Uplink Throughput of TDMA Cellular Systems with Multicell Processing and Amplify-and-Forward Cooperation Between Mobiles

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***Abstract*— Cooperation between base stations and collaborative transmission between mobile terminals are two technologies currently under study as promising paradigms for next generation communications systems. In this paper, we provide a first look to the interplay between these two approaches by studying the per-cell achievable sum-rate (throughput) of different cooperative protocols under a simplified model for the uplink of a TDMA cellular system. The analysis is limited to non-regenerative (Amplify-and-Forward) cooperation schemes between terminals for their simplicity and appeal to a practical implementation.**

A closed form expression for the (asymptotic) achievable rate of multicell processing combined with Amplify-and-Forward collaboration at the terminals is derived for an AWGN (i.e., no fading) scenario. Moreover, the impact of fading is investigated numerically, allowing to draw some conclusions on the impact of multicell diversity (or macrodiversity) on the performance of collaborative schemes among the terminals. In particular, we show that while AF cooperation is generally advantageous for single cell processing (i.e., with no collaboration between base stations), its benefits when combined with multicell processing are limited to the regime of low to moderate transmission rates.

***Index Terms*— Cooperative communications, cellular networks, mesh networks, hybrid networks.**

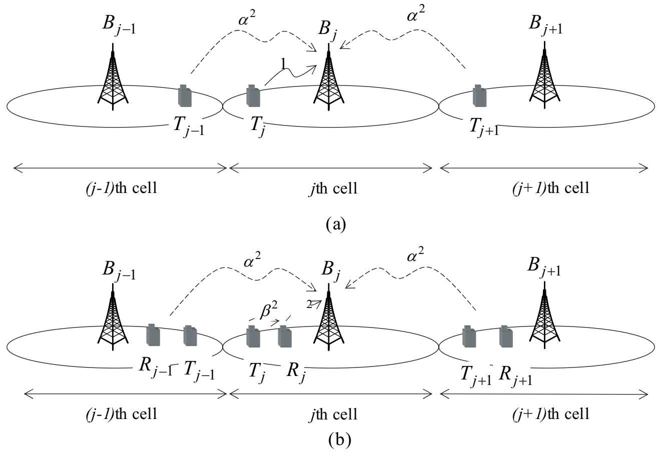

Fig. 1. (a) Linear cellular model proposed by Wyner [3]. (b) Extended model with cooperative transmission between terminals. T_j, B_j and R_j represent source terminal, base station and relay within the jth cell.

I. Introduction

An increasing number of applications, ranging from high data rate ubiquitous access to the Internet to emergency and disaster relief, is shifting the attention of the communications community from the standard cellular paradigm to other more promising network structures. Examples include: *multihop cellular systems*, in which the traditional cellular network infrastructure is extended by allowing transmission to/from mobile terminals (MTs) through multiple wireless hops [1] [2]; *cellular systems with multicell processing*, whereby the cellular infrastructure is modified in order to permit cooperation between different base stations (BSs) for either joint coding/precoding in the downlink or decoding/equalization in the uplink [3][1]; *ad hoc/mesh networks,* generally defined as an infrastructureless collection of wireless nodes that communicate with each other by either direct or multihop transmission. In all the above network structures, collaboration between different nodes (meaning either BSs or MTs) plays a key role.

In this paper, we focus on a combination of the first two network structures listed above. In particular, we are interested in investigating the interplay between cooperation among MTs on one hand and multicell processing (i.e., collaboration among BSs) on the other, in the uplink of a cellular system. Collaboration between MTs in a cellular network in the form of multihop transmission has been proposed in [1] as a means to increase coverage, mitigating unfairness in QoS and guaranteeing service even in emergency conditions. On the other hand, multicell processing has been investigated, among others, in [3] [5] [6], showing that relevant performance improvement is expected under the assumption of ideal collaboration among the BSs.

The analytical framework of this paper is inspired by the cellular model proposed by Wyner in [3] and later adopted in a relevant number of references (see, e.g., [6]-[9]). According

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[1] A similar setting is sometimes referred to in the literature as *distributed antenna systems* [4].

to the linear variant of this model[2], cells are arranged in a linear geometry and only adjacent cells interfere with each other. Moreover, intercell interference is described by a single parameter $\alpha \in [0, 1]$, defining the gain experienced by signals travelling to interfered cells (see fig. 1-(a)). Notwithstanding its simplicity, this model is able to capture the essential structure of a cellular system and it allows to get insight into the system performance. The results in this paper can be seen as an extension of previous works on the Wyner's model, where the novel contribution concerns the introduction of collaborative communications between MTs.

The goal of this paper is to provide a first look at the interplay between multicell processing (or macrodiversity [10]) and cooperative transmission. Ideal collaboration between BSs (multicell processing) is considered as in [3] [5]: the BSs ($\{B_j\}$ in fig. 1-(b)) are assumed to be able to jointly decode the signals received at different cell-sites, having exchanged the necessary information through noiseless low-latency interconnections (for more practical distributed cooperative schemes between BSs see, e.g., [9] [10]). Therefore, the BSs work effectively as a distributed antenna array. Moreover, our analysis is limited to a specific form of collaboration between MTs, namely Amplify-and-Forward (AF) relaying, also referred to as non-regenerative relaying [11] [12]. According to this approach, the MTs acting as relays (e.g., R_j in fig. 1-(b)) do not attempt to decode the signal transmitted by the active MT (e.g., T_j) but simply forward the received signal after amplification. The AF method is appealing for its simplicity that makes it an interesting candidate for practical systems. Finally, we focus on intra-cell TDMA scheduling, i.e., we assume that in each cell there is only one active source MT at each time instant. Intra-cell TDMA strategy has been shown to be optimal in the absence of fading but suboptimal for fading channels [3] [6].

In order to maintain the simplicity of Wyner's model, only two additional parameters are introduced in the model to investigate the effects of AF collaboration (see fig. 1-(b)). In particular, each active MT is assumed to be assigned a relay station that receives average power $\beta^2 \geq 1$ (the power received by the intended base station is normalized to one) and conveys power $\delta^2 \geq 1$ to the intended base station. Therefore, in the extended Wyner model considered in this paper, there are three parameters of interest:

- the intercell gain $\alpha \in [0, 1]$, ruling the effectiveness of multicell processing (as in the original paper [3]), and
- the gains $\beta \geq 1$ and $\delta \geq 1$ on the links source-relay and relay-base station, respectively, that determine the performance of AF collaborative communications between terminals.

Notice that the model considered above implicitly considers source MTs located at the edges of the cell, for which convenient relay MTs on the path to the base station are assumed to be available. This scenario is of particular interest since it provides a case study where cooperation between

[2]Following the analysis of this paper, the analysis could be extended to the planar variant of Wyner's model [3]. The treatment of this case is however significantly more involved in terms of notation and analysis and is not pursued here.

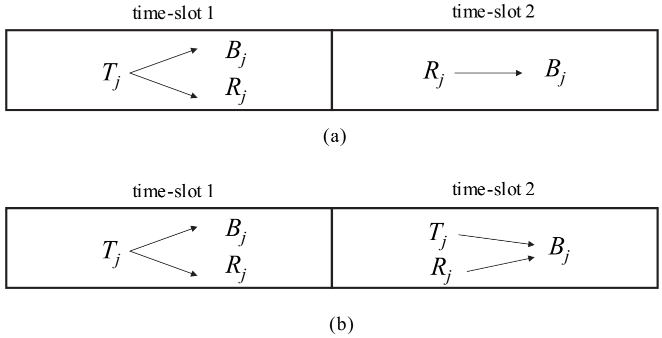

Fig. 2. The two-slot transmission scheme prescribed by AF collaborative schemes between terminals: (a) AF scheme proposed in [11] (referred in the text as OAF); (b) AF scheme proposed in [12] (referred in the text as NAF).

terminals (relaying) is expected to be the most beneficial (see, e.g. [7]).

A. AF collaboration between terminals

Two alternative AF techniques have been proposed in [11] and [12] respectively. Following [14], we refer to the latter scheme [12] as Nonorthogonal AF (NAF) and, by converse, to the former [11] as Orthogonal AF (OAF). Both methods prescribe a two-slot transmission and differ in the transmission policy within the second time-slot, as illustrated in fig. 2. The first slot is used by each terminal (say T_j) to broadcast its message to the desired BS (B_j) and to the relay MT (R_j), whereas in the second slot: (*i*) according to OAF [11], the relay forwards the signal received in the first time-slot to the BS after amplification while the source MT remains silent (fig. 2-(a)); (*ii*) with NAF, the relay behaves as for OAF but the source MT now transmits simultaneously a new independent message[3] (fig. 2-(b)). Both techniques require full channel state information available at the receiver side and no channel state information is assumed to be available at the transmitter. In the context of a single relay channel, [14] shows that, while more complex, NAF outperforms OAF and is optimal from the point of view of the diversity-multiplexing tradeoff within the class the class of AF protocols.

B. Main contributions

The main contributions of this work can be summarized as follows:

1) introduction of an extended Wyner's model for the uplink of a cellular systems with AF collaborative communications;
2) derivation of a closed form (asymptotic) expression for the per-cell achievable sum-rate (throughput) of a TDMA cellular system with multicell processing and either OAF or NAF collaborative communication in an *AWGN channel* (i.e., with no fading). This expression extends the result derived in [3] under the assumption of multicell processing without terminal cooperation;

[3]Notice that in order to perform a fair comparison between OAF and NAF, the powers transmitted in the second time-slot by the source MT and the relay MT are halved so as to maintain the same total transmitted power.

3) investigation of the interplay between diversity obtained through multicell reception (or macrodiversity) and cooperative diversity in a *fading channel* through numerical results. The main conclusion of this study are as follows: *i*) AF collaboration is a promising tool to improve the perfomance of the uplink of cellular systems that employ separate decoding at each BS (*single cell processing*). In particular, for single cell processing, while OAF collaboration is advantageous over direct transmission only for low rates, NAF is beneficial under broad conditions[4]. This result is in accordance to the study of [14] in the context of a single relay channel; *ii*) AF collaboration can be advocated in the context of multicell processing (with α large enough) only for low to moderate transmission rates. In fact, whenever the diversity introduced by multicell processing is significant (i.e., for α large), the additional benefits of cooperative AF diversity are overcome by the increased noise level due to non-regenerative relaying. More specifically, both OAF and NAF are outperformed by direct transmission at sufficiently large spectral efficiencies, with NAF always outperforming OAF (except at very low transmission rates). As a remark, it is expected that this latter conclusion could be different for more refined form of collaborations such as Decode-and-Forward [11].

II. Problem Formulation

As explained in the previous Section, this paper considers the uplink of a linear Wyner's cellular model with intracell TDMA. The system layout is illustrated in fig. 1, where the upper part (a) refers to the scenario where no cooperation between MTs is allowed, and the lower part (b) sketches the case where transmission between an active MT and its BS takes place through AF cooperation by a relay node. In each of the M cells, deployed according to a linear geometry, due to the intracell TDMA protocol considered in this paper, there is only one active source MT at each time. The BSs are denoted as $\{B_j\}_{j=1}^M$, the source MTs, one for each cell, as $\{T_j\}_{j=1}^M$, and the MT acting as relays are referred to as $\{R_j\}_{j=1}^M$. It is assumed that each active terminal T_j has available a relay terminal R_j for cooperation. Flat Rayleigh fading is assumed; moreover, the complex circularly symmetric Gaussian distributed channel gains are independent identically distributed and identified by their subscript. In particular, the channels between source MTs and BSs have power

$$\mathrm{E}[|h_{T_j B_{j+i}}|^2] = 1 \quad \text{for } i = 0; \; \alpha^2 \text{ for } i = \pm 1; \; 0 \quad \text{otherwise}, \tag{1}$$

where$\alpha \in [0, 1]$ is the interference factor. Channel gains between source MTs and relay MTs read

$$\mathrm{E}[|h_{T_j R_{j+i}}|^2] = \beta^2 \text{ for } i = 0; \; 0 \quad \text{otherwise}, \tag{2}$$

with $\beta^2 \geq 1$. Notice that in (2) it is assumed that a relay is not able to receive the transmissions of MTs belonging to adjacent cells. This assumption is reasonable if the relays are MTs, but it may be questionable if the relays are fixed wireless stations placed at heights comparable to the BSs (as in the scenario investigated in [7]). A more reasonable assumption in this case would be that of setting the powers $\mathrm{E}[|h_{T_j R_{j\pm 1}}|^2]$ equal to the intercell factor α^2. The analysis under this setting could be derived from the treatment presented below. The extension is conceptually straightforward and it will not be further illustrated here since the conclusions would be qualitatively similar but the notation would be more involved. Finally, the channels between relay MTs and BSs can be written as

$$\mathrm{E}[|h_{R_j B_{j+i}}|^2] = \delta^2 \quad \text{for } i = 0; \; \alpha^2 \quad \text{for } i = \pm 1; \; 0 \quad \text{otherwise}. \tag{3}$$

Notice that, as a notational convention the channels from "virtual" cells 0 and $M + 1$ are assumed to be zero. To complete the set of assumptions, each MT (either source or relay) is assumed to transmit an energy per symbol equal to[5] E_s and each receiver (either relay or BS) is modelled as having a Gaussian white one-sided noise spectral density equal to N_0.

In the following Sections, the per-cell achievable sum-rate (throughput) of the system in fig. 1 is investigated for: (*i*) *single cell* processing (Sec. III): each BS processes the received signal independently from the other cell-sites, treating intercell interference as Gaussian noise. Both scenarios with direct transmission and AF collaborative transmission are studied; (*ii*) *multicell* processing (Sec. IV): the signals received by the M BSs are jointly processed, assuming a noiseless low-latency interconnection between the BSs. As in the previous case, both scenarios with and without AF cooperation between MTs are studied. Moreover, Sec. V derives a closed form expression for the per-cell achievable sum-rate of multicell processing in AWGN channels. The analysis is then corroborated by numerical results in Sec. VI, allowing to draw some conclusions on the interplay between multicell diversity and cooperative diversity under the assumption of AF transmission.

III. Single Cell Processing

In this Section, the scenario where each BS, say B_j, processes the received signal independently from the other BSs $\{B_i\}_{i \neq j}$ is considered. The per-cell achievable rate for direct transmission between active MTs $\{T_j\}$ and their corresponding BSs $\{B_j\}$ is derived in Sec. III-A, whereas AF collaborative communications (both OAF and NAF) is covered in Sec. III-B.

A. Direct transmission

Here we consider the scenario in fig. 1-(a), where direct transmission between MTs and BSs takes place. The discrete-time baseband signal received in each time instant by the BS B_j ($j = 1, ..., M$) can be written as (discrete-time dependence is omitted for simplicity of notation)

$$y_j = \sum_{i=-1}^{1} h_{T_{j+i} B_j} x_{j+i} + n_j = h_{T_j B_j} x_j + w_j + n_j, \tag{4}$$

with x_j denoting the signal transmitted by the MT T_j, that is assumed to be taken from a Gaussian codebook

[4]OAF can outperform NAF for very low spectral efficiencies.

[5]apart from the second time-slot under the NAF protocol, where the available power is shared by the source MT and the relay.

with $\mathrm{E}[|x_j|^2] = E_s$. The additive Gaussian thermal noise has power $\mathrm{E}[|n_j|^2] = N_0$ and the remaining term $w_j = \sum_{i=-1,i\neq 0}^{1} h_{T_{j+i}B_j} x_{j+i}$ accounts for *intercell interference*. In single-cell processing, the interference w_j is regarded at the BS B_j as additive Gaussian noise with power $\mathrm{E}[|w_j|^2] = E_s(|h_{T_{j-1}B_j}|^2 + |h_{T_{j+1}B_j}|^2)$. Therefore, the compound additive Gaussian noise $w_j + n_j$ has power $\mathrm{E}[|w_j|^2] + N_0$. Recalling that the BS is assumed to have knowledge of the channel gains $h_{T_{j+i}B_j}$ with $i = -1, 0, 1$, the ergodic per-cell achievable sum-rate measured in $bit/s/Hz$ reads

$$R_{SC-DT} = \mathrm{E}_h \left[\log_2 \left(1 + \mathrm{SNR} \frac{|h_{T_jB_j}|^2}{1 + W_j} \right) \right], \quad (5)$$

with $\mathrm{E}_h[\cdot]$ denoting the ensemble average with respect to the fading distribution , $\mathrm{SNR} = E_s/N_0$ the signal to noise ratio and

$$W_j = \frac{\mathrm{E}[|w_j|^2]}{N_0} = \mathrm{SNR}(|h_{T_{j-1}B_j}|^2 + |h_{T_{j+1}B_j}|^2). \quad (6)$$

Notice that (5) assumes that the channel coherence time is small enough so that the transmitted codeword spans a large (theoretically infinite) number of channel states (i.e., for delay tolerant applications or fast-varying channels). Furthermore, from (6) the interference-limited behavior of single cell transmission is apparent by letting $\mathrm{SNR} \to \infty$: in this case, we have $R_{SC-DT} \to \mathrm{E}_h \left[\log_2 \left(1 + \frac{|h_{T_jB_j}|^2}{|h_{T_{j-1}B_j}|^2 + |h_{T_{j+1}B_j}|^2} \right) \right]$, independent of SNR (for a closed form expression, see [8]).

B. Amplify-and-Forward (AF) transmission

In this case, the system is exemplified in fig. 1-(b) and transmission is time-slotted according to fig. 2. In particular, as explained above, we consider two different AF schemes (OAF [11] and NAF [12]), whose transmission policies differ only in the second time-slot. The main result is this Section is the derivation of the per-cell achievable rates attained by the OAF and NAF cooperative schemes in a single cell scenario (equations (13) and (15) respectively).

As for direct transmission, in the first time-slot, under both OAF and NAF protocols, the active terminals $\{T_j\}_{j=1}^{M}$ transmit Gaussian symbols x_j with $\mathrm{E}[|x_j|^2] = E_s$ and, therefore, the signal received by the BS B_j is (4). However, the signals transmitted in the first time-slot by terminals T_j are also overheard by the relay nodes R_j, that receive $y_{R_j} = h_{T_jR_j} x_j + n_{R_j}$, where the noise term n_{R_j} has power $\mathrm{E}[|n_{R_j}|^2] = N_0$. The signal model in the second time-slot differs for the two considered AF schemes, as detailed in the following.

1) Orthogonal AF (OAF) [11] : According to the OAF technique, in the second time-slot the relay scales the received signal y_{R_j} in order to keep the average transmitted energy per symbol equal to E_s, and then forwards the resulting symbol. More precisely, in the second time-slot, the relay forwards $\mu_j y_{R_j} = \mu_j h_{T_jR_j} x_j + \mu_j n_{R_j}$ with

$$\mu_j^2 = \frac{E_s}{E_s |h_{T_jR_j}|^2 + N_0}. \quad (7)$$

Therefore, the signal received by the jth BS B_j in the second time-slot is (prime refers to quantities transmitted in the second time-slot) $y'_j = \sum_{i=-1}^{1} h_{R_{j+i}B_j} (\mu_{j+i} y_{R_{j+i}}) + n'_j$, with n'_j denoting thermal noise at B_j, assumed to be independent of the noise in the first time-slot and with power $\mathrm{E}[|n'_j|^2] = N_0$. In order to get some insight into the signal structure of y'_j, it is useful to define the *equivalent channel gains* experienced by the signals x_{j+i} with $i = -1, 0, 1$ toward the BS B_j in the second time-slot (see fig. 3). These equivalent channel gains account for the signal paths from the source MTs T_{j+i} to the BS B_j through the corresponding relays R_{j+i} and read:

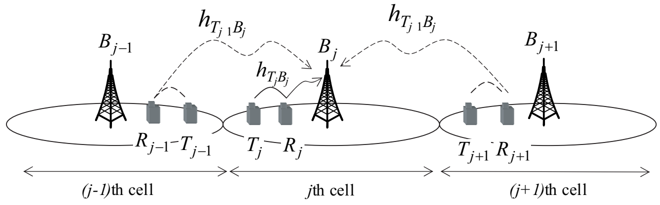

Fig. 3. Definition of the equivalent channels (8) in the second time-slot for AF protocols.

$$h'_{T_jB_j} = \mu_j h_{T_jR_j} h_{R_jB_j}, \quad h'_{T_{j\pm1}B_j} = \mu_{j\pm1} h_{T_{j\pm1}R_{j\pm1}} h_{R_{j\pm1}B_j}. \quad (8)$$

Using the definitions in (8), the signal received by B_j can be written as

$$y'_j = \sum_{i=-1}^{1} h'_{T_{j+i}B_j} x_{j+i} + n'_j = h'_{T_jB_j} x_j + w'_j + z_j + n'_j, \quad (9)$$

where w'_j accounts for intercell interference, $w'_j = \sum_{i=-1,i\neq0}^{1} h'_{T_{j+i}B_j} x_{j+i}$, and z_j is the thermal noise generated at the relays and forwarded to the BS, $z_j = \sum_{i=-1}^{i=1} \mu_{j+i} h_{R_{j+i}B_j} n_{R_{j+i}}$. Therefore, for single cell processing, the equivalent noise power in the second time-slot reads $N_0(1 + W'_j + Z_j)$, where

$$W'_j = \mathrm{E}[|w'_j|^2]/N_0 = \mathrm{SNR}(|h'_{T_{j-1}B_j}|^2 + |h'_{T_{j+1}B_j}|^2) \quad (10)$$

and

$$Z_j = \mathrm{E}[|z_j|^2]/N_0 = \sum_{i=-1}^{1} \mu_{j+i}^2 |h_{R_{j+i}B_j}|^2. \quad (11)$$

The equivalent additive Gaussian noise at the BS in the two slots is correlated as (recall (4) and (9))

$$\begin{aligned} \rho &= \mathrm{E}[(w_j + n_j)(w'_j + z_j + n'_j)^*]/N_0 = \quad (12) \\ &= \mathrm{SNR}(h_{T_{j-1}B_j} h'^*_{T_{j-1}B_j} + h_{T_{j+1}B_j} h'^*_{T_{j+1}B_j}). \end{aligned}$$

It follows that the resulting ergodic per-cell achievable rate can be expressed as shown in (13) on the next page, where the pre-log scaling by $1/2$ depends on the two-slot structure of the AF transmission [11].

2) Nonorthogonal AF (NAF) [12] : According to this scheme, proved in [14] to be optimal from a multiplexing-diversity trade-off standpoint, the source MTs T_j transmit two independent messages, x_j in the first time-slot and x'_j in the second time-slot. On the other hand, the relays R_j forward the signal received in the first time-slot exactly as explained in the previous Section for OAF. In order to present a fair comparison between different techniques, the powers transmitted in the second slot by source and relay MTs are halved in such a way that the total transmitted power is not increased. It follows

$$R_{SC-OAF}=\frac{1}{2}\mathrm{E}_h\left[\log_2\left(1+\mathrm{SNR}\begin{bmatrix} h^*_{T_jB_j} & h'^*_{T_jB_j}\end{bmatrix}\cdot\begin{bmatrix} 1+W_j & \rho \\ \rho & 1+Z_j+W'_j\end{bmatrix}^{-1}\begin{bmatrix} h_{T_jB_j} \\ h'_{T_jB_j}\end{bmatrix}\right)\right], \tag{13}$$

that the signal received by the relay y_{R_j} is scaled by $\mu_j/\sqrt{2}$ before forwarding, and the power of the Gaussian symbol x'_j is $E_s/2$. This introduces some technicalities in the derivation of the signal model, as discussed in the following.

According to the discussion in the previous Section, the received signal in the second time-slot is (to uniform the notation we set $\mathrm{E}[|x'_j|^2]=E_s$, the power scaling is accounted for as shown below)

$$y'_j=\frac{1}{\sqrt{2}}h'_{T_jB_j}x_j+\frac{1}{\sqrt{2}}h_{T_jB_j}x'_j+\tilde{w}'_j+\frac{1}{\sqrt{2}}z_j+n'_j, \tag{14}$$

where n'_j accounts for thermal noise, z_j is defined as in the previous Section and the intercell interference $\tilde{w}'_j$ contains the additional interference contribution of the source MTs of adjacent cells as compared to w_j: $\tilde{w}'_j=\frac{1}{\sqrt{2}}w'_j+\frac{1}{\sqrt{2}}\sum_{i=-1,i\neq 0}^{1}h_{T_{j+i}B_j}x'_{j+i}$. Therefore, the equivalent noise power in the second time-slot modifies as $N_0+\frac{1}{2}W'_j+\frac{1}{2}W_j+\frac{1}{2}Z_j$. The ergodic per-cell achievable sum-rate of this scheme then reads as shown in (15) on the next page, with ρ denoting the correlation of the equivalent noise in the two time-slots, as defined in (12).

IV. Multicell Processing

Here we extend the results of the previous Section to a scenario where the signals received by all the BSs are jointly processed in order to decode the transmitted signals ($\{x_j\}_{j=1}^M$ for direct transmission and OAF; $\{x_j,x'_j\}_{j=1}^M$ for NAF).

A. Direct transmission

The discrete-time signal received by the jth BS is (4). Differently from the single cell case, multicell processing treats adjacent cells' contribution w_j as useful signal and not as an additive noise term. Since the signals received by the M BSs are jointly processed in order to detect the $M\times 1$ transmitted vector $\mathbf{x}=[x_1\cdots x_M]^T$, it is convenient to gather the received signals in a $M\times 1$ vector $\mathbf{y}=[y_1\cdots y_M]^T$ and restate the model (4) according to a matrix formulation

$$\mathbf{y}=\mathbf{H}\mathbf{x}+\mathbf{n}, \tag{16}$$

where the $M\times M$ channel matrix $\mathbf{H}$ is tridiagonal due to the Wyner model assumption about intercell interference [3]:

$$\mathbf{H}=\begin{bmatrix} h_{T_1B_1} & h_{T_2B_1} & 0 & \cdots \\ h_{T_1B_2} & h_{T_2B_2} & \ddots & \cdots \\ 0 & \ddots & \ddots & h_{T_MB_{M-1}} \\ \vdots & 0 & h_{T_{M-1}B_M} & h_{T_MB_M}\end{bmatrix} \tag{17}$$

and $\mathbf{n}=[n_1\cdots n_M]^T$. This expression clearly highlights the fact that signals from all the cells, collected in vector $\mathbf{y}$, are jointly considered for detection of the transmitted vector $\mathbf{x}$. The ergodic per-cell achievable rate of this scheme reads:

$$R_{MC-DT}=\frac{1}{M}\mathrm{E}_h\left[\log_2\left|\mathbf{I}_M+\mathrm{SNR}\mathbf{H}^H\mathbf{H}\right|\right]. \tag{18}$$

Bounds on this expressions can be found in [6].

B. Amplify-and-Forward (AF) transmission

The derivation of the per-cell achievable rates for both OAF and NAF collaborative schemes in a multicell scenario builds on the analysis of Sec. III-B. The distinctive feature here is that, differently from single cell processing, intercell interference terms are treated as additional signals, as explained above in the context of direct transmission. The signal received at the BS B_j in the first slot can be written as (16) for both AF protocols. The signal model for the second slot is detailed in the following according to the two AF schemes of interest.

1) Orthogonal AF (OAF): For OAF, as explained in Sec. III-B.1, the received signal by BS B_j can be expressed as (9). However, differently from single cell processing, in multicell processing, the intercell interference term w_j in (9) is treated as signal useful for decoding. Therefore, similarly to (16), the $M\times 1$ vector $\mathbf{y}'=\begin{bmatrix} y'_1 & \cdots & y'_M\end{bmatrix}^T$ reads $\mathbf{y}'=\mathbf{H}'\mathbf{x}+\mathbf{n}'+\mathbf{z}$, with $\mathbf{H}'$ being a tridiagonal matrix with the same form as (17), containing the equivalent channel gains (8), and $\mathbf{n}'=[n'_1\cdots n'_M]^T$, $\mathbf{z}=[z_1\cdots z_M]^T$. The correlation matrix of the equivalent additive Gaussian noise in $\mathbf{n}'+\mathbf{z}$ reads (recall (11)) $\mathrm{E}[(\mathbf{n}'+\mathbf{z})(\mathbf{n}'+\mathbf{z})^H]=N_0\mathbf{I}_M+N_0\mathbf{R}_z$, where $\mathbf{R}_z$ is a pentadiagonal Toeplitz correlation matrix with $[\mathbf{R}_z]_{j,j+i}=\mathrm{E}[z_jz^*_{j+i}]/N_0$:

$$\mathrm{E}[z_jz^*_{j+i}]=\begin{cases} Z_j & i=0 \\ \mu_j^2h_{R_jB_j}h_{R_jB_{j\pm1}}+ \\ +\mu_{j\pm1}^2h_{R_{j\pm1}B_j}h_{R_{j\pm1}B_{j\pm1}} & i=\pm1 \\ \mu_{j\pm1}^2h_{R_{j\pm1}B_j}h_{R_{j\pm1}B_{j\pm2}} & i=\pm2\end{cases}. \tag{19}$$

Finally, defining the $2M\times 1$ vector $\bar{\mathbf{y}}=[\mathbf{y}^T\ \ \mathbf{y}'^T]^T$, the $2M\times M$ channel matrix $\bar{\mathbf{H}}=\left[\mathbf{H}^T\ \mathbf{H}'^T\right]^T$ and the $2M\times 1$ noise vector $\bar{\mathbf{n}}=[\mathbf{n}^T\ \ (\mathbf{n}'+\mathbf{z})^T]$, the compound model for the received signal over the two time slots can be stated as $\bar{\mathbf{y}}=\bar{\mathbf{H}}\bar{\mathbf{x}}+\bar{\mathbf{n}}$, where the noise correlation matrix reads:

$$\bar{\mathbf{R}}_n=\frac{1}{N_0}\mathrm{E}[\bar{\mathbf{n}}\bar{\mathbf{n}}^H]=\begin{bmatrix} \mathbf{I}_M & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{I}_M+\mathbf{R}_z\end{bmatrix}. \tag{20}$$

The ergodic per-cell achievable sum-rate is then:

$$R_{MC-OAF}=\frac{1}{2M}\mathrm{E}_h\left[\log_2\left|\mathbf{I}_{2M}+\mathrm{SNR}\bar{\mathbf{H}}^H\bar{\mathbf{R}}_n^{-1}\bar{\mathbf{H}}\right|\right]. \tag{21}$$

2) Nonorthogonal AF (NAF): In the case of NAF, as discussed in Sec. III-B.2, the received signal by the BS B_j is (14). Therefore, following the same reasoning of the previous Sections, the $M\times 1$ vector $\mathbf{y}'$ can be written as

$$\mathbf{y}'=\frac{1}{\sqrt{2}}\mathbf{H}'\mathbf{x}+\frac{1}{\sqrt{2}}\mathbf{H}\mathbf{x}'+\mathbf{n}'+\frac{1}{\sqrt{2}}\mathbf{z}, \tag{22}$$

with $\mathbf{x}'=[x'_1\cdots x'_M]^T$. Notice that the equivalent noise has correlation matrix $\mathrm{E}[(\mathbf{n}'+\frac{1}{\sqrt{2}}\mathbf{z})(\mathbf{n}'+\frac{1}{\sqrt{2}}\mathbf{z})^H]=N_0\mathbf{I}_M+\frac{1}{2}N_0\mathbf{R}_z$. Therefore, the ergodic per-cell achievable sum-rate of this scheme is shown in (23) on the next page.

$$R_{SC-OAF} = \frac{1}{2}\mathrm{E}_h\left[\log_2\left(1+\mathrm{SNR}\begin{bmatrix} h_{T_jB_j} & 0 \\ \frac{h'_{T_jB_j}}{\sqrt{2}} & \frac{h_{T_jB_j}}{\sqrt{2}}\end{bmatrix}^H \cdot \begin{bmatrix} 1+W_j & \rho \\ \rho & 1+\frac{1}{2}W'_j+\frac{1}{2}W_j+\frac{1}{2}Z_j\end{bmatrix}^{-1} \cdot \begin{bmatrix} h_{T_jB_j} & 0 \\ \frac{h'_{T_jB_j}}{\sqrt{2}} & \frac{h_{T_jB_j}}{\sqrt{2}}\end{bmatrix}\right)\right], \quad (15)$$

$$R_{MC-NAF} = \frac{1}{2M}\mathrm{E}_h\left[\log_2\left(1+\mathrm{SNR}\begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \frac{1}{\sqrt{2}}\mathbf{H}' & \frac{1}{\sqrt{2}}\mathbf{H}\end{bmatrix}^H \cdot \begin{bmatrix} \mathbf{I}_M & \mathbf{0}_M \\ \mathbf{0}_M & \mathbf{I}_M+\frac{1}{2}\mathbf{R}_z\end{bmatrix}^{-1}\begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \frac{1}{\sqrt{2}}\mathbf{H}' & \frac{1}{\sqrt{2}}\mathbf{H}\end{bmatrix}\right)\right]. \quad (23)$$

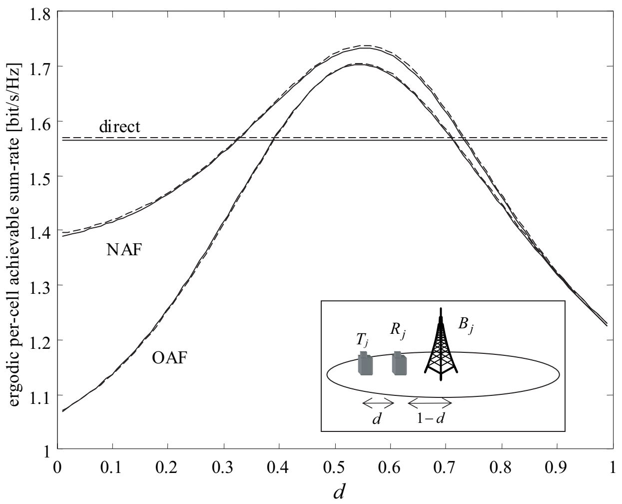

Fig. 4. Per-cell achievable rate for direct transmission and for the AF collaborative schemes NAF and OAF over an AWGN channel. Dashed lines represent the asymptotic closed form expressions (25), (26) and (27) ($\mathrm{SNR} = 3dB$, $\alpha = -5dB$, $\gamma = 3$).

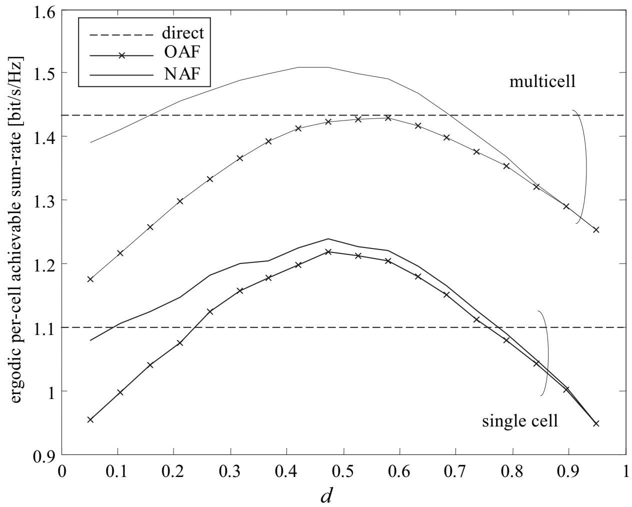

Fig. 5. Ergodic per-cell achievable rates of different transmission schemes for single and multicell processing versus the normalized distance d ($\mathrm{SNR} = 3dB$, $\alpha = -10dB$, $\gamma = 3$).

V. Analysis of the Per-Cell Rate with no Fading

In [3], a closed form expression for the asymptotic (in the number of BSs M) per-cell achievable rate R_{MC-DT} (18) with multicell processing and direct transmission is evaluated under the assumption of an AWGN channel[6]. In this Section, we extend this result to multicell processing with AF collaboration. More precisely, we calculate the asymptotic per-cell achievable rate of the OAF and NAF collaborative schemes over an AWGN channel.

With no fading, the channel gains (1), (2) and (3) read (define for notational convenience $\beta = 1/d^{\gamma/2}$ and $\delta = 1/(1-d)^{\gamma/2}$):

$$h_{T_jB_{j+i}} = \begin{cases} 1 & i=0 \\ \alpha & i=\pm 1 \\ 0 & \text{otherwise}\end{cases}, \quad h_{T_jR_{j+i}} = \begin{cases} \beta & i=0 \\ 0 & \text{otherwise}\end{cases},$$
$$h_{R_jB_{j+i}} = \begin{cases} \delta & i=0 \\ \alpha & i=\pm 1 \\ 0 & \text{otherwise}\end{cases}. \quad (24)$$

For direct transmission, since the matrix $\mathbf{H}^H\mathbf{H}$ in the expression of the achievable rate (18) is Toeplitz, using the results from [13], we have that [3]

$$\lim_{M\to\infty} R_{MC-DT} = \int_0^1 \log_2(1+\mathrm{SNR}((1+2\alpha^2)+ +4\alpha\cos(2\pi\theta)+2\alpha^2\cos(4\pi\theta)))d\theta. \quad (25)$$

For the OAF and NAF cooperative cases, a similar approach can be followed in order to evaluate an asymptotic closed form expression of the achievable rate R_{MC-OAF} (21) and R_{MC-OAF} (23). In particular, as shown in the Appendix, we have

$$\lim_{M\to\infty} R_{MC-OAF} = \frac{1}{2}\int_0^1 \log_2\left(\mathrm{SNR}\frac{r_0+2\sum_{n=1}^3 r_n\cos(n\pi\theta)}{s_0+2\sum_{n=1}^2 s_n\cos(n\pi\theta)}\right)d\theta \quad (26)$$

and

$$\lim_{M\to\infty} R_{MC-NAF} = \frac{1}{2}\int_0^1 \log_2\left(\mathrm{SNR}\frac{u_0+2\sum_{n=1}^3 u_n\cos(n\pi\theta)}{v_0+2\sum_{n=1}^2 v_n\cos(n\pi\theta)}\right), \quad (27)$$

where the definition of constants r_n, s_n, u_n and v_n can be found in the Appendix (equations (37), (32), (38) and (39) respectively).

[6]Since TDMA is optimal in an AWGN scenario with no cooperation, R_{MC-DT} is in this case the capacity of the channel [3].

VI. Numerical Results

Here, some numerical results are presented in order to corroborate the analysis carried out in the previous Sections. Towards the goal of obtaining a better insight into the performance of scenarios where collaboration between MTs is allowed, we specialize the results of the previous Sections to a simple geometric model (see the box in fig. 4). The relay station R_j is assumed to be on a line that connects the active MT T_j to the BS B_j. Normalizing to one the distance between T_j and B_j, the relay R_j is located at a distance from T_j equal to $0 \leq d \leq 1$, where $1-d$ is the normalized distance of R_j to the BS B_j. The average channel gains between active terminals T_j and corresponding relays R_j, namely β^2, and between relay terminals R_j and relative BSs B_j, namely δ^2, are defined by d and by the path loss exponent $\gamma > 1$ as:

$$\beta^2 = \frac{1}{d^\gamma} \tag{28a}$$

$$\delta^2 = \frac{1}{(1-d)^\gamma}. \tag{28b}$$

Let us first consider an AWGN channel in order to validate the analytical results (25)-(27). Fig. 4 compares the exact per-cell achievable rates of direct transmission (18), OAF (21) and NAF (23) with the asymptotic formulas (25), (26) and (27) respectively, versus the normalized distance d for $M = 20$, SNR $= 3dB$, $\gamma = 3$ and $\alpha = -10dB$. Even for a number of cells as small as $M = 20$, the asymptotic expressions (25)-(27) describe accurately the per-cell achievable sum-rate. Moreover, it can be seen that collaboration yields benefits for a range of values of d around 0.5. However, as thoroughly discussed in the next Section in the context of fading channels, this performance gain is in fact limited to sufficiently small spectral efficiencies: by increasing the SNR or the intercell factor α, direct transmission tends to outperform AF cooperation.

We now turn to the study of a fading scenario. Where not stated otherwise, relevant parameters are selected as $M = 10$, SNR $= 3dB$, $\gamma = 3$ and $\alpha = -10dB$. We first consider the effect of the normalized relay distance d in fig. 5, where the ergodic per-cell achievable rate of different schemes under both single and multiple cell processing is plotted versus d. The benefits, if any, of collaborative AF communication are achieved when the relay is placed halfway at $d \simeq 0.5$.

Focusing on single cell processing, from fig. 5, both OAF and NAF outperform direct transmission for a wide range of values of d. Setting the distance to $d = 0.5$, fig. 6 then shows the ergodic per-cell achievable sum-rate versus the SNR. From this figure, the interference-limited behavior of single cell processing is apparent. Moreover, it can be concluded that by increasing SNR (or equivalently the achievable rate), the performance gain of OAF decreases while NAF retains its benefits. These results are in accordance to the analyses in [11] [14] for the single relay channel.

Collaboration between BSs overcome the intercell interference-limited behavior of single cell processing and allows to obtain considerable larger achievable rates. Furthermore, at large spectral efficiencies, noise forwarding due to AF protocols become deleterious to system performance. In particular, as shown in fig. 6, direct transmission outperforms OAF for rates larger than 1.4 $bit/s/Hz$ and NAF for rates larger than 2 $bit/s/Hz$. It can be then concluded that, when combined with multicell reception, the advantages of AF collaboration are limited to moderate transmission rates. Moreover, notice that for both single and multicell processing OAF can yield larger achievable rates than NAF in the regime of very low spectral efficiencies. As a final remark, altering the path loss exponent γ would not modify qualitatively the conclusions above: increasing γ would however improve the performance of AF collaborative schemes (due to the increased power gain attainable).

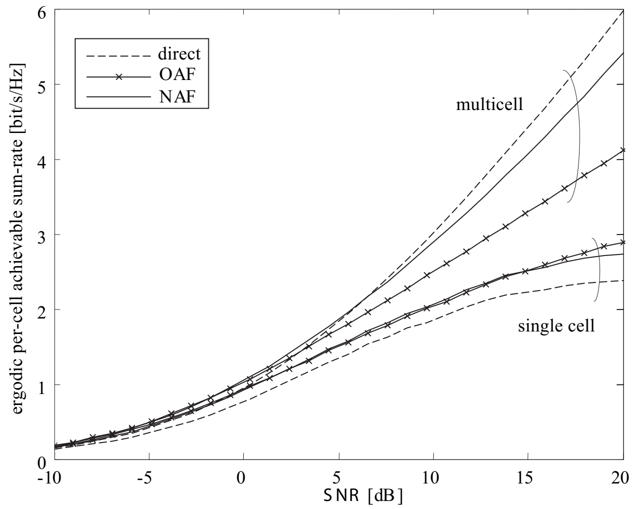

Fig. 6. Ergodic per-cell achievable rates of different transmission schemes for single and multicell processing versus SNR ($\alpha = -10dB$, $d = 0.5$, $\gamma = 3$).

The interplay between gains obtained from multicell processing (or from macrodiversity) and from cooperative AF transmission between terminals is better illustrated in fig. 7. Herein, the ergodic rates under single and multicell processing are plotted versus the intercell gain factor α for $d = 0.5$. Parameter α has opposite effects on the two scenarios: while for single cell processing increasing α causes a performance degradation due to larger intercell interference, multicell reception benefits from a stronger signal paths toward adjacent cells (unless the SNR is very large [6]). Moreover, in accordance to the discussion above, AF collaboration is always effective for single cell processing, with OAF outperforming NAF for low spectral efficiencies (i.e., large α). On the other hand, when the intercell gain α is large enough, the performance improvement due to collaboration among BSs renders the additional noise forwarded to the BS by relays a limiting factor to the performance and direct transmission outperforms AF protocols.

VII. Concluding remarks

In this paper, a first look at the interplay between multicell processing (i.e., cooperation between base stations) and cooperation between terminals has been presented. We considered the uplink of a TDMA cellular system, modelled according to an extended Wyner's model, and focused on AF collaborative

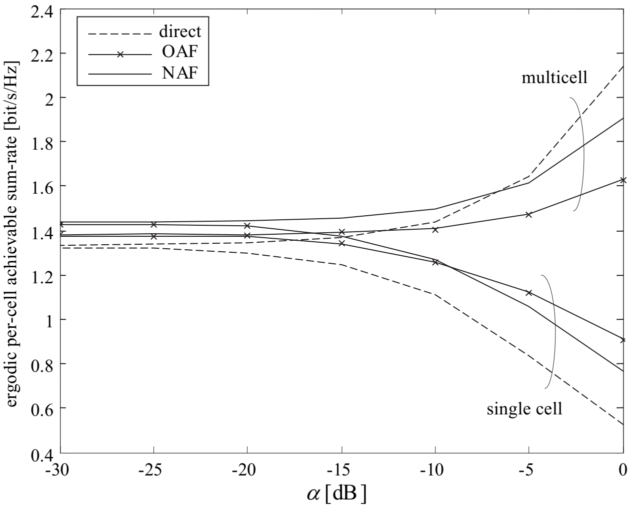

Fig. 7. Ergodic per-cell achievable rates of different transmission schemes for single and multicell processing versus the intercell gain α ($\mathrm{SNR} = 3dB$, $d = 0.5$, $\gamma = 3$).

techniques. These simplified assumptions allowed the computation of a closed form expression for the per-cell achievable rate on a non-faded scenario, that extends the main result in [3]. Based on the analysis and on numerical results, some general conclusions have been drawn:

1. non-regenerative (AF) collaboration is a useful means to improve the performance of the uplink channel for simple systems that employ separate processing at different BSs;

2. when collaboration between BSs is possible and effective (i.e., the intercell attenuation is small enough), multicell diversity renders the benefits of added diversity from non-regenerative relaying negligible and direct transmission can in fact outperform AF.

When interpreting these results, it should be stressed that some of the drawbacks of collaborative communications pointed out above are due to the simple strategy employed, namely AF. It is expected that with more refined forms of collaboration that avoid the noise amplification problem of AF, such as Decode and Forward and variations (see, e.g., [11]), some of the results could be significantly different. Further discussion on this point can be found in [15].

VIII. Appendix: Derivation of (26) and (27)

Using (24), the channel matrices in the first and second time-slot can be easily shown to read $\mathbf{H} = \mathbf{\Psi}_M([1,\ \alpha])$ and $\mathbf{H}' = \mu\beta\cdot\mathbf{\Psi}_M([\delta,\ \alpha])$, where, in order to ease the presentation, we use the notation $\mathbf{\Psi}_M(\mathbf{v})$ to identify a $M \times M$ symmetric band Toeplitz matrix with non-zero elements in the first row (or column) given by the $L \times 1$ vector $\mathbf{v}$, i.e., $[\mathbf{\Psi}_M(\mathbf{v})]_{1,i} = [\mathbf{\Psi}_M(\mathbf{v})]_{i,1} = [\mathbf{v}]_i$, $i = 1, ..., L$. Moreover, we have

$$\mu^2 = \frac{\mathrm{SNR}}{1 + \mathrm{SNR}\beta^2}. \qquad (29)$$

Notice that the correlation matrix $\mathbf{R}_z$ defined by (19) reads $\mathbf{R}_z = \mu^2 \cdot \mathbf{\Psi}_M([\delta^2 + 2\alpha^2,\ 2\alpha\delta,\ \alpha^2])$.

Let us first consider the achievable rate of OAF (21). This can be rewritten for AWGN channels as

$$R_{MC-OAF} = \frac{1}{2M}\left(\log_2\left|\bar{\mathbf{R}}_n + \mathrm{SNR}\cdot\bar{\mathbf{H}}\bar{\mathbf{H}}^H\right| - \log_2|\bar{\mathbf{R}}_n|\right). \qquad (30)$$

In the expression (30), both the noise correlation matrix $\bar{\mathbf{R}}_n$ and the channel correlation matrix $\bar{\mathbf{H}}\bar{\mathbf{H}}^H$ are not Toeplitz, and thus the results in [13] cannot be directly used in order to evaluate the rate (30), as in [3]. For the second term, $\log_2|\bar{\mathbf{R}}_n|$, this problem can be easily overcome by exploiting a property of the determinant of block matrices. Namely, given a block matrix $\mathbf{E} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$, if the matricial blocks $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ commute under matricial product, then the determinant $|\mathbf{E}|$ satisfies $|\mathbf{E}| = |\mathbf{AD} - \mathbf{BC}|$. Therefore, in our case we have $|\bar{\mathbf{R}}_n| = \left|\mathbf{\Psi}_M\left([\mu^2\delta^2 + 2\mu^2\alpha^2 + 1,\ 2\alpha\delta\mu^2,\ \alpha^2\mu^2]\right)\right|$ and using the results in [13] to the Toeplitz matrix we get

$$\lim_{M\to\infty}\frac{1}{2M}\log_2|\bar{\mathbf{R}}_n| = \frac{1}{2}\int_0^1 \log_2\left(s_0 + 2\sum_{n=1}^{2} s_n\cos(2n\pi\theta)\right)d\theta, \qquad (31)$$

with

$$s_0 = \delta^2\mu^2 + 2\alpha^2\mu^2 + 1,\ s_1 = 2\alpha\delta\mu^2,\ s_2 = \alpha^2\mu^2 \qquad (32)$$

On the other hand, in order to find an asymptotic closed form expression for the first term in (30), we need to leverage on some asymptotic properties of the product of symmetric band Toeplitz matrices, as detailed below.

Given two $M \times M$ symmetric band Toeplitz matrices $\mathbf{\Psi}_M(\mathbf{v}_1)$ and $\mathbf{\Psi}_M(\mathbf{v}_2)$ with $\mathbf{v}_1$ and $\mathbf{v}_2$ $L\times 1$ vectors, the product $\mathbf{\Psi}_M(\mathbf{v}_1)\mathbf{\Psi}_M(\mathbf{v}_2)$ is generally not Toeplitz and the commutative property, i.e., $\mathbf{\Psi}_M(\mathbf{v}_1)\mathbf{\Psi}_M(\mathbf{v}_2) = \mathbf{\Psi}_M(\mathbf{v}_2)\mathbf{\Psi}_M(\mathbf{v}_1)$ does not hold in general. However, for M that grows to infinity (for fixed L), following [13], it can be easily proved that $\mathbf{\Psi}_M(\mathbf{v}_1)\mathbf{\Psi}_M(\mathbf{v}_2)$ is asymptotically equivalent to the symmetric Toeplitz band matrix $\mathbf{\Psi}_M(<\mathbf{v}_1, \mathbf{v}_2>)$, where $<\mathbf{v}_1, \mathbf{v}_2>$ denotes the $(2L-1)\times 1$ vector containing the positive lags values of the cross-correlation between sequences $\{[\mathbf{v}_1]_M\ [\mathbf{v}_1]_{M-1}\cdots[\mathbf{v}_1]_1\ [\mathbf{v}_1]_2\cdots[\mathbf{v}_1]_M\}$ and $\{[\mathbf{v}_2]_M\ [\mathbf{v}_2]_{M-1}\cdots[\mathbf{v}_2]_1\ [\mathbf{v}_2]_2\cdots[\mathbf{v}_2]_M\}$. Asymptotic equivalence, rigorously defined in [13], essentially refers to the situation where two matrices are "indistinguishable" as the dimension M increases. In other terms, for increasing dimension M, the number of elements at which the two matrices differ becomes negligible. We will denote this property as follows

$$\mathbf{\Psi}_M(\mathbf{v}_1)\mathbf{\Psi}_M(\mathbf{v}_2) \mathrel{\mathring{=}} \mathbf{\Psi}_M(<\mathbf{v}_1, \mathbf{v}_2>). \qquad (33)$$

The same argument above applies for $\mathbf{\Psi}_M(\mathbf{v}_2)\mathbf{\Psi}_M(\mathbf{v}_1)$, i.e., $\mathbf{\Psi}_M(\mathbf{v}_2)\mathbf{\Psi}_M(\mathbf{v}_1) \mathrel{\mathring{=}} \mathbf{\Psi}_M(<\mathbf{v}_2, \mathbf{v}_1>) = \mathbf{\Psi}_M(<\mathbf{v}_1, \mathbf{v}_2>)$. Therefore, using the transitive property of equivalence we can conclude that, in a sense, symmetric band Toeplitz matrices commute under matricial product asymptotically in the dimension M, meaning that

$$\mathbf{\Psi}_M(\mathbf{v}_2)\mathbf{\Psi}_M(\mathbf{v}_1) \mathrel{\mathring{=}} \mathbf{\Psi}_M(\mathbf{v}_1)\mathbf{\Psi}_M(\mathbf{v}_2). \qquad (34)$$

Using the definitions and results presented above, it is straightforward to prove that

$$\bar{\mathbf{R}}_n + \mathrm{SNR}\bar{\mathbf{H}}\bar{\mathbf{H}}^H = \begin{bmatrix} \mathbf{\Psi}_{11} & \mathbf{\Psi}_{12} \\ \mathbf{\Psi}_{21} & \mathbf{\Psi}_{22} \end{bmatrix}, \qquad (35)$$

where we have defined for simplicity of notation $\boldsymbol{\Psi}_{11} = \boldsymbol{\Psi}_M([K_1,\, 2\alpha\mathrm{SNR},\, \alpha^2\mathrm{SNR}])$, $\boldsymbol{\Psi}_{12} = \mu\beta\mathrm{SNR}\cdot\boldsymbol{\Psi}_M([\delta+2\alpha^2,\, \alpha\delta+\alpha,\, \alpha^2])$, $\boldsymbol{\Psi}_{21} = \mu\beta\mathrm{SNR}\cdot\boldsymbol{\Psi}_M([\delta+2\alpha^2,\, \alpha\delta+\alpha,\, \alpha^2])$, $\boldsymbol{\Psi}_{22} = \mathrm{SNR}\boldsymbol{\Psi}_M([K_2,\, 2\alpha\delta,\, \alpha^2])$, $K_1 = \mathrm{SNR}+2\alpha^2\mathrm{SNR}+1$ and $K_2 = 2\alpha^2+\delta^2+\frac{1}{\mathrm{SNR}}$. Now, recalling the asymptotic properties (33) and (34) and using the properties on the determinant of block matrices introduced above, after straightforward but tedious manipulations we get

$$\lim_{M\to\infty}\frac{1}{2M}\log_2|\bar{\mathbf{R}}_n+\mathrm{SNR}\bar{\mathbf{H}}\bar{\mathbf{H}}^H| \tag{36}$$
$$= \frac{1}{2}\int_0^1 \log_2\left(\mathrm{SNR}\left(r_0+2\sum_{n=1}^{4} r_n\cos(2n\pi\theta)\right)\right)d\theta,$$

with

$$\begin{aligned} r_0 &= 2\alpha^4\mathrm{SNR}+8\alpha^2\delta\mathrm{SNR}+K_1K_2 \\ &\quad -\beta^2\mu^2\mathrm{SNR}\left(2\alpha^4+2\left(\alpha+\alpha\delta\right)^2+\left(\delta+2\alpha^2\right)^2\right) && \text{(37a)} \\ r_1 &= 2\alpha^3\mathrm{SNR}+2\alpha^3\delta\mathrm{SNR}+2\alpha\delta K_1+2\alpha K_2\mathrm{SNR}+ && \text{(37b)} \\ &\quad -\beta^2\mu^2\mathrm{SNR}\left(2\alpha^2\left(\alpha+\alpha\delta\right)+2\left(\alpha+\alpha\delta\right)\left(\delta+2\alpha^2\right)\right) \\ r_2 &= 4\alpha^2\delta\mathrm{SNR}+\alpha^2K_1+\alpha^2K_2\mathrm{SNR} \\ &\quad -\beta^2\mu^2\mathrm{SNR}\left(\left(\alpha+\alpha\delta\right)^2+2\alpha^2\left(\delta+2\alpha^2\right)\right) && \text{(37c)} \\ r_3 &= 2\alpha^3\mathrm{SNR}(1+\delta-\beta^2\mu^2(1+\delta)) && \text{(37d)} \\ r_4 &= \alpha^4\mathrm{SNR}(1-\beta^2\mu^2). && \text{(37e)} \end{aligned}$$

Finally, combining (31) and (36) in (30), the expression of the achievable sum-rate for OAF (26) is proved.

The derivation of the sum-rate for NAF (27) follows the same steps presented above for OAF. The resulting coefficients can be found to equal to:

$$\begin{aligned} u_0 &= 2\alpha^4\mathrm{SNR}+4\alpha^2\mathrm{SNR}\left(\delta+1\right)+K_1K_3 \\ &\quad -\frac{1}{2}\beta^2\mu^2\mathrm{SNR}\left(2\alpha^4+2\left(\alpha+\alpha\delta\right)^2+\left(\delta+2\alpha^2\right)^2\right) && \text{(38a)} \\ u_1 &= 2\alpha^3\mathrm{SNR}+\alpha^3\mathrm{SNR}\left(\delta+1\right)+\alpha\left(\delta+1\right)K_1+2\alpha\mathrm{SNR}K_3+ \\ &\quad -\frac{1}{2}\beta^2\mu^2\mathrm{SNR}\left(2\alpha^2\left(\alpha+\alpha\delta\right)+2\left(\alpha+\alpha\delta\right)\left(\delta+2\alpha^2\right)\right) && \text{(38b)} \\ u_2 &= 2\alpha^2\mathrm{SNR}\left(\delta+1\right)+\alpha^2K_1+\alpha^2\mathrm{SNR}K_3 \\ &\quad -\frac{1}{2}\beta^2\mu^2\mathrm{SNR}\left(\left(\alpha+\alpha\delta\right)^2+2\alpha^2\left(\delta+2\alpha^2\right)\right) && \text{(38c)} \\ u_3 &= \alpha^3\mathrm{SNR}(3+\delta-\beta^2\mu^2(1+\delta)) && \text{(38d)} \\ u_4 &= \alpha^4\mathrm{SNR}\left(1-\frac{1}{2}\beta^2\mu^2\right) && \text{(38e)} \end{aligned}$$

with $K_3 = \frac{1}{2}+2\alpha^2+\frac{1}{2}\delta^2+\frac{1}{\mathrm{SNR}}$. Moreover, we obtain:

$$v_0 = \frac{1}{2}\mu^2\delta^2+\mu^2\alpha^2+1,\; v_1 = \alpha\delta\mu^2,\; v_2 = \frac{1}{2}\mu^2\alpha^2. \tag{39}$$

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