_p}\}$

$N_I \sim N(0, \sigma_{N_I}^2)$ with $\sigma_{N_I}^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)|^2 df$

- Similarly, if $M = 1$, we have:

$$V_{I,1}(T_p) = m_1 + N_I$$

where $m_1 = Re\{x_{z,1}(t) * h(t)|_{t=T_p}\}$

$N_I \sim N(0, \sigma_{N_I}^2)$ with $\sigma_{N_I}^2 = \frac{N_0}{2} \int_{-\infty}^{+\infty} |H(f)|^2 df$

Remark: The filter $H(f)$ affects both the signal part (m_0 and m_1) and the noise ($\sigma_{N_I}^2$)