Throughput of Cellular Uplink with Dynamic User Activity and Cooperative Base-Stations

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Abstract—The throughput of a linear cellular uplink with a random number of users, different power control schemes, and cooperative base stations is considered in the large system limit where the number of cells is large for non-fading Gaussian channels. The analysis is facilitated by establishing an analogy between the cellular channel per-cell throughput with joint multi-cell processing (MCP), and the rate of a deterministic inter-symbol interference (ISI) channel with flat fading. It is shown that, under certain conditions, the dynamics of cellular systems (i.e., a random number of users coupled with a given power control scheme) can be interpreted, as far as the uplink throughput is concerned, as the flat fading process of the equivalent ISI channel. The results are used to demonstrate the benefits of MCP over the conventional single cell processing approach as a function of various system parameters in the presence of random user activity.

I. INTRODUCTION

Wireless communications systems in general and cellular systems in particular are of major interest as they allow the provision of continuous services to mobile users. In recent years a considerable research effort has been devoted to the development of new technologies for providing better services and extending system coverage. In this context, the use of joint multi-cell processing (MCP) has been identified as a key tool for enhancing system performance. Since its introduction by Wyner in [1], many aspects of MCP has been studied (see [2] and references therein for a survey of recent results on MCP). Here we are interested in studying the impact of users with dynamic activity (i.e., each user is active with a certain probability in each time slot) on the performance of cellular uplink with MCP. Early attempts to deal with random number of users in cellular systems focused on single cell processing (SCP) (e.g., [3]), and were based on the notion of a random multiple-access channel (MAC) [4]. In a recent work [5], the per-cell throughput of a simple infinite linear cellular uplink with a single dynamic user per cell is analyzed. The analysis relies on the special topology of the model in which interference stems from a single neighboring cell only. In a parallel work [6], the authors use similar tools to consider also the resulting rate statistics to derive outage performance for the same cellular uplink.

In this work, we extend the results of [5], derived for a single user per cell, and study the case of more than one dynamic user per cell in the large-system limit. In particular, we calculate the per-cell throughput supported by a simple linear infinite non-fading cellular uplink model, in which the number of users in each cell is a binomially distributed random variable (r.v.), and all the BSs jointly decode their received signals to recover the users’ messages. To facilitate analytical treatment we use a linear variant of the Wyner cellular model family where each user “sees” only a finite (but arbitrary) number of BSs [1]. The main analytical tool used here is a recent result by Tulino et al. that provides expressions for the achievable rates of a linear time invariant (LTI) inter-symbol interference (ISI) channel with flat fading applied to its output symbols [7]. By establishing an analogy between the per-cell throughput of the cellular uplink and the rate of an ISI channels (similarly to Wyner [1]), we show that results of [7] can be applied to the cellular setup to address the dynamic setting at hand. In particular, the path gain between a user and the BSs are interpreted as the ISI channel coefficients and the cellular power control scheme determines the fading statistics of the equivalent ISI channel. We use the results to demonstrate the benefits of MCP over SCP for a cellular system with dynamic user activity. In a related work [8], achievable rates for an output-erasure ISI channel were derived and used to calculate the per-cell throughput of a cellular uplink with MCP and base-stations subjected to backhaul failures.

II. SYSTEM MODEL

We consider a linear cellular Gaussian (no fading) uplink, where M identical cells are arranged along a line [1]. Each cell includes a BS and K identical mobile terminals (MTs). While ignoring boundary effects, it is assumed that each MT’s transmissions are received by \( L_1 + L_2 + 1 \) BSs only: its local
BS, the $L_1$ adjacent BSs on its left, and the $L_2$ adjacent BSs on its right (See Fig. 1 for the special case of $L_1 = 0$, $L_2 = 1$, and $K = 2$). In particular, the signals of the $m_{th}$ BS are received at the $m_{2^{nd}}$, BS with signal path gain $\alpha_{m_1-m_2} \in \mathbb{C}$. We further assume perfect symbol and block synchronization, and that the total cell transmission power is subjected to an average power constraint $P$. As mentioned above we consider a dynamic model in which during each slot (or transmission block) MTs are randomly and independently active with probability $(1-q)$. The activity r.v.'s are assumed to be identically distributed (i.i.d.) among the users but are non-ergodic along the time index for each user. Finally, in order to satisfy a per-cell power constraint any power allocation must satisfy

$$\sum_{k=1}^{K} p_{m,k} \leq P, \quad \forall m.$$  

(2)

III. PRELIMINARY

The main analytical tool we use in this work is reported in a recent work [7], in which Tulino et al. study the capacity of a deterministic inter-symbol interference (ISI) flat fading channel (depicted in Fig. 2). The channel includes a unit power stationary Gaussian input $x_i$ with power spectral density (PSD) $S_c(f)$, which enters a linear time invariant filter $H(f)$. The output of the latter is then multiplied by a flat fading i.i.d. process $\sqrt{A_i}$ where $\gamma$ is a non-negative constant, and corrupted by zero mean unit power white Gaussian noise $z_i$. Assuming that only the decoder is aware of the filter coefficient, the fading process, the constant $\gamma$ and the statistics of the input and noise signals, the ergodic input-output mutual information is proved in [7] to be given by

$$I(\gamma) = \int_0^1 \log_2 \left(1 + \gamma \beta S(f)\right) df +$$

$$+ \mathbb{E} \left( \log_2 \left(1 + \gamma \nu |A|^2\right) \right) - \log_2 \left(1 + \gamma \beta \nu\right),$$  

(3)

where $S(f) = S_c(f) |H(f)|^2$ is the filter output PSD, and $\beta$ and $\nu$ are the unique positive solutions to

$$\mathbb{E} \left( \frac{1}{1 + \gamma \nu |A|^2} \right) = \frac{1}{1 + \gamma \beta \nu} = \int_0^1 \frac{1}{1 + \gamma \beta S(f)} df.$$  

(4)

\footnotetext{1}{It is noted that this result is actually taken from the presentation slides and is more compact than that reported in the conference proceedings [7].}

In the special case in which $A \in \{0,1\} \sim \mathcal{B}(1-q)$ (an output erasure channel [5][8]), the mutual information (3) reduces to

$$I(\gamma) = \int_0^1 \log_2 \left(1 + \gamma \beta S(f)\right) df + d(\bar{q}|1-\beta),$$

(5)

where $d(\bar{q}|1-\beta)$ is the relative entropy (in bits) between $\mathcal{B}(1-q)$ and $B(\beta)$, and $0 \leq \beta \leq 1 - \bar{q}$ is the unique solution to

$$\frac{\bar{q}}{1-\beta} = \int_0^1 \frac{1}{1 + \gamma \beta S(f)} df.$$  

(6)

IV. MULTICELL PROCESSING

With MCP, the BSs send their received signals to a central processor (CP) via an ideal backhaul network. The CP collects the received signals and jointly decodes the MTs’ messages. We also assume that the CP is aware of the activity pattern of all the MTs. Observing (1), the received signal vector at the BSs for an arbitrary symbol of an arbitrary block is given by

$$Y = HXE + Z,$$  

(7)

where $X$ is the MTs’ $MK \times 1$ transmit vector $X \sim \mathcal{C}\mathcal{N}(0,Q)$ and $Q = \text{diag}\{p_{1,1}, \ldots, p_{M,K}\}$. $E = \text{diag}\{e_{1,1}, \ldots, e_{M,K}\}$ is the diagonal $MK \times MK$ activity matrix, $H$ is the $M \times MK$, $L_1 + L_2 + 1$ block diagonal channel transfer matrix $[H]_{m,(n-1)K+1} = \alpha_{n-m}$, $k = 1,2, \ldots, K$, and $Z$ is the $M \times 1$ noise vector $Z \sim \mathcal{C}\mathcal{N}(0,I)$.

With optimal MCP, the throughput per cell is a r.v., given in the large system limit by

$$R_{\text{mcp}} = \lim_{M \to \infty} \frac{1}{M} \log_2 \det \left( I + HEQE^\dagger H^\dagger \right).$$  

(8)

A close inspection of the covariance matrix $G = I + HEQE^\dagger H^\dagger$ reveals the following.

**Proposition 1** With MCP the $MK$ user MAC of (7) is equivalent in terms of the throughput per cell to the following $M$ user MAC

$$\tilde{Y} = \tilde{H}\tilde{E}\tilde{X} + Z,$$  

(9)

where $\tilde{X}$ is an $M \times 1$ vector $\tilde{X} \sim \mathcal{C}\mathcal{N}(0,I)$, $\tilde{E}$ is a diagonal $M \times M$ matrix $[E]_{m,m} = \sqrt{\sum_{k=1}^{K} e_{m,k} p_{m,k}}$, and $\tilde{H}$ is an $M \times M$ matrix $[H]_{i,j} = \alpha_{i-j}$.

**Proof:** To prove this claim it is enough to show that the covariance matrices of the two Gaussian vectors conditioned on the activity r.v.’s (expressions (7) and (9)) are equal. This is easily achieved by a straightforward matrix multiplication,
i.e., by showing that $\mathbf{HEQE}^\dagger \mathbf{H}^\dagger = \mathbf{H} \tilde{\mathbf{E}} \mathbf{E}^\dagger \mathbf{H}^\dagger$. (See also [1, Lemma 3.1] for a similar claim.)

It is concluded that the system is equivalent in terms of throughput to a system with a single user per-cell. Moreover, the transmissions of the $m$th cell virtual user are multiplied by the r.v. $\tilde{\mathbf{E}}_{m,m} = \tilde{\mathbf{e}}_{m,m} \triangleq \sqrt{\sum_{k=1}^{K} \mathbb{E}[\tilde{e}_{m,k}] p_{m,k}}$ which is a function of the activity pattern and the power control scheme of the original system.

Now, the ground is set for presenting our main observation.

**Proposition 2** In the large system limit ($M \to \infty$) the per-cell throughput of the $MK$ user MAC of (7), is equal to the capacity of the deterministic ISI channel with flat fading of (3), with $S_e(f) = 1$, $\gamma = 1$, filter coefficients $\{\alpha_l\}_{l=-L_1}^{L_2}$ (i.e., $H(f) = \sum_{l=-L_1}^{L_2} \alpha_l e^{-j2\pi fn}$), and flat fading $A$ distributed as the i.i.d. diagonal elements of $\tilde{\mathbf{E}}$.

**Proof:** (outline) Following similar argumentation as those made in [9] we claim that in the large system limit the per-cell throughput $R_{mcp}$ (expression (8)) converges almost surely (a.s.) to its expected value

$$R_{mcp} = \lim_{M \to \infty} \mathbb{E} \left( \frac{1}{M} \log_2 \det \left( \mathbf{I} + \tilde{\mathbf{E}} \mathbf{E}^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{E}} \right) \right),$$

(10)

where the expectation is taken over the i.i.d. diagonal entries of $\tilde{\mathbf{E}}$. Next, we address the fact that the virtual flat fading process $\tilde{\mathbf{E}}$ affects the MTs’ signals $\tilde{\mathbf{X}}$ and not the outputs of the BSs $\mathbf{H} \tilde{\mathbf{X}}$. By recalling that

$$\det \left( \mathbf{I} + \tilde{\mathbf{E}} \mathbf{E}^\dagger \mathbf{H}^\dagger \mathbf{H} \tilde{\mathbf{E}} \right) = \det \left( \mathbf{I} + \tilde{\mathbf{E}} \mathbf{E}^\dagger \mathbf{H} \tilde{\mathbf{E}} \right),$$

(11)

it is evident that for i.i.d. input $\tilde{\mathbf{X}}$ the resulting throughput is the same, regardless whether the virtual flat fading affects the input signal or the output signal. Examining the right-hand-side (RHS) of (11) it is concluded that up to a transpose conjugate of the inter-cell interference coefficient vector $\{\alpha_l\}$ the per-cell throughput of the cellular uplink and the rate of the ISI channel are equivalent.

It is noted that the last proposition holds for other user activity pattern statistics. For example the number of users per-cell can be assumed to be unbounded and drawn according to a Poisson distribution (see [3]). Next, we consider several possible power control policies that determine the actual fading distribution of the equivalent ISI channel.

**A. No Power Control**

When no power control (NPC) is used, each active user (i.e., $e_{m,k} = 1$) is transmitting with a fixed power $p_{m,k} = P/K$. Hence, the fact that the active users can increase their transmission power while the cell still meets its total power constraint, is ignored. For NPC it is easily verified that $\{e_{m,m}\}_{m=1}^{M}$ are i.i.d. r.v.’s $e_{m,m} = \sqrt{L_m} P/K$ where $\{L_m\}_{m=1}^{M}$ are i.i.d. Binomial r.v.’s $L_m \sim \text{Bernoulli}(K, 1-q)$. It can be shown that for large $K$ and fixed $P$ the virtual fading process consolidates and the per-cell throughput converges (and is upper bounded for any $K$, not necessarily large) to that of a static system ($K$ active users in each cell) but with power penalty of $P(1-q)$.

**B. Adaptive Power Control**

According to the adaptive power control (APC) scheme each active user in the $m$th cell transmits using power $p_{m,k} = P/K_m$, where $K_m = \sum_{k=1}^{K} e_{m,k}$ is the number of active users in the $m$th cell. In this case it is easily verified that the total cell power constraint is satisfied and that $\{e_{m,m}\}_{m=1}^{M}$ are i.i.d. r.v.’s $e_{m,m} = \sqrt{L_m} P/K_m$ where $\{L_m\}_{m=1}^{M}$ are i.i.d. Bernoulli r.v.’s $L_m \sim \text{BER}(1-q^K)$. As with NPC, it is easily shown that for large $K$ and fixed $P$ the virtual fading process consolidates and the per-cell throughput converges (and is upper bounded for any $K$, not necessarily large) to that of a static system with no power penalty.

**C. Cognitive Power Control**

For the cognitive power control (CPC) policy we use the convention that inactive users are assumed to be aware of all the active users’ messages (see [10]). Accordingly, each inactive user divides its power and transmits the active users’ messages in a coherent manner. Straightforward calculations yield that the optimal power of the virtual user is

$$P_0^c = \left\{ \begin{array}{ll} (K-L+1)P & 0 < L \leq K \\ 0 & L = 0 \end{array} \right.$$

(12)

Hence, $\{e_{m,m}\}_{m=1}^{M}$ are i.i.d. r.v.’s $e_{m,m} = \sqrt{P_0^c}$ where $\{L_m\}_{m=1}^{M}$ are i.i.d. Binomial r.v.’s $L_m \sim \text{BER}(K,K-1-q)$. It is easily verified that the per-cell throughput in this case is upper bounded for any $K$ by that of a static system but with power gain of $P(1+Kq - (K+1)q^K)$.

**V. SINGLE-CELL PROCESSING**

For comparison purposes we consider single cell processing (SCP) schemes. Here, each BS is aware of the activity pattern of its cell’s users and these of the interfering cells’ users only. On the other hand it is aware of the codebooks of its cell’s users only while it is oblivious of the codebooks of the interfering cells’ users. Hence, each BS treats the signals stemming from the interfering cells as Gaussian noise (conditioned on the activity pattern). Accordingly, the sum-rate of the $m$th cell is an r.v. given by

$$R_{scp}(m) = \log_2 (1 + \text{SNR}(m)),$$

(14)

where $\text{SNR}(m)$ is the signal to interference plus noise ratio at the $m$th BS. Hence, the per cell throughput of the system is given by

$$R_{scp} = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} R_{scp}(m) = \mathbb{E} \left[ \log_2 (1 + \text{SNR}) \right],$$

(15)

where

$$\text{SNR} = \frac{|\alpha_l|^2 e_l^2}{1 + \sum_{l=1}^{L_2} |\alpha_l|^2 e_l^2},$$

(16)

and the expectation is taken over the arbitrary i.i.d. r.v.’s $\{e_l\}_{l=-L_1}^{L_2}$ which are distributed as the i.i.d. diagonal entries of $\tilde{\mathbf{E}}$ according to the specific power control scheme being used (i.e., NPC, APC, or CPC). The second equality of
\[ \beta = \frac{q^K \sqrt{2P(|\alpha_0|^2 + |\alpha_1|^2) + P^2(|\alpha_0|^4 + |\alpha_1|^4) - 2P^2|\alpha_0|^2|\alpha_1|^2 (1 - 2q^{2K}) + 1 - q^{2K}P(|\alpha_0|^2 + |\alpha_1|^2) - 1}}{q^{2K}P^2\left(|\alpha_0|^2 - |\alpha_1|^2\right)^2 - 1} \]  

(13)

(15) holds almost surely (a.s.) and is achieved by the strong law of large numbers (SLLN). The latter is applicable here since the interference in each cell stems from no more than \(L_1\) neighboring cells to the left and \(L_2\) neighboring cells to the right. Thus, SINR\((m_1)\) and SINR\((m_2)\) are i.i.d. for \(|m_1 - m_2| > L_1 + L_2 + 1\).

VI. SOFT HANDOFF MODEL

Here we focus on the simplest instance of the considered model. According to the soft-handoff (SHO) model, depicted in Fig. 1, inter-cell interference stems from one adjacent cell only (see [11]). In this case \(L_1 = 0, L_2 = 1\), and only \(\alpha_0\) and \(\alpha_1\) are non-zero. The power spectral density is given by

\[ S(f) = (|\alpha_0|^2 + |\alpha_1|^2) + 2|\alpha_0||\alpha_1|\cos(2\pi f + \phi) , \]

where \(\phi = \angle (\alpha_1\alpha_0^*)\). The integrals on the RHS of (3) and (4) reduce for the SHO model to

\[ \int_0^1 \log_2(1 + \gamma \beta S(f)) df = \log_2 \left( \frac{a + \sqrt{a^2 - b^2}}{2} \right) , \]  

(18) and

\[ \int_0^1 \frac{1}{1 + \gamma \beta S(f)} df = \frac{1}{\sqrt{a^2 - b^2}} , \]  

(19)

respectively, where

\[ a = 1 + \gamma \beta(|\alpha_0|^2 + |\alpha_1|^2) \quad \text{and} \quad b = 2\gamma \beta|\alpha_0||\alpha_1| . \]  

(20)

Expressions (18) and (19) hold for all power control schemes (or any equivalent fading distribution).

In case where APC is applied, hence \(A \in \{0, 1\} \sim B(1 - q^K)\) and \(\gamma = P\), the result can be expressed in closed form (using (5), (6), (18), and (19)), and the MCP rate is given by

\[ R_{\text{mcp}} = \log_2 \left( \frac{a + \sqrt{a^2 - b^2}}{2} \right) + d(q^K |1 - \beta) , \]  

(21)

where \(\beta\) is given explicitly by (13).

VII. NUMERICAL RESULTS

In this section, we present numerical results for various settings of interest. For all settings, results of the three power control schemes (NPC, APC and CPC) for both MCP and SCP approaches are presented. Unsurprisingly, for each power control scheme it is observed that joint MCP is always beneficial over SCP. Moreover, for MCP, CPC is beneficial over APC, which in turn is beneficial over NPC. This is because for MCP the resulting channel, given an activity pattern, is a MAC channel whose sum-rate increases without bound with the total transmit power. The same relations are also observed for SCP under all tested conditions. We further note that all the rates presented here are plotted using both analytical expressions (continuous and dashed lines for MCP and SCP respectively) and Monte-Carlo (MC) simulations (marked by asterisks). Examining Figures 3-6 an excellent match between the MC and exact results is observed for all cases over a wide range of system parameters.

In Figure 3, the per-cell throughputs supported by the SHO model \((\alpha_0 = 1, \alpha_1 = 0.5)\), for \(K = 5\) users per-cell, and non-activity probability \(q = 0.3\).

Fig. 3. Per-cell throughput vs. the total cell power \(P\) supported by the SHO model \((\alpha_0 = 1, \alpha_1 = 0.5)\), for \(K = 5\) users per-cell, and non-activity probability \(q = 0.3\).

Fig. 4. Per-cell throughput vs. the non-activity probability \(q\) supported by the SHO model \((\alpha_0 = 1, \alpha_1 = 0.5)\), for \(K = 5\) users per-cell, and total cell power \(P = 5\) [dB].

Due to the SLLN this amounts to dividing the per-cell rate by \((1 - q)K\)
that the per-active user rates of all schemes increase with $q$. Obviously, the MCP rates coincide for $q = 0$. (The same applies to the SCP rates.) In Figure 5 we demonstrate the important role played by the number of users per-cell $K$ for the same setting of Figure 3. Examining the figure, the benefit of MCP-CPC over all other schemes is observed. This is because its resulting average power increases without bound with $K$ (under fixed total cell power), while the other MCP schemes result in a bounded power and all SCP schemes yield bounded SINRs. Finally, in Fig. 6 the per-cell throughputs supported by the SHO model ($\alpha_0 = 1, \alpha_1 = \alpha$) for $q = 0.3$, and total cell power $P = 5$ [dB], are plotted as functions of the inter-cell interference factor $\alpha$. Obviously, for each power scheme the rates of MCP and SCP coincide when no inter-cell interference is present ($\alpha = 0$). Also notable is that all SCP rates decrease with increasing $\alpha$. This is because the inter-cell interference power increases with $\alpha$ while the useful signal power is unchanged. In contrast, the MCP rates increase with $\alpha$ for the SHO model. It is noted that the latter does not hold in general when interferences stem from more than one cell.

VIII. CONCLUDING REMARKS

In this work we have studied the performance of a non-fading cellular uplink system with cooperative BSs and random numbers of users per cell. Using the simple Wyner model family and focusing on the large system limit, we have established an analogy between the per-cell throughput of the dynamic cellular uplink and the achievable rate of an LTI ISI channel with flat fading, under certain conditions. In particular we have shown that the power control scheme being used in the cellular uplink determines the fading statistics of the ISI channel. Using a recent result regarding the achievable rate of the ISI channel [7], expressions for the cellular throughput are provided. Moreover, for cases where interference stems from only one adjacent cell, the rate is explicitly given, revealing analytically the impact of different system parameters. Finally, we have demonstrated the benefits of joint MCP over the conventional SCP approach for several cases of interest. For instance, we have shown that combining cooperative BSs and cognitive MTs provides a dramatic increase in system performance.

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