ECE 744: Final Fall 2014

Please provide clear and complete answers by detailing your derivations.

1. (**1 point**) Consider the optimization problem

$$\begin{array}{rl} \text{minimize} & 3e^{-x_1} + e^{-x_2} \\ \text{s.t.} & x_1^2 + x_2^2 = 2 \end{array}$$

Is it convex? If not, obtain an equivalent convex problem (i.e., a convex problem that has the same optimal value and solutions).

2. (**2 points**) We are given the problem

$$\begin{array}{rl} \text{minimize} & c^T x \\ \text{s.t.} & a^T x \leq b \end{array}$$

where $c \neq 0$ and $a \neq 0$.
a. Calculate both p^* and X_{opt} as a function of c and a, and substantiate your derivation by means of a sketch.
b. Check that the solution (when it exists) derived at the previous point satisfies the KKT conditions.

3. (**1 point**) Consider the problem of allocating the information rates x_i, $i = 1, ..., n$, to each one of n data users. The rates must satisfy some constraints that are defined by a convex set $\mathcal{C}$, i.e., $x \in \mathcal{C}$. The allocation is done by maximizing the function $\sum_{i=1}^{n} \ln(x_i)$. Write down the optimality conditions for this problem and interpret the result (Hint: This optimization is said to provide proportional fairness).

4. (**1 point**) Prove that the classification margin for a linear classifier $t(x) = w^T x + b$ at a (correctly classified) data point (x_0, y_0) with $x_0 \in \mathbb{R}^n$ and $y_0 \in \{-1, 1\}$ is given as $y_0 t(x_0)/||w||_2$.

5. (**3 point**) Consider the so called inverse waterfilling problem

$$\begin{array}{rl} \text{minimize} & \displaystyle\sum_{i=1}^{n} \log \frac{a_i}{x_i} \\ \text{s.t.} & 0 \leq x_i \leq a_i \\ & \displaystyle\sum_{i=1}^{n} x_i \leq D \end{array}$$

where $a_i > 0$ and $D > 0$ are parameters. (This problem finds application in the quantization of n Gaussian sources of variance a_i for $i = 1, ..., n$.)
a. What can be said about the existence of an optimal primal and dual solution?
b. Write the KKT conditions.

c. Find a (the?) solution.

6. (**3 point**) Consider the (simple!) problem

$$\text{minimize} \qquad |a^T x + b|$$

with $x \in \mathbb{R}^n$.
a. Write the epigraph form.
b. Calculate the dual function for the epigraph form.
c. Comment on strong duality and constraint qualifications for the epigraph form.