Cyclotron resonance and Hall effect in semiconductors
with both types of carriers

Two cyclotron frequencies:

\[ \omega_{ce} = \frac{eB}{m_e} \quad \text{for electrons} \]
\[ \omega_{ch} = \frac{eB}{m_h} \quad \text{for holes} \]

Cyclotron resonance is used to obtain information on effective masses/shape of energy surfaces

Suppose the constant energy surface is an ellipsoid in revolution.
\( \mathbf{B} \) is applied at some angle

The cyclotron frequency is

\[ \omega_{c} = eB \left[ \frac{\cos^2 \theta}{m_t} + \frac{\sin^2 \theta}{m_r m_t} \right]^{1/2} \]

-depends on effective masses and angle \( \theta \)
measuring \( \omega_{c} \) at various angles gives the effective masses

Lecture 10

High electric field and hot electrons

Electron drift velocity in Ge vs. electric field for different crystallographic orientations at 300 K (from Landolt-Boernstein - A. Neukermans, G. S. Kino, Phys. Rev. B 7 2693 (1973)).

\[ J = nev = ne \mu_e \mathcal{E} \]
\[ \frac{dE}{dt} = \left( \frac{dE}{dt} \right)_L + \left( \frac{dE}{dt} \right)_H \]
\[ = -e\mathcal{E}v - \frac{E(T_e) - E(T)}{\tau_E} = 0 \]
\( \tau_E \) - energy relaxation time
\( v \) - electron drift velocity

\[ E(T_e) = \frac{3}{2} k_B T_e \quad E(T) = \frac{3}{2} k_B T \]
\[ \Rightarrow \quad T_e = T + \frac{2}{3} \frac{e^2 \mathcal{E} \mu_e}{k_B} \]

- used to determine carrier concentration and mobility
Negative differential conductance and Gunn effect

Conduction band in GaAs

• In the lower $\Gamma$ valley, electrons exhibit a small effective mass and very high mobility, $\mu_1$.
• In the satellite L valley, electrons exhibit a large effective mass and very low mobility, $\mu_2$.
• The two valleys are separated by a small energy gap, $\Delta E$, of approximately 0.31 eV.

Optical absorption processes

1. The fundamental absorption process

   direct process at $k = 0$ (zone center)
   powerful method to determine the energy gap $E_g$

2. Exciton absorption

   fundamental absorption (theory)
3. Free carrier absorption

Intraband transition – like in metals
can occur even when the photon energy is below the bandgap
depends on free carrier concentration – more significant in doped semiconductors

4. Absorption involving impurities

- Neutral donor → conduction band
- Valence band → neutral acceptor
- Valence band → ionized donor
- Ionized acceptor → ionized donor

Photoconductivity

Phenomenon in which a material becomes more conductive due to the absorption of electromagnetic radiation

"Dark" conductivity: \( \sigma_0 = e(n_0\mu_e + p_0\mu_h) \)

Light absorbed: electron-hole pairs created; carrier concentrations increased by \( \Delta n, \Delta p \)

New conductivity: \( \sigma = \sigma_0 + e\Delta n (\mu_e + \mu_h) \)

\( \frac{\Delta \sigma}{\sigma_0} = \frac{\sigma - \sigma_0}{\sigma_0} = \frac{e\Delta n (\mu_e + \mu_h)}{\sigma_0} \)

Two opposite processes affecting \( \Delta n \):
- Generation of free carriers due to absorption, rate \( g \)
  \[ \frac{dn}{dt} = g - \frac{n - n_0}{\tau'} \]
- Recombination; lifetime of carriers \( \tau' \)

In steady state \( \frac{dn}{dt} = 0 \Rightarrow \Delta n = n - n_0 = gt' \)

Evaluate \( g \) per unit volume through absorption coefficient \( \alpha \) and slab thickness \( d \):

\[ g = \frac{\alpha d N(\omega)}{V} \]

\( N(\omega) \) – number of photons incident per unit time: \( N(\omega) = \frac{I(\omega)A}{\hbar\omega} \)

Then

\[ \Delta n = \frac{\alpha I(\omega)}{\hbar\omega \tau'} \]

Change in conductivity:

\[ \frac{\Delta \sigma}{\sigma_0} = e \frac{\alpha I(\omega)\tau'(\mu_e + \mu_h)}{\hbar\omega \sigma_0} \]

\( \frac{\Delta \sigma}{\sigma_0} \propto \alpha \) and \( \frac{\Delta \sigma}{\sigma_0} \propto I(\omega) \)

Numerical estimate: if \( \tau' \approx 10^{-4} \) s, \( I \approx 10^{-4} \) watts/cm², and \( \hbar\omega \approx 0.7 \) eV
(for Ge), get \( \Delta n \approx 5 \times 10^{14} \) cm⁻³
**Luminescence**

Radiative recombination of charge carriers

Classification by excitation mechanisms:
- photoluminescence
- electroluminescence
- cathodoluminescence
- thermoluminescence
- chemiluminescence

Same physical processes as for absorption, but in opposite direction

**Summary**

- Conductivity of semiconductors: \( \sigma = ne\mu_e + pe\mu_h \)
  - mobility: \( \mu_e = \frac{e\tau_e}{m_e} \)
- Cyclotron resonance is used to obtain information on effective masses.
- Hall coefficient: \( R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(n\mu_e + p\mu_h)^2} \) Hall measurements are used to determine carrier concentration and mobility.
- In high electric field, the carriers acquire significant energy and become "hot". This affects mobility and can cause current instabilities (e.g. Gunn effect caused by negative differential conductivity due to inter-valley transfer)
- Mechanisms of optical absorption and luminescence.
  - Fundamental absorption occurs above the bandgap.
- photoconductivity – increase of conductivity by generation of additional carriers by electromagnetic radiation

**Carrier Diffusion**

In general, total current in a semiconductor involves both electrons and holes (in the presence of both a concentration gradient and an electric field):

\[
\vec{j}_n = ne\mu_n \vec{E} + eD_n \nabla n
\]

\[
\vec{j}_p = pe\mu_p \vec{E} - eD_p \nabla p
\]

The second term in the above equations is the diffusion current (Fick’s law). It arises from non-uniform carrier density.

In one dimension, for the negative carrier:

\[
j_n = ne\mu_n E + eD_n \frac{\partial n}{\partial x}
\]

At equilibrium, the drift and diffusion currents are equal:

\[
j_n = 0 \Rightarrow 0 = ne\mu_n E + eD_n \frac{\partial n}{\partial x}
\]

\[
\Rightarrow \text{can write } e \frac{\partial V}{\partial x} = - \frac{\partial E_z}{\partial x}
\]

**Electric field**  
\( E = - \frac{\partial V}{\partial x} \)  
(V - potential)

By applying a field, all energies will be pushed up by the potential \( V \):
Have
\[ n \sim N_e e^{\frac{\mu E_c}{k_B T}} \Rightarrow \frac{\partial n}{\partial x} = \frac{\partial n}{\partial E_c} \frac{\partial E_c}{\partial x} = \frac{N_e}{k_B T} e^{\frac{\mu E_c}{k_B T}} \frac{\partial E_c}{\partial x} = \frac{n}{k_B T} \frac{\partial E_c}{\partial x} = -\frac{e n E}{k_B T} \]

Substitute this into the diffusion equation,
\[ \frac{\partial p}{\partial t} = n e \mu_n E + e D_n \frac{\partial n}{\partial x} \]
Get
\[ n e \mu_n E - \frac{e^2 n D_n E}{k_B T} = 0 \Rightarrow D_n = \frac{n e \mu_n k_B T}{e} \text{ Einstein relation} \]

Similarly, for holes
\[ D_p = \frac{n e \mu_p k_B T}{e} \]

1) **Stationary solution for** \( E = 0 \):
\[ \frac{\partial p}{\partial t} = 0 \]
\[ D_p \frac{\partial^2 p}{\partial x^2} - \frac{p - p_0}{\tau'_p} = 0 \]
let \( p - p_0 = p_1 \). Then \( p_1 = p - p_0 = Ae^{-\gamma (D_p \tau'_p)^{1/2}} \)
The excess concentration decays exponentially with \( x \).
The distance \( L_D = (D_p \tau'_p)^{1/2} \) is called the diffusion length
Effective diffusion velocity:
\[ v_D = \frac{L_D}{\tau'_p} = \left( \frac{D}{\tau'_p} \right)^{1/2} \]
Diffusion current:
\[ J_D = e p_1 v_D = e p_1 \left( \frac{D}{\tau'_p} \right)^{1/2} \]

**Diffusion equation for one carrier type**
\[ J_p = -e D_p \frac{\partial p}{\partial x} + pe \mu_p E \text{ (holes, one dimension)} \]

Variation of \( p(x) \) in time is given by continuity equation:
\[ \frac{\partial p}{\partial t} = G_p - U_p - \frac{1}{e} \frac{\partial J_p}{\partial x} \]

Recombination term:
\[ U_p = \left( \frac{\partial p}{\partial t} \right)_{\text{Recomb}} = -\frac{D p - p_0}{\tau'_p} \]
Then
\[ \frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \frac{\partial (p E)}{\partial x} - \frac{p - p_0}{\tau'_p} \text{ - Diffusion equation} \]

1) **Stationary solution for a uniform field** \( E \neq 0 \):
\[ \frac{\partial^2 p}{\partial x^2} - \frac{\mu_p E}{D_p} \frac{\partial p_1}{\partial x} - \frac{p_1}{L_D^2} = 0 \Rightarrow p_1 = Ae^{-\gamma s \sqrt{L_D}} \]
Where \( \gamma = \sqrt{1 + s^2} - s \) and \( s = \frac{\mu_p E L_D}{2D_p} \)
\( \gamma < 1 \Rightarrow \text{effective diffusion length } L_D/\gamma \text{ is larger} \)
Summary of the semiconductors section

- Semiconductors are mostly covalent crystals; They are characterized by moderate energy gap (~0.5 – 2.5 eV) between the valence and conduction bands.
- When impurities are introduced, additional states are created in the gap. Often these states are close to the bottom of the conduction band or top of the valence band.
- Intrinsic carrier concentration: strongly depends on temperature.
- Fermi level position in intrinsic semiconductor: 
  \[ \frac{E_F}{k_B T} \left( \frac{m_e m_h}{2 \pi \hbar^2} \right) \left( m_e m_h \right)^{3/4} e^{-E_F/(2k_BT)} = p = n_i \]

Summary of the semiconductors section

- In a doped semiconductor, many impurities form shallow hydrogen-like levels close to the conductive band (donors) or valence band (acceptors), which are completely ionized at room T:
  \[ n = N_d \quad \text{or} \quad p = N_a \]
- Conductivity of semiconductors: \[ \sigma = n e \mu_e + p e \mu_h \]
- Mobility: \[ \mu_e = \frac{e \tau_e}{m_e} \quad \mu_h = \frac{e \tau_h}{m_h} \]
- Magnetic field effects:
  - Cyclotron resonance is used to obtain information on effective masses.
  - Hall coefficient: \[ R_H = \frac{p \mu_h^2 - n \mu_e^2}{e(n \mu_e + p \mu_h)^2} \]
  - Hall measurements are used to determine carrier concentration and mobility.

Summary of the semiconductors section

- In high electric field, the carriers acquire significant energy and become "hot". This affects mobility and can cause current instabilities (e.g. Gunn effect caused by negative differential conductivity due to inter-valley transfer).
- Mechanisms of optical absorption and luminescence:
  - band-to-band
  - excitonic
  - free carrier
  - impurity-related

Fundamental absorption occurs above the bandgap.
- photoconductivity – increase of conductivity by generation of additional carriers by electromagnetic radiation
- Diffusion. Basic relations are Fick's law and the Einstein relation

Basics of selected semiconductor devices:

- p-n junctions.
- Bipolar transistors.
- Tunnel diodes.
- Semiconductor lasers
Charge density near the junction is not uniform: electrons (majority carriers) from the n-side and holes (majority carriers) from the p-side will migrate to the other side through the junction.

These migrated particles leave the ionized impurities behind: a charged region is formed.

In equilibrium, at zero bias, the chemical potential has to be the same at both sides: bending of the conduction and valance bands

Before junction is formed:

\[ E_c \quad \mu \]

\[ E_v \quad \mu \]

After junction is formed:

\[ E_c \quad \mu \]

\[ E_v \quad \mu \]

The band edge shift across the junction is called the built-in voltage \( V_{bi} \).

Recall that for carrier concentration we had

\[
n = 2 \left( \frac{m_e kT}{2 \pi \hbar^2} \right)^{3/2} e^{(\mu - E_c)/kT} = N_c e^{(\mu - E_c)/kT}
\]

\[
p = 2 \left( \frac{m_h kT}{2 \pi \hbar^2} \right)^{3/2} e^{(E_v - \mu)/kT} = N_v e^{(E_v - \mu)/kT}
\]

For \( n \)-type semiconductor \( (n, N_D \gg p, N_A) \) \( n = N_D \Rightarrow \mu_n = E_c - kT \ln \frac{N_c}{N_D} \)

For \( p \)-type semiconductor \( (p, N_A \gg n, N_D) \) \( p = N_A \Rightarrow \mu_p = E_v + kT \ln \frac{N_v}{N_A} \)

Use the above to calculate \( V_{bi} \)

\[
\mu_n = \mu_p \Rightarrow E_{cn} - kT \ln \frac{N_c}{N_D} = E_{cp} - E_g + kT \ln \frac{N_v}{N_A} \Rightarrow
\]

\[
V_{bi} = E_{cp} - E_{cn} = E_g - kT \ln \frac{N_c}{N_D} - kT \ln \frac{N_v}{N_A} = E_g + kT \ln \frac{N_A N_D}{N_c N_v}
\]

Note that

\[
n_i^2 = 4 \left( \frac{m_e kT}{2 \pi \hbar^2} \right)^{3/2} \left( \frac{m_h kT}{2 \pi \hbar^2} \right)^{3/2} e^{-E_g/kT} = N_c N_v e^{-E_g/kT}
\]

\[
\Rightarrow \quad V_{bi} = kT \ln \frac{N_A N_D}{n_i^2}
\]
Reverse bias

draws electrons and holes away from the n-side and holes from the p-side.
The depletion width grows and the junction resistance increases.

Forward bias

“pushes” electrons in the n-side and holes in the p-side towards the junction. The depletion width will become thinner \(\rightarrow\) current flows

I-V characteristics of a p-n junction

Assume positive \(V\) when it is forward bias:

There are two currents from two types of majority carriers, \(j_n\) and \(j_p\)

\(E \sim 0\) at area outside the depletion layer \(\Rightarrow\) mostly diffusion current outside the depletion layer.

Diffusion current needs inhomogeneity in carrier density. This is indeed the case because of recombination.

Can write (subscripts indicate \(n\) and \(p\) sides, respectively):

\[
\frac{\partial p_n}{\partial x} = \frac{\partial n_n}{\partial x} \quad \frac{\partial p_p}{\partial x} = \frac{\partial n_p}{\partial x}
\]

Current equation in neutral region (i.e. away from the depletion layer) is given by the continuity equations:

\[
\frac{\partial n}{\partial t} = G_n - U_n + \frac{1}{e} \frac{\partial J_n}{\partial x} \quad \frac{\partial p}{\partial t} = G_p - U_p - \frac{1}{e} \frac{\partial J_p}{\partial x}
\]

For steady case, \(\frac{\partial n}{\partial t} = \frac{\partial p}{\partial t} = 0\)

Current equations (last lecture):

\[
\frac{\partial n}{\partial t} = n e \mu_n \bar{E} + e D_n \nabla n \quad \frac{\partial p}{\partial t} = p e \mu_p \bar{E} - e D_p \nabla p
\]

\(E \sim 0\) outside the depletion layer \(\Rightarrow\) \(j_n = e D_n \frac{\partial n}{\partial x} \quad j_p = -e D_p \frac{\partial p}{\partial x}\)

Combine these with the continuity equations:

\[
D_n \frac{\partial^2 n}{\partial x^2} + G_n - U_n = 0 \quad D_p \frac{\partial^2 p}{\partial x^2} + G_p - U_p = 0
\]
Sufficient to solve only one of these equations, because
\[ \frac{\partial n}{\partial x} = \frac{\partial p}{\partial x} \Rightarrow j_n = -\frac{D_n}{D_p} j_p = \frac{D_n}{D_p} \]
Also, assume there is no external excitation, i.e., \( G_n = G_p = 0 \).

The second equation becomes (last lecture):
\[ D_p \frac{\partial^2 p_n}{\partial x^2} - \frac{p_n - p_{n0}}{\tau^*_p} = 0 \]

General solution:
\[ p_n = \frac{A}{\sqrt{D_p\tau^*_p}} e^{x/(D_p\tau^*_p)^{1/2}} + B e^{-x/(D_p\tau^*_p)^{1/2}} + p_{n0} \]

Now consider the boundary condition in solving this equation.

In the depletion layer,
At n side: \( n = N_c e^{(\mu_n - E_g)/k_BT} \)
\[ np = N_c N_v e^{(\mu_n - \mu_p - E_g)/k_BT} \]
At p side: \( p = N_v e^{(E_v - \mu_p)/k_BT} \)

The current depends on \( x \) because of the recombination process.
The current through the diode depends on the geometry (e.g. length) of the diode.

For simplicity, we can define the current as the current at the depletion layer \( (x=x_n) \) because the depletion layer is thin and there is not too many recombination in this region, i.e. \( L_p >> x_n + x_p \), and similarly \( L_n >> x_n + x_p \).

Current at the depletion layer boundary in the n-side:
\[ j \bigg|_{x-x_n} = \frac{e(D_p - D_n)p_{n0}}{L_p} \left[ e^{x_{eV}/k_BT} - \frac{x_{eV}}{L_p} \right] \]
Similarly calculate current at the depletion layer boundary in the p-side
\[ j \bigg|_{x=x_p} = \frac{e(D_n - D_p)p_{n0}}{L_p} \left[ e^{x_{eV}/k_BT} - \frac{x_{eV}}{L_p} \right] \]
Total current: \[ j = j_s \left[ e^{\frac{eV}{k_BT}} - 1 \right] \]

where \[ j_s = \frac{e(D_p - D_n)p_{n0}}{L_p} + \frac{e(D_n - D_p)n_{p0}}{L_p} \left( \frac{e}{L_p} \right) (D_p - D_n) \]

- saturation current

Omar uses another (equivalent) form:

\[ (j_p)_{x=x_p} = \frac{eD_pP_{n0}}{L_p} \left[ e^{\frac{eV}{k_BT}} - 1 \right] \quad (j_n)_{x=x_n} = \frac{eD_nn_{p0}}{L_n} \left[ e^{\frac{eV}{k_BT}} - 1 \right] \]

So the total current is \[ j = j_s \left[ e^{\frac{eV}{k_BT}} - 1 \right] \]

where the saturation current is \[ j_s = e \left( \frac{D_n n_{p0}}{L_n} + \frac{D_p P_{n0}}{L_p} \right) \]

or, since \[ n_{n0}P_{n0} = n_{p0}P_{p0} = n_i^2 \]

\[ j_s = e n_i^2 \left( \frac{D_n}{L_n P_{p0}} + \frac{D_p}{L_p n_{n0}} \right) \]

- saturation current in terms of majority career concentrations

Since \[ n_i^2 = N_c N_v e^{\frac{-E_v}{k_BT}} \]

\( j_s \) may be reduced by choosing larger bandgap material.

Bipolar junction transistor

Emisor circuit – forward biased
Collector circuit – reverse biased

Emitter current:
\[ I_e = I_{e0} e^{\frac{eV}{kT}} \]

injects holes into the base (n-region)

Holes diffuse through the base; some of them decay.

Collector current: \[ I_c = I_{e0} + \alpha I_e \] \( \alpha \) - fraction of holes that survive

\( I_{e0} \) is very small \( \rightarrow \) ca write \( I_c \approx \alpha I_e \)

Voltage drop across the load is: \[ V_l = \alpha R_l I_e \] amplification

Voltage gain \[ \frac{dV_l}{dV_e} = \frac{dV_l}{dI_e} \frac{dI_e}{dV_e} = \frac{\alpha R_l I_e}{kT/e} \]

Power gain:
\[ \frac{dP_i}{dP_e} = \frac{2\alpha^2 R_l I_e}{1 + \ln(I_c/I_{e0})kT/e} \]

If we take \( I_e = 10 \text{ mA}, I_{e0} = 10 \mu\text{A}, kT = 25 \text{ meV at 300 K, } \alpha \approx 1, \)
and \( R_l = 2 \text{ k}\Omega, \)

get voltage and power gains \( \approx 800 \) and \( 200, \) respectively.

Fundamental limitation of bipolar junction transistor – low frequency – determined by diffusion of holes (electrons in npn case) into the base

The high-frequency limit beyond which the device cannot function properly, usually lies in the range of tens – hundreds of MHz

Other types of transistors are needed for higher-frequency range
**Tunnel diode** (very high doping levels)

- **a)** equilibrium
- **b)** Reverse bias - large tunneling current
- **c)** Small forward bias - some tunneling current
- **c)** Large forward bias - no tunneling current

↔️ I-V characteristics

Tunneling process is very fast – can operate at high frequencies (e.g. 10 GHz)

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**Summary of the semiconductors section**

- **p-n junction:** both electrons and holes diffuse across the junction – potential barrier develops, called *built in voltage* $V_{bi}$:
  \[
  V_{bi} = kT \ln \frac{N_A N_D}{n_i^2}
  \]

- The junction acts as rectifier. The current vs applied voltage $V$ is
  \[
  I = I_0 (e^{V/eV_T} - 1) \quad \text{Forward: } I \approx I_0 e^{V/eV_T} \quad \text{Reverse: } I \approx -I_0
  \]

- Bipolar junction transistor – two back to back junctions: emitter is forward biased, collector is reverse biased

  Works as amplifier: when a signal is applied at the emitter, a current pulse passes through the base-collector circuit. The voltage gain is:

  \[
  \frac{dV_I}{dV_e} = \frac{dI_e}{dV_e} = \frac{aR_I I_e}{kT/e}
  \]

- Tunnel diode is realized when the doping levels in a p-n junction are very high, so the junction width is very small – tunneling occurs.

  A region of negative differential resistance exists in forward bias.

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**Gunn diode**

**Gunn effect** (discovered by J.B Gunn in 1963):

Above some critical voltage, corresponding to $E$-field of 2 - 4 kV/cm (in GaAs), the current becomes an oscillating function of time.

Cause for this behavior – negative differential resistance

Gunn "diode" is a bulk device:

Oscillation frequency is given by the transit time of electrons through the device:

\[
 f_0 = \frac{v_d}{L}
\]

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**Gunn-Oscillation**

Assume that a small local perturbation in the net charge arises at $t = t_0$

This results in non-uniform electrical field distribution

The electrons at point A, experiencing an electric field $E_{L1}$, will now travel to the anode with velocity $v_A$.

The electrons at point B is subjected to an electrical field $E_{H1}$. They will therefore drift towards the anode with velocity $v_2 < v_A$.

⇒ The initial charge perturbation will therefore grow into a dipole domain, known as a *Gunn domain*.

Gunn domains will grow while propagating towards the anode until a stable domain has been formed.

www2.hlphys.uni-linz.ac.at/mmm/uebungen/gunn_web/gunn_effect.htm
Body is commonly tied to ground (0 V)

• When the gate is at a low voltage
  – p-type body is at low voltage
  – Source-body and drain-body diodes are OFF (reverse bias)
  – Depletion region between n and p bulk: no current can flow, transistor is OFF

• When the gate is at a high voltage
  – Positive charge on gate of MOS capacitor
  – Negative charge attracted to oxide in the body (under the gate)
  – Inverts channel under the gate to n-type
  – Now current can flow through this n-type channel between source and drain
  – Transistor is ON

Emission of Light by Semiconductor Diodes

In a forward-biased p-n junction fabricated from a direct band gap material, the recombination of the electron-hole pairs injected into the depletion region causes the emission of electromagnetic radiation - a light emitting diode

If mirrors are provided and the concentration of the electron hole pairs (called the injection level) exceeds some critical value → a semiconductor laser

Edge-emitting laser  vertical cavity surface-emitting laser