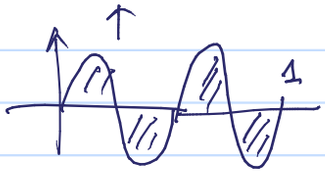
1. a. $E_0 = A^2$
$E_1 = A^2$ $\Rightarrow A = \sqrt{E_b}$

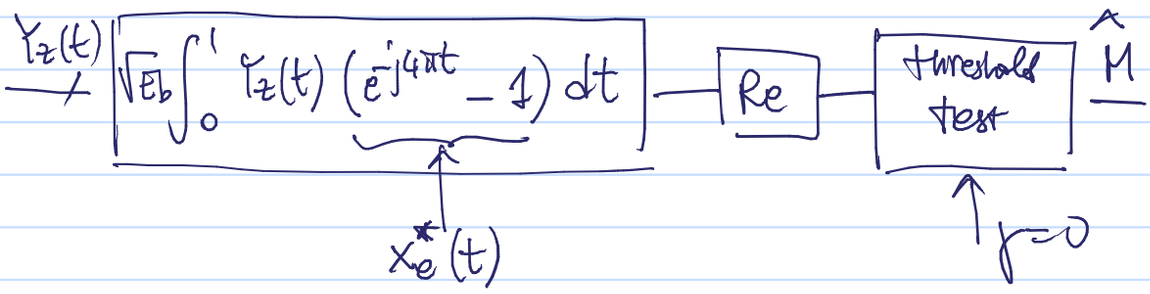
b. $\Delta_E(0,1) = E_0 + E_1 - 2\sqrt{E_0 E_1}\,\mathrm{Re}(\rho_{01}) = 2E_b$

$$\rho_{01} = \frac{\cancel{E_b}\int_0^1 e^{j4\pi t}\,dt}{\cancel{E_b}} = \int_0^1 \cos(4\pi t)\,dt + j\int_0^1 \sin(4\pi t)\,dt$$

$$= 0$$



c. $\gamma = \frac{E_1 - E_0}{2} = 0$



d. $P_B(E) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = Q(\sqrt{10})$

e. $m_0 = \mathrm{Re}\{x_{z,0}(t) * \underset{\delta(t)}{h(t)}\}|_{t=1} = \mathrm{Re}\{x_{z,0}(1)\} = \sqrt{E_b}$

$m_1 = \mathrm{Re}\{x_{z,1}(t) * h(t)\}|_{t=1} = \mathrm{Re}\{x_{z,1}(1)\} = \sqrt{E_b}\cos(4\pi)$
$= \sqrt{E_b}$

This is clearly not a good filter since $m_0 = m_1$ and hence $P_B(E) = 1/2$.

2. $$x_{z,0}(t) = \begin{cases} A(1+j) & \text{for } 0 \le t \le T_p/2 \\ 0 & \text{elsewhere} \end{cases} \qquad A \geq 0$$

$$x_{z,1}(t) = \begin{cases} A(-1-j) & \text{for } 0 \le t \le T_p/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_{z,2}(t) = \begin{cases} A(1+j) & \text{for } T_p/2 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

$$x_{z,2}(t) = \begin{cases} A(-1-j) & \text{for } T_p/2 \le t \le T_p \\ 0 & \text{elsewhere} \end{cases}$$

a. $$\cancel{2}A^2 \cdot \frac{T_p}{\cancel{2}} = 2E_b \Rightarrow A = \sqrt{2\frac{E_b}{T_p}}$$

b. Conditional distance spectre for all messages:

$$A^2 \int_0^{T_p/2} \left| 1+j + (1+j) \right|^2 dt = \cancel{8}^{4} A^2 \frac{T_p}{\cancel{2}} = 4A^2 T_p = 8E_b$$

$$E_0 + E_2 - 2\,\underbrace{\cancel{\mathrm{Re}(\rho_{02})}}_{0} \sqrt{E_0 E_2} = 4E_b$$

$$\{(8E_b, 1), (4E_b, 2)\}$$

c. Union bound

$$P_{WUB}(E) = \frac{1}{4}\left(\boxed{2}\, Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \boxed{2}\, Q\left(\sqrt{\frac{4E_b}{N_0}}\right)\right)$$

Union bound approximation

$$P_{WUB}(E) \simeq \frac{\boxed{2}}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

d. 4-PSK $\quad P_{WUB}(E) \simeq \frac{1}{4} \cdot 6 \cdot Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

$$\uparrow$$

$$A_E(\min) = 4E_b$$

Same union bound approximation

e. 4-PSK is preferable since the modulation scheme in this problem requires double the bandwidth