Abstract—Spectrum leasing via cooperation refers to the possibility for primary users to lease part of the spectral resources to secondary users in exchange for cooperation. This paper proposes a novel implementation of this concept in which secondary cooperation aims at improving the secrecy of the primary link. In particular, a secondary transmission with multiple antennas creates interference on both primary and eavesdropping receivers but an appropriately designed beamformer may impair more the eavesdropper’s reception and thus enhance primary secrecy. The optimal design of the secondary beamformer that maximizes the primary secrecy rate while guaranteeing a minimal secondary rate is studied. It is proved that the problem can be solved in a domain that includes only two real numbers irrespective of the number of antennas. Numerical results show that the proposed spectrum leasing strategy increases the primary secrecy rate compared to the case of no spectrum leasing for a wide range of secondary minimum rate constraints.

Index Terms—Physical security, spectrum leasing, Pareto boundary, interference channel.

I. INTRODUCTION

Cognitive radio networks make efficient use of the spectrum by allowing the coexistence of secondary devices in a bandwidth occupied by a primary network. Among the proposals for the implementation of cognitive radio, a spectrum leasing framework whereby the primary network leases part of the spectral resources to secondary users in exchange for cooperation has been proposed in [1]–[3]. Such previous work has considered the scenario where secondary nodes provide cooperation in the form of relaying of primary packets in return for the possibility to transmit own data in leased spectral resources. The primary system benefits in terms of achievable rates [1] or reliability [2], while the secondary nodes earn the possibility to access the spectrum.

This paper explores an alternative implementation of the concept of spectrum leasing via cooperation. The main idea is that secondary transmissions, if allowed by the primary network on the same spectral resource as the primary [4], create interference at the primary receiver, but nevertheless may have an overall positive effect on the primary system performance. In particular, we are interested in the secure communications of primary networks via cooperation of the secondary transmitter. To investigate this, we consider that primary system coexists with a multi-antenna secondary transmitter and an eavesdropper. If the secondary transmission creates more interference to the eavesdropper than to the primary receiver, then the primary secrecy rate is improved thanks to the secondary transmission, i.e., the secondary nodes obtain access to the spectrum and the primary system can achieve a larger secrecy rate.

When there exists an eavesdropper, the secrecy capacity between a transmitter and an intended receiver was studied in [5]–[7]. The secrecy rate, however, is equal to zero or is very low when the channel quality between the transmitter and the intended receiver is not good. To improve the secrecy rate, an information theoretic analysis of secrecy capacity in the presence of a helper node was studied in [8], [9]. In these works, the sole role of the helper node is that of increasing the main link’s secrecy rate.

In this paper, we study the design of the beamforming vector at the secondary transmitter with the aim of maximizing the primary rate while guaranteeing that the secondary system achieves a minimum required rate. This problem accounts for a spectrum leasing scenario in which, on the one hand, the primary user may be willing to allow the secondary transmit as long as its secrecy rate is increased, while, on the other hand, the secondary user assists the primary link under the constraint of a minimum achievable rate for itself. The main contributions of this work are: i) proposal of a spectrum leasing scheme via cooperation based on enhancement of the primary secrecy rate; ii) analysis of the optimal beamforming vector maximizing the primary secrecy rate while guaranteeing minimal secondary rates. As for point ii), we will show that the optimization of the beamforming vector can be carried out in a reduced space that includes only two real numbers irrespective of the number of antennas. Moreover, we demonstrate that the proposed spectrum leasing method improves the secrecy rate of the primary networks for a wide range of secondary minimum rate constraints.

Notation: Lower case and upper case boldface denotes vectors and matrices, respectively. $[.]^*$ denotes conjugate transpose. $\|a\|$ denote the Euclidean norm of $a$. $P_X$ and $P_X^\perp$ denote the projection matrix onto the column space of $X$ and orthogonal complement of the column space of $X$. 
respectively, i.e., $P_X = X (X^* X)^{-1} X^*$ and $P_X^+ = I - P_X$.

For given $e$ where $e_m \in \{+1, -1\}$, $a \geq b$ if $a_m e_m \geq b_m e_m$ for all $m$. $\mathbb{R}_+$ denotes the set of non-negative real numbers.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The system model under consideration in this paper is illustrated in Fig. 1. The system consists of a primary transmitter, a primary receiver, a passive eavesdropper, a secondary transmitter and a secondary receiver. The secondary transmitter has multiple antennas and all other nodes have a single antenna. We assume that the channel gains directly connected to nodes are available by exploiting the reciprocity of channels. For example, the secondary transmitter perfectly knows $h_{sp}, h_{se}, h_{ss}$ and the receivers know the relevant receiver-side channel state information. Moreover, the secondary transmitter uses a beamforming strategy, i.e., a scalar coding strategy that results in unit-rank input covariance matrix.

The intended receiver and the eavesdropper treat the interference from the helping interferer as an additive noise like [9]. Notice that restricting the operation at the eavesdropper to treating interference as noise is the worst case scenario, i.e., one may want to secure primary communications irrespective of the operations carried out at the eavesdropper such as performing interference cancellation (joint decoding) [8]. The power constraints are given by $P_p$ for the primary link and $P_s$ for the secondary link. Given the assumptions above, the following rate is achievable by the primary link with perfect secrecy

$$R_p (w) = \log \left( 1 + \frac{|h_{pp}|^2 P_p}{\sigma_p^2 + |w^* h_{sp}|^2} \right) - \log \left( 1 + \frac{|h_{pe}|^2 P_p}{\sigma_e^2 + |w^* h_{se}|^2} \right),$$

where $[A]^+ = \max(A, 0)$, $w$ denotes the beamforming vector at the secondary transmitter, $h_{ij}$ or $h_{ij}^*$ are the channel coefficient or $N \times 1$ channel vector between nodes, and $\sigma_p^2$ and $\sigma_e^2$ are the noise variance at the primary receiver and the eavesdropper, respectively. Moreover, the rate achievable by the secondary link is given by

$$R_s (w) = \log \left( 1 + \frac{|w^* h_{ss}|^2}{\sigma_e^2 + |h_{ps}|^2 P_p} \right),$$

where $\sigma_e^2$ is the noise variances at the secondary receiver. Note that the power constraint should be satisfied in the design of $w$ such as

$$\|w\|^2 \leq P_s.$$  

The problem of spectrum leasing via enhanced physical-layer secrecy can be formulated as the maximization of the primary secrecy rate subject to the power constraint and the secondary quality of service (QoS) constraint. The latter is given by imposing that the secondary rate is larger than a given threshold. This corresponds to the following formulation:

$$\max_w \quad R_p (w) \quad \text{s.t.} \quad \|w\|^2 \leq P_s, \quad R_s (w) \geq R_{\text{min}},$$

where $R_{\text{min}}$ is the minimum rate accepted by the secondary user in exchange for cooperation with the primary. To ensure feasibility of (4), one can set

$$R_{\text{min}} = \alpha R_{s,\text{max}},$$

where $\alpha \in [0, 1]$ and $R_{s,\text{max}} = R_s (\sqrt{\frac{P_s}{|h_{ss}|^2}})$.

III. REFERENCE RESULTS

In this section, we present some reference results that will be useful for comparison with the performance of spectrum leasing. In particular, we first consider the case where the secondary user is not present and then the case where the secondary user’s only goal is that of optimizing the primary secrecy rate. The first clearly provides a lower bound on the secrecy rate achievable via spectrum leasing, while the second provides an upper bound since it removes the secondary QoS constraint.

A. No Spectrum Leasing

When spectrum leasing is not feasible, or equivalently when $P_s = 0$ in (4), the secrecy rate of the primary user is given by [7]

$$R_p = \log \left( 1 + \frac{|h_{pp}|^2 P_p}{\sigma_p^2} \right) - \log \left( 1 + \frac{|h_{pe}|^2 P_p}{\sigma_e^2} \right).$$

This rate clearly sets a lower bound on the primary secrecy rate achievable, since the presence of the secondary with multiple antennas can always increase the primary secrecy rate [9].
B. Channel with a Helping Interferer

When the secondary transmitter does not impose any QoS constraints, i.e., \( \alpha = 0 \) in (5), we can consider the secondary user merely as a helper for the primary link. This scenario was studied in [9] and it clearly provides an upper bound to the achievable primary secrecy rates from (4) for any \( \alpha \). In [9], based on the results in [11], it was shown that the optimal solution of (4) when \( \alpha = 0 \) has the form

\[
\mathbf{w}_{\text{opt}, \text{HI}} = \sqrt{P_s} \frac{\lambda \mathbf{w}^Z + (1 - \lambda) \mathbf{w}^\text{MRT}}{\| \lambda \mathbf{w}^Z + (1 - \lambda) \mathbf{w}^\text{MRT} \|},
\]

where \( 0 \leq \lambda \leq 1 \), \( \mathbf{w}^Z = \frac{P_{\mathbf{h}_{sp}}}{\| \mathbf{h}_{sp} \|} \mathbf{h}_{sc} \), and \( \mathbf{w}^\text{MRT} = \frac{P_{\mathbf{h}_{sp}}}{\| \mathbf{h}_{sp} \|} \mathbf{h}_{sc} \). In other words, the optimal beamformer \( \mathbf{w} \) at the helper can be obtained as a real linear combination of two vectors, namely \( \mathbf{w}^Z \) and \( \mathbf{w}^\text{MRT} \). \( \mathbf{w}^Z \) is the beamforming strategy designed to create no interference at the primary receiver. Instead, beamformer \( \mathbf{w}^\text{MRT} \) generates as much interference to the eavesdropper as the secondary transmitter can.

It is worth noting that (7) entails the solution to (4)-(5) when \( \alpha = 0 \) that can be obtained by simply optimizing a single real number rather than an \( N \times 1 \) complex vector. It was also proved in [11] that (7) can also be written as a real linear combination of \( \mathbf{h}_{sp} \) and \( \mathbf{h}_{sc} \) as follows:

\[
\mathbf{w}_{\text{opt}, \text{HI}} = \beta_1 \frac{P_{\mathbf{h}_{sp}}}{\| \mathbf{h}_{sp} \|} \mathbf{h}_{sc} + \beta_2 \frac{P_{\mathbf{h}_{sp}}}{\| \mathbf{h}_{sp} \|} \mathbf{h}_{sc},
\]

where \( \beta_1, \beta_2 \in \mathbb{R} \) and \( \beta_1^2 + \beta_2^2 = P_s \).

IV. Spectrum Leasing for Enhanced Secrecy

In this section, we discuss the optimal solution of problem (4). A solution parameterized by three complex numbers is first introduced. Then, we provide that a simpler parameterization that depends on only two real numbers by using the concept of channel gain region [12].

A. Complex Parametrization of the Optimal Solution of (4)

The proposition below shows that the optimal beamforming vector according to (4)-(5) can be found by optimizing over three complex numbers rather than an \( N \times 1 \) complex vector \( \mathbf{w} \). This result is based on the complex parameterization for the beamforming vectors that correspond the Pareto boundary in general interference channels [11].

**Proposition 1:** The beamforming vector that solves (4), \( \mathbf{w}_{\text{opt}} \), is a complex linear combination of \( \mathbf{h}_{sp}, \mathbf{h}_{sc} \) and \( \mathbf{h}_{as} \). That is, the solution exists on the vector space spanned by \( \{ \mathbf{h}_{sp}, \mathbf{h}_{sc}, \mathbf{h}_{as} \} \).

**Proof:** Since the proof is similar with the that of Proposition 1 in [11], we show the sketch of the proof only. The proof goes by contradiction. Suppose that the solution of (4) is given by \( \mathbf{w}_{\text{opt}} \). If we assume that \( \mathbf{w}_{\text{opt}} \) does not belong to the vector space spanned by \( \{ \mathbf{h}_{sp}, \mathbf{h}_{sc}, \mathbf{h}_{as} \} \), then we can find a new beamforming vector \( \mathbf{w}_{\text{new}} \) that lies on the space and has the following properties: i) same power with respect to \( \mathbf{w}_{\text{opt}} \), i.e., \( \| \mathbf{w}_{\text{opt}} \|^2 = \| \mathbf{w}_{\text{new}} \|^2 \); ii) primary and secondary rates are the same or higher. Firstly, suppose that the beamforming \( \mathbf{w}_{\text{new}} \) is obtained by projecting the vector \( \mathbf{w}_{\text{opt}} \) into the subspace generated by vectors \( \{ \mathbf{h}_{sp}, \mathbf{h}_{sc}, \mathbf{h}_{as} \} \). Since the projection of the vector results in the decreasing of the magnitude, \( \mathbf{w}_{\text{new}} \) can add another terms that contribute to the two rates of interest while keeping the power constraint, i.e., same power as \( \mathbf{w}_{\text{opt}} \). Finally, we can find an another beamforming vector, \( \mathbf{w}_{\text{new}} \), that increases the rates of interest, thus, it contradicts the optimality of the \( \mathbf{w}_{\text{opt}} \) and the solution of (4) must exist on the vector space at hand.

B. Real Parametrization of the Optimal Solution of (4)

As discussed in Section III-B, the solution of (4)-(5) with \( \alpha = 0 \) can be found via a parameterization with a single real parameter. Here we show that a similar result holds true for any \( \alpha \). In particular, we show that optimization (4) can be performed without loss of generality on two real numbers.

**Proposition 2:** The beamforming vector that solves (4) has the form

\[
\mathbf{w}_{\text{opt}} = \sqrt{P_s} \mathbf{v}_{\text{max}} \{ Z \}, \quad Z = \lambda_1 \mathbf{h}_{sc}^* \mathbf{x}_1 + \lambda_2 \mathbf{h}_{as} \mathbf{x}_2 - \lambda_3 \mathbf{h}_{sp} \mathbf{x}_3^*, \quad \lambda_k \in [0, 1], \quad \sum_{k=1}^3 \lambda_k = 1,
\]

where \( \mathbf{v}_{\text{max}} \{ Z \} \) denotes the principal eigenvector of \( Z \).

**Proof:** The proof of Proposition 2 requires to introduce the concept of power gain region and will be discussed below.\(^2\) It is worth noting that (2) includes three real numbers, but they can be obtained with the combinations of two real numbers since the sum of three numbers should be one.

C. Power Gain Region and Proof of Proposition 2

In order to prove Proposition 2, we review the concept and main results related to power gain region as introduced in [12]. Assume that there are a single transmitter and \( K \) receivers, the transmitter has \( N \) antennas, and each receiver has a single antenna. For beamforming transmission strategies, the achieved power gain at the \( k \)-th receiver is defined as

\[
x_k(\mathbf{w}) = |\mathbf{w}^* \mathbf{h}_k|^2,
\]

where \( \mathbf{w} \) is a beamforming vector at the transmitter and \( \mathbf{h}_k \) is a channel vector between the transmitter and \( k \)-th receiver, \( k \in \mathcal{K}, \mathcal{K} = \{1, 2, \ldots, K\} \). Then, the power gain region with a transmit power constraint, i.e., \( \| \mathbf{w} \|^2 \leq P \), is defined as the set of all achievable power gains as follows:

\[
\Omega = \left\{ (x_1(\mathbf{w}), x_2(\mathbf{w}), \ldots, x_K(\mathbf{w})) | \| \mathbf{w} \|^2 \leq P \right\}.
\]

At this point, we introduce a new and important definition, i.e., the outer boundary of the power gain region in direction \( \mathbf{e}, \mathbf{e} \in \{-1, +1\}^N \). Given \( \mathbf{e} \), we can define the outer boundary of \( \Omega \) in direction \( \mathbf{e} \) as follows:

\[
\mathcal{B}^\mathbf{e} \Omega = \left\{ x' | x' \geq \mathbf{e} \mathbf{x}, \mathbf{x} \in \Omega, \mathbf{x} \in \mathbb{R}^N \right\},
\]

\(^2\)We assume that the secondary transmitter has more than three antennas. When the number of antennas is less than three, the solution introduced in this work should be changed [12]. But for simplicity, we restrict our interest to the case of more than three antennas.
where \( x = [x_1(w), x_2(w), \ldots, x_K(w)]^T \). In other words, \( \Omega \) has total \( 2^K \) number of outer boundaries each corresponds to every \( e \) and each outer boundary consists of maximum values that are equal to or bigger than any values in \( \Omega \) in direction \( e \). For instance, when \( K = 2 \), the power gain region can be drawn as Fig. 2 for different vectors \( e \). In particular, if we set \( e = [1+1] \), we get \( B^e\Omega \) that is shown in right-upper part in Fig. 2. This corresponds to the case of conventional rate region. In contrast, when \( e = [-1 + 1] \), \( B^e\Omega \) is drawn as the right-lower part in Fig. 2 and similarly for \( e = [-1 + 1] \). Regarding the beamforming vectors that correspond to the outer boundary points of the power gain region, we have the following result.

**Lemma 1**: A point \( x(w) \) is on \( B^e\Omega \), i.e., \( x(w) \) belongs to \( B^e\Omega \), if and only if \( w \) is given by

\[
  w_{B^e\Omega} = \sqrt{P_{\text{max}}} \left( \sum_{k=1}^{K} \lambda_k e_k h_k^* h_k^* \right),
\]

for some \( \lambda_k \in [0, 1] \), \( \sum_{k=1}^{K} \lambda_k = 1 \) [12].

The detail proof of Lemma 1 is shown in [12]. We just bring the results in this work. Regarding the solution of (4), if we prove that the optimal solution of (4) lies on the outer boundary of the power gain region in some direction, then Lemma 1 can be directly used to find the optimal solution of (4). Therefore, we will show that the solution of (4) exists on the outer boundary of the power gain region such as

\[
\Omega = \left\{ (w^* h_{sp}^2, w^* h_{se}^2, w^* h_{ss}^2) \mid \| w \|^2 \leq P_p \right\}
\]

in direction \( e = [-1 + 1] \).

**Proof of Proposition 1**: The proof is by contradiction. Assume that the optimal beamforming vector \( w_{\text{opt}} \) is not on the boundary points of the power gain region. Then, by definition of the boundary region (12), we can find another beamforming vector \( w_{\text{new}} \) that satisfies one of following three cases and lies on the boundary region in direction \( e = [-1 + 1] \).

1. \( |w_{\text{opt}}^* h_{sp}^2| > |w_{\text{new}}^* h_{sp}^2|, |w_{\text{opt}}^* h_{se}^2| = |w_{\text{new}}^* h_{se}^2|, |w_{\text{opt}}^* h_{ss}^2| = |w_{\text{new}}^* h_{ss}^2| \)
2. \( |w_{\text{opt}}^* h_{sp}^2| < |w_{\text{new}}^* h_{sp}^2|, |w_{\text{opt}}^* h_{se}^2| = |w_{\text{new}}^* h_{se}^2|, |w_{\text{opt}}^* h_{ss}^2| = |w_{\text{new}}^* h_{ss}^2| \)
3. \( |w_{\text{opt}}^* h_{sp}^2| = |w_{\text{new}}^* h_{sp}^2|, |w_{\text{opt}}^* h_{se}^2| = |w_{\text{new}}^* h_{se}^2|, |w_{\text{opt}}^* h_{ss}^2| < |w_{\text{new}}^* h_{ss}^2| \)

For the cases 1) and 2), \( R_p(w_{\text{new}}) \) is higher than \( R_p(w_{\text{opt}}) \) and \( R_s(w_{\text{new}}) \) is equal to \( R_s(w_{\text{opt}}) \). Thus, the existence of such \( w_{\text{new}} \) contradicts the optimality of \( w_{\text{opt}} \) and the optimal solution of (4) must lie on the outer boundary of the power gain region. For the case 3), \( R_p(w_{\text{opt}}) \) is equal to \( R_p(w_{\text{new}}) \) and \( R_s(w_{\text{new}}) \) is higher than \( R_s(w_{\text{opt}}) \). Moreover, if \( w_{\text{opt}} \) satisfies all of constraints in (4), \( w_{\text{new}} \) also satisfies all of constraints, thus, \( w_{\text{new}} \) that lies on the boundary is a solution for this case. Only for case 3), there might exist other beamforming vectors that are not on the boundary but satisfies all constraint. The proof is concluded by using Lemma 1.

V. **Simulation results**

Fig. 3 compares the achievable secrecy rates i) with spectrum leasing obtained by Proposition 2, ii) with a helping interferer, iii) without spectrum leasing and iv) the upper bound as a function of \( P_p^s/\sigma_p^2 = P_s^p/\sigma_s^2 \), which is defined as SNR hereinafter. We use 1/4 and 1/2 for the minimum rate constraint, \( \alpha \), in (5). First, we note that the secrecy rate without spectrum leasing reaches a performance floor as SNR increases. This is clear from (6) by taking \( \text{SNR} \to \infty \). This is not the case in the presence of a helping interferer since in this case the interferer can focus its beam solely on the eavesdropper as explained in Section III-B. With spectrum leasing, one can reap most of the benefits of a helping interferer while still serving the needs of the secondary user. Specifically, we observe that even when the rate of secondary is guaranteed to more than half of maximum achievable rate, the secrecy rate with spectrum leasing is comparable, especially at the low SNR regime. Obviously, the secrecy rate is decreased as \( \alpha \) increases.

In Figs. 4 and 5, we illustrate the effect of the minimum secondary rate constraint on the achievable secrecy rate with spectrum leasing. These are obtained by two specific channel realizations as examples. Fig. 4 corresponds to a case where the channel from the secondary transmitter, to the secondary receiver and to the primary receiver are fairly aligned, while for Fig. 5 the channels are quite orthogonal. When the channel vector to the secondary receiver is highly correlated with the channel vector to the primary receiver, increasing \( \alpha \) results in more interference at the primary receiver. Thus, the secrecy rate is also decreased rapidly as the secondary rate requirement \( \alpha \) is increased, as seen from Fig. 4. Fig. 5 shows that this effect does not occur when the channels to the primary receiver and to the secondary receiver are almost orthogonal. Fig. 6 instead illustrates the achievable secrecy rate for increasing \( \alpha \) as averaged over fading channels. It is observed that the average performance lies between two extreme examples in Figs. 4 and 5.\(^3\)

\(^{3}\)In some cases such as the case of Fig. 4, the secondary user gets benefit from the cooperation; while the primary user’s secrecy will be degraded. The interaction between the primary user and the secondary user can be considered to solve this problem, but for simplicity we restrict our interest to no interaction scenario.
Fig. 3. Average achievable secrecy rate with and without spectrum leasing versus SNR ($N = 4$).

Fig. 4. Achievable secrecy rates for with spectrum leasing on the secrecy rate ($N = 4$, SNR = 10 dB, high correlation between $h_{sp}$ and $h_{ss}$).

Fig. 5. Achievable secrecy rates for with spectrum leasing on the secrecy rate ($N = 4$, SNR = 10 dB, low correlation between $h_{sp}$ and $h_{ss}$).

Fig. 6. Average achievable secrecy rate as a function of the secondary rate requirement level $\alpha$ ($N = 4$, SNR = 10 dB).

VI. CONCLUSION

We proposed an interference management technique to increase the secrecy rate of the primary networks by using spectrum leasing. Based on the framework of the power gain region, the beamforming vector that maximizes the primary secrecy rate while maintaining the minimum rate constraint for the secondary link was studied. In future work, we will consider the effect of limited knowledge of the channel information to the eavesdropper for a practical implementation and multiple antennas at receivers [13].

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