Newton’s Laws

I. If no net force acts on a body, then the body’s velocity cannot change.

II. The net force on a body is equal to the product of the body’s mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction ($F_{12} = -F_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration:

\[
\overrightarrow{F}_{\text{tot}} = 0 \iff \overrightarrow{a} = 0 \text{ (constant velocity)}
\]

\[
\overrightarrow{F}_{\text{tot}} = m\overrightarrow{a} \text{ for any object}
\]

\[
F_{\text{tot},x} = ma_x \quad F_{\text{tot},y} = ma_y \quad F_{\text{tot},z} = ma_z
\]

---

Forces:

- **Gravitational Force:** $\overrightarrow{F}_g = mg$ down to the ground
- **Tension Force:** $\overrightarrow{T}$ along the string
- **Spring Force:** $F_s = -kx$
- **Normal Force:** $\overrightarrow{N}$ perpendicular to the support
- **Friction Force**
  - Static: maximum value $f_s = \mu_{st}N$
    - opposite to the component of other forces parallel to the support
  - Kinetic: value $f_k = \mu_{kin}N$
    - opposite to the velocity, parallel to the support

\[
\mu_{st} > \mu_{kin}
\]
Relative Motion/Reference Frames

Relative Velocity: Rowing a Boat

You can row a boat at $v_{\text{row}} = 3 \text{ m/s}$, and you want to go straight across a river which flows with $v_{\text{river}} = 2 \text{ m/s}$. At what angle should you row?

$$\vec{v}_{\text{boat}} = \vec{v}_{\text{row}} + \vec{v}_{\text{river}}$$

you want $\vec{v}_{\text{boat}}$ in y-direction to go straight across.

Rowing a Boat (continued)

$$\vec{v}_{\text{boat}} = \vec{v}_{\text{row}} + \vec{v}_{\text{river}}$$

you want $\vec{v}_{\text{boat}}$ in y direction

$$\vec{v}_{\text{row}} = \vec{v}_{\text{boat}} \cos \theta$$

need $v_{\text{row},x} = -v_{\text{river},x}$

$48.2$ degrees; $v_{\text{boat}} = 2.2 \text{ m/s}$

Inertial Frames:

$$T = mg$$
Lecture 7a Andrei Sirenko, NJIT

Non-inertial Frame
Pseudo Forces

Inertial Frame

Inertial Frame;
There are no Pseudo Forces:
\[ T \cos \theta = mg \]
\[ T \sin \theta = ma \]
\[ a = g \cdot \tan \theta \]

Non-Inertial Frame;
There is a Pseudo Force: \( ma \)
Newton's Laws do not work !!!

Combination of Forces:
Net Force
Dealing with Multiple Forces

If multiple forces are acting on the same object, the net force determines the acceleration.

\[ F_{\text{net}} = F_1 + F_2 \]
\[ F_{\text{net}} = ma \]

Use a free body diagram to keep track of the forces on one object.
Uniform Circular Motion

Centripetal acceleration

\[ a = \frac{v^2}{r} \]

Period

\[ T = \frac{2\pi r}{v} \]

Centripetal force: \( F = ma \)

\[ F = \frac{mv^2}{r} \]

Top view:

Net Force and Centripetal Force

Centripetal force is a combination of:

- Gravitational Force: \( mg \) down to the ground
- Tension Force: \( T \) along the string
- Normal Force: \( N \) perpendicular to the support
- Static Friction Force maximum value \( F_{fr}^{max} = \mu_s N \)

Net Force and Centripetal Force:

\[ ma = T \]

\[ ma = \frac{mv^2}{R} = T \]
Centripetal Force and Kinetic Friction Force:

Kinetic friction does not affect Centripetal acceleration directly

Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.
What is her centripetal acceleration?

\[ v = 100 \text{ m} / 12 \text{ s} = 8.33 \text{ s}; \quad R = 100/\pi = 31.8 \text{ m} \]
\[ a = (8.33)^2/31.8 \text{ m/s}^2 = 2.2 \text{ m/s}^2 \]

Uniform Circular Motion

Centripetal acceleration

\[ a = \frac{v^2}{r} \]
Period

\[ T = \frac{2\pi r}{v} \]

Problem #1

m = 5 kg
L = 5 m
R = 2 m
Find v, T, and a
Problem solving tactics:

\[ m = 5 \text{ kg} \]
\[ L = 5 \text{ m} \]
\[ R = 2 \text{ m} \]

Find \( v, T, \) and \( a \)

\[ \sin \theta = \frac{R}{L} = 0.4; \quad \tan \theta = \frac{R}{L}/\left(1-(\frac{R}{L})^2\right)^{\frac{1}{2}} = 0.44 \]

\[ X: \quad ma = T \cdot \sin \theta \]
\[ Y: \quad ma = 0 = -mg + T \cdot \cos \theta \]

\[ ma = mg \cdot \sin \theta / \cos \theta = mg \cdot \tan \theta \]
\[ T = mg / \cos \theta \quad a = g \cdot \tan \theta \]
\[ v = (aR)^{\frac{1}{2}} \]

Centripetal Force originates from the tension force!

Circular motion:

\[ ma = mv^2/R \]
\[ a = v^2/R \]

\[ T = 5 \text{ kg} \cdot 9.8 \text{ m/s}^2 / (1-(2m/5m)^2)^{\frac{1}{2}} = 53 \text{ N} \]
\[ a = 4.3 \text{ m/s}^2; \quad ma = 5 \text{ kg} \cdot 4.3 \text{ m/s}^2 = 21 \text{ N}; \quad v = (4.3 \cdot 2)^{\frac{1}{2}} \text{ m/s} = 2.9 \text{ m/s} \]
Net Force and Centripetal Force

Problem #2

\[ R = 20 \text{ m}; \ \mu_{st} = 0.5 \]
Angle 10°
Find \( v_{\text{max}} \)

\[ R \]

\[ \theta \]

\[ F_{\text{centripetal}} = \frac{m v^2}{r} \]

\[ v^2 = \frac{r}{m} \]
in the centripetal acceleration

\[ F = m \frac{v^2}{r} \]

\[ \mu_{st} \]

\[ N \]

\[ \theta \]

\[ X: \ ma = N \sin \theta + \mu_{st} N \cos \theta \]

\[ Y: \ 0 = N \cos \theta - mg - \mu_{st} N \sin \theta \]
Problem #2

\( R = 20 \text{ m}; \ \mu_{st} = 0.5 \)

Angle 10°

Find \( v_{\text{max}} \)

\[ \begin{align*}
X: & \quad ma = N \left( \sin \theta + \mu_{st} \cos \theta \right) \\
Y: & \quad N = mg / \left( \cos \theta - \mu_{st} \sin \theta \right) \\
ma & = m \frac{v_{\text{max}}^2}{R}
\end{align*} \]

\( ma = v_{\text{max}}^2 / R \)

\( m v_{\text{max}}^2 / R = mg \left( \sin \theta + \mu_{st} \cos \theta \right) / \left( \cos \theta - \mu_{st} \sin \theta \right) \)

\( v_{\text{max}} = \frac{12.8 \text{ m/s}}{29 \text{ m/s}} \approx 29 \text{ m/s} \)

Mass \( m \) disappeared !!!

QZ # 7 Analyze the previous problem

1. \( R = 20 \text{ m}; \ \mu_{st} = 0.5; \ \text{Angle} \ 10° \)
What is going to happen to the static friction force for the case when the velocity of the track is doubled: \( v = v_{\text{max}} \times 2 \)
What is going to happen to the track? (describe)

2. \( R = 20 \text{ m}; \ \mu_{st} = 0.5, \ v = 5 \text{ m/s}; \ m = 3000 \text{ kg}, \ \text{and} \ \theta = 0 \)
What is the value and direction of the static friction force?
9. Two blocks (X and Y) are in contact on a horizontal frictionless surface. A 36-N constant force is applied to X as shown. The magnitude of the force exerted by Y on X is:

\[ a = \frac{F}{m_x + m_y} = \frac{36N}{20kg + 40kg} = 1.5 \text{ m/s}^2 \]

\[ F_y = 36N - 4 \times 1.5 \times 1.5 = 30N \]

A) 15 N  
B) 6.0 N  
C) 20 N  
D) 30 N  
E) 36 N

11. A block is at rest on a horizontal plank of wood. The plank is slowly lifted at one end while the other end stays on the floor. If the coefficient of static friction between the block and the plank is 0.5, what is the steepest angle the plank can have before the block begins to slide without being pushed?

\[ \theta = \tan^{-1} \frac{f_s}{m g} = \tan^{-1} 0.5 = 26.5^\circ \]

A) 22^\circ  
B) 37^\circ  
C) 45^\circ  
D) 53^\circ  
E) 59^\circ

13. Two blocks with the weights of 70-N and 35-N are connected by a string as shown. If the pulley is massless and the surface is frictionless, the magnitude of the acceleration of the 70-N block is:

A) 1.6 m/s^2  
B) 3.3 m/s^2  
C) 4.9 m/s^2  
D) 6.7 m/s^2  
E) 9.8 m/s^2

\[ \text{Acceleration should have the same magnitude for both blocks} \]

\[ a = \frac{mg}{m_1 + m_2} = \frac{35N}{70kg + 35kg} = 0.5 \text{ m/s}^2 \]

14. A horizontal force F is gradually increased until the 40 kg block begins moving to the right. The 10 kg block cannot move because of the cord attaching it to the wall at left. For what force F does the lower block just start to move?

\[ F = f_s + f_{s+2} \]

\[ f_s = (10 \times g + 40 \times g) \times 0.03 \]

\[ f_{s+2} = 10 \times g 	imes 0.15 \]

\[ F \geq 50kg \times 9.8 m/s^2 \times 0.03 + 10kg \times 9.8 m/s^2 \times 0.15 = 161.7 N \]