b. \( H(X) = -0.35 \log_2(0.35) - 0.3 \log_2(0.3) - 0.25 \log_2(0.25) - 0.1 \log_2(0.1) \)

\[ = 1.88 \]

\[ L(C) = E[l(X)] = 0.35 \cdot 1 + 0.3 \cdot 2 + 0.25 \cdot 3 + 0.1 \cdot 3 \]

\[ = 2 \]

\[ L(C) - H(X) = 0.12 \] redundancy due to the fact that the pmf is not dyadic.
c. \[ \text{Var}(l(X)) = E[l(X)^2] - [E[l(X)]]^2 \]

\[ E[l(X)^2] = 0.35 + 0.3 \cdot 4 + 0.25 \cdot 9 + 0.1 \cdot 9 \]

\[ = 4.7 \]

\[ \Rightarrow \text{Var}(l(X)) = 4.7 - 4 = 0.7 \]
\[
L(C) = E[l(X)] = 2
\]
\[
\text{var}(X) = E[\{l(X)^2\}] - E[l(X)]^2 = 0
\]
2. We need $H(X|Y, k) = 0$

The dotted area corresponds to $H(k)$

This area corresponds to $H(X|Y)$ since $H(X|Y, k) = 0$

From the figure,

$$H(k) = H(X|Y) + I(Y; k) + H(k|X, Y) \geq 0$$

concluding the proof.
3. \( c(n) = \underbrace{1 \cdots 1}_{n-1 \text{ ones}} 10 \Rightarrow l(n) = n \) (unary coding)

\[
\mathbb{E} [l(N)] = \mathbb{E} [N] = H(N).
\]
4. a. Neyman-Pearson test:

\[
\log \frac{p(x^n, y^n)}{p(x^n)p(y^n)} = \sum_{i=1}^{n} \log \frac{p(x_i, y_i)}{p(x_i)p(y_i)} > \delta \\
\rightarrow \text{threshold}
\]

b. \( \frac{1}{n} \log \frac{p(x^n, y^n)}{p(x^n)p(y^n)} \xrightarrow{n \to \infty} \mathbb{E}_{p(x|y)} \left[ \log \frac{p(x)}{p(x)p(y)} \right] = I(X;Y) \)

c. We have

\[
\Pr \left[ \left| \frac{1}{n} \log \frac{p(x^n, y^n)}{p(x^n)p(y^n)} - I(X;Y) \right| > \epsilon \right] \to 0 \text{ as } n \to \infty
\]

Therefore

\[
\Pr \left[ \frac{1}{n} \log \frac{p(x^n, y^n)}{p(x^n)p(y^n)} < I(X;Y) - \epsilon \right] \to 0 \text{ as } n \to \infty
\]

so choosing

\[ \gamma = n \left( I(X;Y) - \epsilon \right) \]

guarantees a vanishing probability of false alarm as \( n \to \infty \).
5. \[ X \xrightarrow{f} W \xrightarrow{k \text{ bits}} \]

\[ k \geq H(W) \geq H(X) \]