Constrained Codes for Joint Energy and Information Transfer with Receiver Energy Utilization Requirements

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***Abstract*—In various wireless systems, such as sensor RFID networks and body area networks with implantable devices, the transmitted signals are simultaneously used both for information transmission and for energy transfer. In order to satisfy the conflicting requirements on information and energy transfer, this paper proposes the use of constrained run-length limited (RLL) codes in lieu of conventional unconstrained (i.e., random-like) capacity-achieving codes. The receiver's energy utilization requirements are modeled stochastically, and constraints are imposed on the probabilities of battery underflow and overflow at the receiver. It is demonstrated that the codewords' structure afforded by the use of constrained codes enables the transmission strategy to be better adjusted to the receiver's energy utilization pattern, as compared to classical unstructured codes. As a result, constrained codes allow a wider range of trade-offs between the rate of information transmission and the performance of energy transfer to be achieved.**

***Index Terms*—Energy transfer, constrained codes, energy harvesting.**

I. Introduction

Various modern wireless systems, such as sensor RFID networks and body area networks with implantable devices, challenge the conventional assumption that the energy received from an information bearing signal cannot be reused. This has motivated recent research activity on the optimal *resource allocation* in the presence of information and energy transfer for various network topologies, see, e.g., [1]-[6].

Unlike [1]-[6] and references therein, this work focuses on the *code design* for systems with joint information and energy transfer. We focus on a point-to-point link as shown in Fig. 1, in which the receiver's energy requirements are modeled as a random process. The statistics of this process generally depend on the specific application to be run at the receiver, e.g., sensing or radio transmission. The performance in terms of energy transfer is measured by the probabilities of overflow and underflow of the battery at the receiver. The probability of overflow measures the efficiency of energy transfer by accounting for the energy wasted at the receiver. Instead, the probability of underflow is a measure of the fraction of the time in which the application run at the receiver is in outage due to the lack of energy.

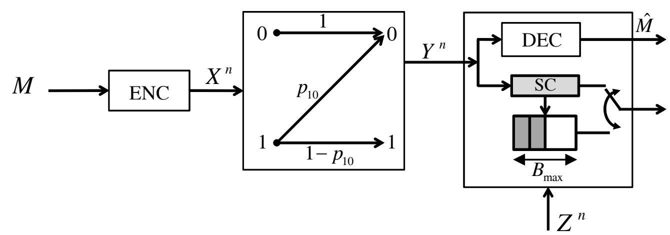

Fig. 1. Point-to-point link with information and energy transfer (SC: Supercapacitor).

Classical codes, which are designed with the only aim of maximizing the information rate, are unstructured (i.e., random-like). As a result, they do not allow to control the timing of the energy transfer, and hence to optimize the probability of overflow and underflow. With this in mind, here it is proposed to adopt constrained run-length limited (RLL) codes [7] in lieu of conventional unconstrained codes.

Constrained RLL codes have been traditionally studied for applications related to magnetic and optical storage [7]. The application to the problem at hand of energy transfer has been previously studied in the context of point-to-point RFID systems in [8], although no analysis of the information-energy trade-off was provided. In contrast, in this work, a thorough analysis is provided of the interplay between information rate and energy transfer in terms of probabilities of battery overflow and underflow. The analysis reveals that, by properly choosing the parameters that define RLL codes depending on the receiver's utilization requirements, constrained codes allow to greatly improve the system performance in terms of simultaneous energy and information transfer.

II. System Model

We consider the point-to-point channel illustrated in Fig. 1. We assume that at each discrete time i, the transmitter can either send an "on" symbol $X_i = 1$, which costs one unit of energy, or an "off" signal $X_i = 0$, which does not require any energy expenditure. The receiver either obtains an energy-carrying signal, which is denoted as $Y_i = 1$, or receives no

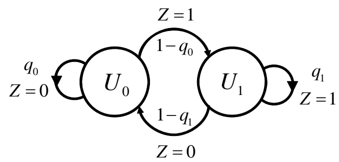

Fig. 2. Energy utilization model at the receiver.

useful energy, which is represented as $Y_i = 0$. The channel is memoryless, and has transition probabilities as shown in Fig. 1. Accordingly, p_{10} represents the probability that energy is lost when propagating between transmitter and receiver[1]. At the receiver side, upon reception of an energy-carrying signal $Y_i = 1$, the energy contained in the signal is harvested. The harvested energy is temporarily held in a supercapacitor and, if not used in the current time interval i, is stored in a battery, whose capacity limited to $B_{\max}$ energy units.

The receiver's energy utilization is modeled as a stochastic process $Z_i \in \{0, 1\}$, so that $Z_i = 1$ indicates that the receiver requires one unit of energy at time i, while $Z_i = 0$ implies that no energy is required by the receiver at time i. This process is not known at the transmitter and evolves according to the Markov chain shown in Fig. 2. The probability that $Z_i = 0$ when in state U_0 is referred to as q_0 and the probability that $Z_i = 1$ in state U_1 is denoted as q_1.

Due to the finite capacity of the battery, there may be battery overflows and underflows. An overflow event takes place when energy is received and stored in the supercapacitor (i.e., $Y_i = 1$), but is not used by the receiver (i.e., $Z_i = 0$) and the battery is full (i.e., $B_i = B_{\max}$), so that the energy unit is lost; instead, an underflow event occurs when energy is required by the receiver (i.e., $Z_i = 1$) but the supercapacitor and the battery are empty (i.e., $B_i = 0$ and $Y_i = 0$). In the rest of this section we define all the parts of the system in Fig. 1 in detail.

A. Transmitter

The transmitter aims at communicating a message M, uniformly distributed in the set $\left[1 : 2^{nR}\right]$, reliably to the decoder, while at the same time guaranteeing desired probabilities of battery overflow and underflow (see Sec. II-B). Note that n is the codeword length and R represents the information rate in bits per channel use, while the constraints on the probabilities of overflow and underflow represent the requirements on energy transfer.

The codewords $x^n(m)$, with $m \in \left[1 : 2^{nR}\right]$, of a type-$i$ RLL code satisfy run-length constraints on the number of consecutive symbols i, where $i = 0$ or $i = 1$[2]. To elaborate, let d and k be integers such that $0 \leq d \leq k$. We say that a finite length binary sequence $x^n(m)$ satisfies the type-0 (d, k)-RLL constraint if the following two conditions hold (see Fig. 3): *(i)* the runs of 0's have length at most k; and *(ii)* the runs of 0's between successive 1's have length at least d; note that the first and last runs of 0's are allowed to have lengths smaller than d.

[1]A more general model would allow also for a non-zero probability p_{01} of receiving energy when no energy is transmitted. This could be interpreted as the probability of harvesting energy from the environment (see [5]). We do not consider this extension in this work.

[2]Classical RLL codes as discussed in, e.g., [7] are type-0, but here we find it useful to extend the definition to include also type-1 RLL codes.

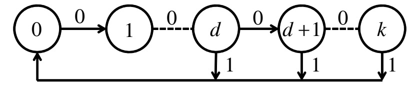

Fig. 3. The codewords of a type-0 (d, k)-RLL code must be outputs of the shown finite-state machine.

Therefore, a type-0 (d, k)-RLL code is such that the codewords include sufficiently long stretches of zero-energy symbols 0, via the selection of d, thus limiting battery overflow, but not too infrequently, via k, thus partly controlling also battery underflow. As a result, type-0 (d, k)-RLL codes are suitable for overflow-limited regimes in which controlling overflow events is most critical. A type-1 (d, k)-RLL code is defined in the same way, upon substitution of all "0" for "1" and vice versa in the definition above, and is hence well suited for underflow-limited regimes. We focus here on type-0 codes; a more general treatment can be found in [9].

B. Receiver

The received signal Y^n is used by the decoder both to decode the information message M encoded via the constrained code at the transmitter and to perform energy harvesting. Let B_i denote the number of energy units available in the battery at time i. At the ith time period, the decoder first receives signal Y_i, and stores its energy (if $Y_i = 1$) temporarily in a supercapacitor (see Fig. 1). Then, if $Z_i = 1$, the receiver attempts to draw one energy unit from the supercapacitor or, if the latter is empty, from the battery. If the energy in the supercapacitor is not used, it is stored in the battery in the next time slot. As a result, the amount of energy in the battery evolves as

$$B_{i+1} = \min\left(B_{max}, (B_i + Y_i - Z_i)^+\right), \qquad (1)$$

where $(a)^+ = \max(0, a)$.

When the receiver harvests a unit of energy, $Y_i = 1$, no energy is used, $Z_i = 0$, and the battery is full, $B_i = B_{max}$, we have an overflow event. To keep track of the overflow events, we define a random process O_i such that $O_i = 1$ if the event $\{B_i = B_{max}, Y_i = 1, \text{ and } Z_i = 0\}$ occurs and $O_i = 0$ otherwise. This can be expressed as

$$O_i = 1\{B_i = B_{max}, Y_i = 1 \text{ and } Z_i = 0\}. \qquad (2)$$

When the receiver wishes to use a unit of energy, $Z_i = 1$, and both the supercapacitor and the battery are empty, $Y_i = 0$ and $B_i = 0$, we have an underflow event. To describe underflow events, we introduce a random process U_i such that $U_i = 1$ if the event $\{B_i = 0, Y_i = 0 \text{ and } Z_i = 1\}$ takes place and $U_i = 0$ otherwise. This can be expressed as

$$U_i = 1\{B_i = 0, Y_i = 0 \text{ and } Z_i = 1\}. \qquad (3)$$

We define the probability of underflow as $\Pr\{\mathcal{U}\} = \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathrm{E}[U_i]$, and the probability of overflow as $\Pr\{\mathcal{O}\} = \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} \mathrm{E}[O_i]$. We note that the expectation is taken over the distribution of the message M, of the channel and of the receiver's energy utilization process Z^n.

C. Performance Criteria and Problem Formulation

The point-to-point link under study will be evaluated in terms of its performance for both information and energy transfer. A triple (R, P_{of}, P_{uf}) of information-energy requirements is said to be achievable by an encoder-decoder pair if the information transfer at rate R is reliable, i.e., if we have $\limsup_{n\to\infty} \Pr\left[\hat{M} \neq M\right] = 0$, where $\hat{M}$ is the decoded message, and if the energy transfer fulfill the constraints

$$\Pr\{\mathcal{O}\} \leq P_{of}, \quad \text{and} \quad \Pr\{\mathcal{U}\} \leq P_{uf}. \tag{4}$$

We are interested in investigating the set of achievable triples (R, P_{of}, P_{uf}) for different classes of codes, namely unconstrained and (d,k)-RLL constrained. To obtain further insight, in Sec. V, we will consider the problem

$$\begin{aligned} &\text{minimize} \quad \max(P_{of}, P_{uf}) \\ &\text{subject to} \quad (R, P_{of}, P_{uf}) \quad \text{is achievable,} \end{aligned} \tag{5}$$

where R is fixed and the minimization is done over all codes belonging to a certain class.

III. Unconstrained Codes

In this section, we study the information-energy transfer performance of classical unconstrained codes. To this end, we assume that the codewords $x^n(m)$, $m \in \left[1 : 2^{nR}\right]$, are generated independently as i.i.d. Bernoulli process with probability $\Pr[X = 1] = p_x$ and evaluate the corresponding performance on average over the code ensemble. The maximum information rate R achieved by this code is given as

$$R = \mathrm{I}(X;Y) \;=\; \mathrm{H}(p_y) - \frac{p_y}{1-p_{10}}\mathrm{H}(p_{10}), \tag{6}$$

where we have defined the probability $p_y \triangleq \Pr[Y_i = 1] = p_x(1-p_{10})$ and the binary entropy function $\mathrm{H}(a) \triangleq -a\log_2 a - (1-a)\log_2(1-a)$.

We now turn to the evolution of the performance in terms of energy transfer. In order to simplify the analysis and obtain some insight, we first assume the special case for then receiver's energy utilization model in which the process Z^n is i.i.d. and hence $q_1 = 1 - q_0 \triangleq q$. Note that q is the energy usage probability, in that we have $q = \Pr[Z_i = 1]$. The extension of the analysis to the Markov model in Fig. 2 follows along similar lines and is discussed in [9], while related numerical results are reported in Sec. V. If the process Z^n is i.i.d., the battery state evolves according to the birth-death Markov chain shown in Fig. 4. Using standard considerations and recalling (2) and (3), we can then calculate the probability of overflow and underflow respectively, as

$$\Pr\{\mathcal{O}\} \;=\; \pi_{B_{\max}} p_y (1-q) \triangleq \mathrm{O}(p_y), \tag{7}$$

$$\text{and } \Pr\{\mathcal{U}\} \;=\; \pi_0 (1-p_y) q \triangleq \mathrm{U}(p_y), \tag{8}$$

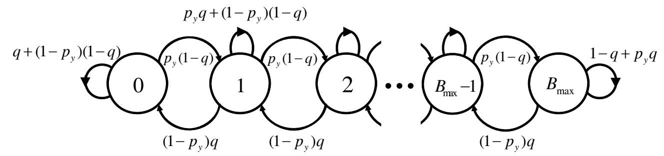

Fig. 4. The birth-death Markov process defining the battery state evolution along the channel uses with unconstrained (i.i.d.) random codes and i.i.d. receiver's energy usage process Z^n (i.e., $q = q_1 = 1 - q_0$).

where π_i is the steady-state probability of state $i \in [0, B_{\max}]$ for the Markov chain in Fig. 4. This can be easily calculated as

$$\pi_i \;=\; \frac{A^i}{1 + A + \ldots + A^{B_{\max}}}, \tag{9}$$

where $A = \frac{p_y(1-q)}{(1-p_y)q}$. The following lemma summarizes our conclusions so far.

Lemma 1. *Given an i.i.d. receiver energy usage process with energy usage probability q, the information-energy triple* (R, P_{of}, P_{uf}) *is achievable with unconstrained (i.i.d.) codes if there exist probability* $p_y \in [0, 1-p_{10}]$ *that satisfy*

$$R \;\leq\; H(p_y) - \frac{p_y}{1-p_{10}} H(p_{10}) \tag{10}$$

$$P_{of} \;\geq\; \mathrm{O}(p_y), P_{uf} \geq \mathrm{U}(p_y), \tag{11}$$

where $\mathrm{O}(p_y)$ *and* $\mathrm{U}(p_y)$ *are defined in (7) and (8), respectively.*

Remark 1. The region (10) is in general not convex, but it can be convexified if one allows for time sharing between codes with different values of p_y.

In order to get further insight into the performance of unconstrained codes, we now assume that the channel is noiseless, i.e., $p_{10} = 0$ and, as a result, we have $Y_i = X_i$ for all $i = 1, ..., n$ and $p_y = p_x$. Moreover, the solution of problem (5) is summarized in the following lemma.

Lemma 2. *The optimal solution* $p_x^\star$ *of problem (5) is given as*

$$q \qquad \text{if } R \leq \mathrm{H}(q) \tag{12a}$$

$$\mathrm{H}^{-1}(R) \qquad \text{if } R > \mathrm{H}(q) \text{ and } q \leq \frac{1}{2} \tag{12b}$$

$$1 - \mathrm{H}^{-1}(R) \qquad \text{if } R > \mathrm{H}(q) \text{ and } q > \frac{1}{2} \tag{12c}$$

where $\mathrm{H}^{-1}(R)$ *is the inverse of the entropy function in the interval* $[0, 1/2]$. *Moreover, the optimal value* $\max(\mathrm{O}(p_x^\star), \mathrm{U}(p_x^\star))$ *of the problem (5) is given by*

$$\frac{(1-q)\,q}{B_{\max}+1} \quad \text{if} \quad R \leq \mathrm{H}(q) \tag{13a}$$

$$\mathrm{O}\left(\mathrm{H}^{-1}(R)\right) \quad \text{if} \quad R > \mathrm{H}(q) \text{ and } q \leq \frac{1}{2} \tag{13b}$$

$$\mathrm{U}\left(1 - \mathrm{H}^{-1}(R)\right) \quad \text{if} \quad R > \mathrm{H}(q) \text{ and } q > \frac{1}{2}. \tag{13c}$$

Proof: Please see [9]. ■

Remark 2. Lemma 2 suggests that, when the rate is sufficiently small, i.e., when $R \leq \mathrm{H}(q)$, problem (5) is solved by "matching" the code structure to the receiver's energy utilization model. This is done, under the given i.i.d. assumption on codes and receiver's energy utilization, by setting $p_x^\star = q$. Instead, when the rate constraint is the limiting factor, one is forced to allow for a mismatch between code properties and receiver's energy utilization model (by setting $p_x^\star \neq q$). Specifically, Lemma 2 identifies two regimes, namely the *overflow-limited regime,* defined by the condition $q \leq 1/2$, and the *underflow-limited regime,* where we have $q > 1/2$. In the former regime, the rate constraint forces p_x to be larger than q, and we have $\max(\mathrm{O}(p_x), \mathrm{U}(p_x)) = \mathrm{O}(p_x)$. In contrast, in the underflow-limited regime, the rate constraint forces p_x to be smaller than q, and we have $\max(\mathrm{O}(p_x), \mathrm{U}(p_x)) = \mathrm{U}(p_x)$. These ideas will be useful when interpreting the gains achievable by constrained codes discussed in Sec. IV.

IV. Constrained Codes

In this section, we study the performance of (d,k)-RLL codes. To this end, as with unconstrained codes, we adopt a random coding approach. Specifically, we take the codewords to be generated independently according to a stationary Markov chain defined on the finite state machine in Fig. 3. It is known that this choice is optimal in terms of capacity (see, e.g., [7], [8]). A stationary Markov chain on the graph of Fig. 3 is defined by the transition probabilities where $\mathcal{P} = \{p_d, p_{d+1}, ..., p_{k-1}\}$ on its edges. We define as C_i the state of the constrained code at time i, prior to the transmission of X_i. Then, the transition probability $p_j = \Pr[C_i = j+1 | C_{i-1} = j]$, for $j = d, ..., k-1$ and $i > 1$. Barring degenerate choices for $\mathcal{P}$, it is easy to see that the Markov chain is irreducible, and hence one can calculate the unique steady-state distribution $\pi_j = \Pr[C_i = j]$ for $j \in [0,k]$ (see, e.g., [7]).

A. Information Rate

In [11, Lemma 5], it was proved that an achievable rate R with (d,k)-RLL codes is given as $R = \mathrm{I}(C_2; Y_2 | C_1)$. Evaluating this expression for type-0 (d,k)-RLL constrained codes leads to

$$\begin{aligned} R =& \mathrm{H}(Y_2|C_1) - \mathrm{H}(Y_2|C_1, C_2) \\ =& \sum_{j=d}^{k-1} \pi_j \left\{ \mathrm{H}((1-p_j)(1-p_{10})) - (1-p_j)\mathrm{H}(p_{10}) \right\}. \quad (14) \end{aligned}$$

B. Energy Transfer

To calculate the probabilities of battery underflow and overflow, namely $\Pr\{\mathcal{U}\}$ and $\Pr\{\mathcal{O}\}$, we focus at first on the special case in which the energy usage process Z^n is i.i.d. with energy usage probability q. We refer to [9] for the analysis of the extension to the Markov model in Fig. 2 , which follows along similar lines, and to Sec. V for the corresponding numerical results.

We use a renewal-reward argument (see, e.g., [10]). We recall that a renewal process is a random process of inter-renewal intervals $I_1, I_2, ...$ that are positive i.i.d. random variables. For our analysis, it is convenient to define the renewal event as $\{C_i = 0\}$, so that a renewal takes place every time the state of the constrained code C_i is equal to 0. This is equivalent to saying that, in the channel use before a renewal event, the transmitted signal X_i equals 1 for type-0 (d,k)-RLL codes. Based on the above, the renewal intervals I_i, for $i \geq 1$, are i.i.d. integer random variables with distribution $p_I(i)$ that can be calculated, given $\mathcal{P}$, as $p_I(i) = 0$ if $i \leq d$ and $i > k+1$; $p_I(i) = 1 - p_d$ if $i = d+1$; $p_I(i) = (1 - p_{i-1}) \prod_{l=d}^{i-2} p_l$ if $d+1 < i \leq k$; and $p_I(i) = \prod_{l=d}^{k-1} p_l$ if $i = k+1$. Moreover, it is useful to define a Markov chain $\tilde{B}_i$ that defines the evolution of the battery as evaluated at the renewal instants (i.e., for values of i for which $C_i = 0$). We refer to the steady-state probability of this Markov chain as $\tilde{\pi}_b$ with $b \in [0, B_{\max}]$. Finally, we define as $\tilde{O}_b$ the random variable that counts the number of overflow events in a renewal that starts with a battery with capacity $b \in [0, B_{\max}]$, and, similarly, we define as $\tilde{U}_b$ the random variable that counts the number of underflow events in a renewal that starts with a battery with capacity $b \in [0, B_{\max}]$.

The transition probabilities for the process $\tilde{B}_i$ are reported in [9], from which the steady state probabilities $\tilde{\pi}_b$ can be calculated (see, e.g., [10]). The next proposition summarizes the main result of the analysis. We use the definition $p(n; i, q) = \binom{i}{n} q^n (1-q)^{i-n}$ with $n = 0, ..., i$, for the probability distribution of a binomial random variable with parameters (i, q).

Proposition 1. *Given an i.i.d. receiver energy usage process with energy usage probability q, the information-energy triple (R, P_{of}, P_{uf}) is achievable with type-0 (d,k)-RLL codes if there exist transition probabilities $\mathcal{P} = \{p_d, p_{d+1}, ..., p_{k-1}\}$ that satisfy*

$$R \leq \sum_{j=d}^{k-1} \pi_j \{ \mathrm{H}((1-p_j)(1-p_{10})) - (1-p_j) H(p_{10}) \}, \quad (15)$$

$$P_{of} \geq \frac{\tilde{\pi}_{B_{\max}} \mathrm{E}\left[\tilde{O}_{B_{\max}}\right]}{\mathrm{E}[I]}, \quad (16)$$

$$\textit{and } P_{uf} \geq \frac{\sum_{b=0}^{B_{\max}} \tilde{\pi}_b \mathrm{E}\left[\tilde{U}_b\right]}{\mathrm{E}[I]}, \quad (17)$$

where we have defined $E[I] = \sum_{i=d+1}^{k+1} i \cdot p_I(i)$, along with

$$\begin{aligned} \mathrm{E}\left[\tilde{U}_b\right] =& \sum_{i=d+1}^{k+1} p_I(i) \left\{ (1-p_{10}) \sum_{l=1}^{i-b-1} p(l+b; i-1, q) \right. \\ & \left. + p_{10} \sum_{l=1}^{i-b} p(l+b; i, q) \right\}, \quad (18) \end{aligned}$$

$$\text{and } \mathrm{E}\left[\tilde{O}_{B_{\max}}\right] = \sum_{i=d+1}^{k+1} p_I(i)(1-p_{10}) p(0; i, q). \quad (19)$$

Proof: Please see [9]. ∎

Remark 3. The right-hand side of (16) evaluates the probability of overflow as the ratio of the average numbers of

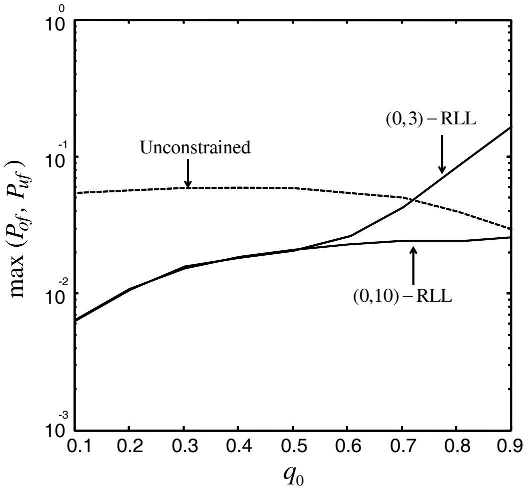

Fig. 5. Maximum between probability of underflow P_{uf} and overflow P_{of} as per problem (5) for unconstrained and type-0 constrained codes versus q_0 with $q_1 = 0$ (see Fig. 2) and $R = 0.1$. To simplify the numerical optimization, the curve for $k = 10$ has been obtained by optimizing only over p_0, p_1, p_2, p_3 and p_9 in $\mathcal{P} = \{p_0, p_1, ..., p_9\}$ and setting $p_3 = p_4 = p_5 = ... = p_8$.

overflow events in a renewal interval over the average length of a renewal interval. The right-hand side of (17) can be similarly interpreted. Note that, by the given definition of renewal events, in order to have an overflow, the initial battery state $\tilde{B}_i$ must be in state $B_{\max}$, whereas underflow events can potentially happen for all states $b \in \{0, ..., B_{\max}\}$. This is reflected by the numerators of (16) and (17).

V. Numerical Results

In this section, we compare the performance of unconstrained and constrained codes using problem (5) as the benchmark. Fig. 5 shows the optimal value of $\max(P_{of}, P_{uf})$ for a noiseless channel, i.e., $p_{10} = 0$ in Fig. 1, when $R = 0.1$ and $q_1 = 0$ versus q_0 (recall Fig. 2). With $q_1 = 0$, the energy usage process Z^n is such that a single energy request (i.e., $Z_i = 1$) is followed by an average of $1/(1 - q_0)$ instants where no energy is required (i.e., $Z_i = 0$). Therefore, as q_0 increases from 0.1 to 0.9, the average length of an interval with no energy usage increases from around 1 to 10. Similar to the discussion in Remark 2 for unconstrained codes, when neglecting the rate constraint, problem (5) is observed to be optimized by matching the code structure to the receiver's energy utilization model. When q_0 is sufficiently small, this can be easily accomplished with type-0 (d, k)-RLL codes with a small k. This is because k defines the maximum possible number of zero symbols X_i sent before a symbol $X_i = 1$. As q_0 increases, and hence the average length of the bursts of zeros grows in the process Z_i, the value of k must be correspondingly increased. This is confirmed by Fig. 5, which shows the significant gain achievable by the use of RLL codes when properly selecting the code parameters.

The impact of the information rate R is illustrated in Fig. 6 for $q_0 = q_1 = 0$ and $p_{10} = 0$. Following the discussion above, when the rate is small, with $q_0 = q_1 = 0$, it is sufficient to

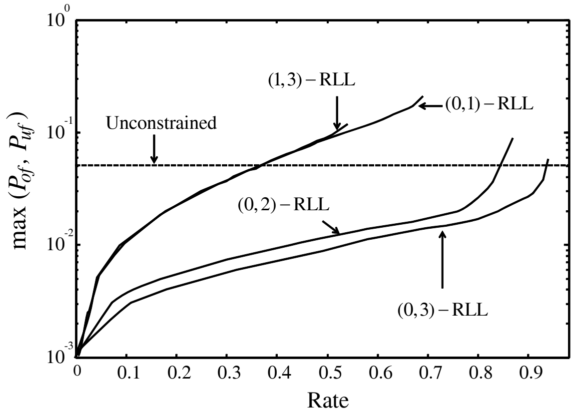

Fig. 6. Maximum between probability of underflow and overflow as per problem (5) for unconstrained and type-0 constrained codes versus the information rate R with $q_0 = q_1 = 0$ (see Fig. 2).

choose a type-0 or type-1 (d, k)-RLL code, since this code matches the statistics of the energy usage process. However, as the rate grows larger, one needs to increase the value of k, while keeping d as small as possible [7, Table 3.1].

VI. Conclusions

We have investigated the use of constrained run-length limited (RLL) codes with the aim of enhancing the achievable performance in terms of simultaneous information and energy transfer. The analysis has demonstrated that constrained codes enable the transmission strategy to be better adjusted to the receiver's energy utilization pattern as compared to classical unstructured codes. Interesting future work includes the investigation of non-binary codes and multi-terminal scenarios.

References

[1] L. R. Varshney, "Transporting information and energy simultaneously," in *Proc. IEEE Int. Symp. Inform. Theory* (ISIT 2008), pp. 1612-1616, Toronto, Canada, Jul. 2008.

[2] P. Grover and A. Sahai, "Shannon meets Tesla: Wireless information and power transfer," in *Proc. IEEE Int. Symp. Inform. Theory* (ISIT 2010), pp. 2363-2367, Austin, TX, Jun. 2010.

[3] R. Zhang and C. Keong Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," in *Proc. IEEE GLOBECOM 2011*, pp. 1-5, Houston, TX, Dec. 2011.

[4] K. Ishibashi, H. Ochiai, and V. Tarokh, "Energy harvesting cooperative communications," in *Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications*, pp. 1819–1823, Sydney, Australia, Sept. 2012.

[5] P. Popovski, A. M. Fouladgar, O. Simeone, "Interactive joint transfer of energy and information," *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 2086-2097, May 2013.

[6] A. M. Fouladgar, O. Simeone, "Information and energy flows in graphical networks with energy transfer and reuse," to appear in *IEEE Wireless Communications Letters,* 2013.

[7] B. Marcus, R. Roth, P. Siegel, *Introduction to Coding for Constrained Systems,* available at: http://www.math.ubc.ca/~marcus/Handbook/

[8] A. I. Barbero, E. Rosnes, G. Yang, and O. Ytrehus. "Constrained codes for passive RFID communication," in *Proc. IEEE Information Theory and Applications Workshop*, pp. 1-9, San Diego, CA, Feb. 2011.

[9] A. M. Fouladgar, O. Simeone, E. Erkip, "Constrained Codes for Joint Energy and Information Transfer," http://arxiv.org/abs/1311.1187, 2013.

[10] R. G. Gallager, *Discrete Stochastic Processes,* Kluwer, Norwell MA, 1996.

[11] E. Zehavi and J. K. Wolf, "On runlength codes," *IEEE Trans. Inform. Theory*, vol. 34, no. 1, pp. 45-54, Jan. 1988.