Superconductivity is a phenomenon occurring in certain materials at low temperatures, characterized by exactly zero electrical resistance. Discovered in 1911 by Heike Kamerlingh Onnes - the resistance of solid mercury abruptly disappeared at the temperature of 4.2 K.

\[ \rho = \frac{m}{ne^2\tau} \]

Zero \( \rho \) means there are some electrons with infinite \( \tau \) – no scattering.

**Superconducting elements**

- More than 20 metallic elements are superconductors
- Cu, Au, Ag, Na, K and magnetically ordered metals (Fe, Ni, Co) are not superconductors
- Certain elements are superconducting at high pressures or as thin films
- Highest \( T_c \) of an element is 9.3 K for Nb
- There are thousands of alloys and compounds that exhibit superconductivity
- The highest \( T_c \) superconductors tend to be poor conductors in the normal state
- Record \( T_c \) is currently \( \sim 138 \) K (a ceramic consisting of Ti, Hg, Cu, Ba, Ca, Sr, and O)

**Persistent current in a superconductor**

Most sensitive method of measuring a small resistance;

\[ V = -L \frac{dI}{dt} = IR \]

\[ I(t) = I_0 e^{-\frac{R}{L}t} \]

This gives an upper value on the value of \( R \):

Resistivity \( \rho < 10^{-26} \, \Omega \cdot \text{cm} \) for a superconductor

Compare with \( \rho < 10^8 \, \Omega \cdot \text{cm} \) for Cu

18 orders of magnitude difference!
**Limitations of persistent current flow**

Persistent currents will flow in a superconductor unless:

1) A sufficiently large magnetic field is applied
2) The current exceeds a certain critical current, \( I_c \) (the Silsbee effect)
3) An AC electric field above a certain frequency is applied

The transition from dissipationless to normal response occurs at \( \omega \sim \Delta/\hbar \), where \( \Delta \) is the energy gap.

Superconductors are poor thermal conductors

Thermal conductivity of lead


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**Perfect diamagnetism**

Second basic property of superconductors – they expel magnetic field completely when in superconducting phase \( (T<T_c) \).

This phenomenon is called the *Meissner effect.*

\[
B = \mu_0 (H + M) = 0
\]

\[
\chi = -\frac{M}{H} = -1
\]

Distinguishes the superconductor from an ideal but normal conductor, for which \( dB/dt = 0 \)

Could explain the expulsion of the flux if a S/C is moved into a magnetic field → motion of metal produces currents, which do not decay because \( R = 0 \)

However, \( R = 0 \) does not explain the flux expulsion when a S/C is already in a magnetic field and is cooled from its normal to S/C state (through \( T_c \))

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**Magnetic properties: type I superconductors**

Superconductors are divided into 2 types, depending on their behavior in a magnetic field.

All superconducting elements are type I, except Nb

Superconductivity is destroyed by the presence of some magnetic field, the *critical field*, \( B_c \). Typically \( B_c \sim \text{tens of mT for type I} \)

\[
B_c(T) = B_c(0) \left[1 - \left(\frac{T}{T_c}\right)^2\right]
\]

Field \( B_c \) does not need to be external:

Critical current \( I_c \) causes a magnetic field \( B_c \), which destroys superconductivity

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**Rough correlation between critical field and \( T_c \)**

B-T phase diagram

Critical field goes to zero as \( T \to T_c \)
Penetration depth

Surface currents expel magnetic flux from type I superconductors. Currents actually penetrate the sample slightly (~100 nm). Magnetic field decreases exponentially inside sample:

\[ B(x) = B_0 e^{-x/\lambda} \]
\[ \lambda(T) = \frac{\lambda(0)}{\sqrt{1 - \left(\frac{T}{T_c}\right)^4}} \]

As \( T \rightarrow T_c \), \( \lambda \) increases and \( B \) penetrates deeper into the sample. At \( T_c \), \( \lambda \rightarrow \infty \), and the sample goes normal.

Type II superconductors

- Alloys (and Nb and V)
- Higher \( T_c, B_c, I_c \) than type I
- More applications e.g. \( \text{Nb}_3\text{Sn} \) magnets
- 2 critical fields:
  \( B < B_{c1} \) - just like a type I
  \( B > B_{c2} \) - normal
- Intermediate fields – sample has both superconducting and normal regions

Isotope effect

- \( T_c \) depends on atomic mass: \( T_c \propto \frac{1}{\sqrt{M}} \)
- More general: \( T_c M^{\alpha} = \text{constant} \)
- Suggests superconductivity is not just an electronic effect

Isotope effect: 1st direct evidence of interaction of electrons and the lattice

Suggests that lattice vibrations play a part in the superconducting process

Experimental evidence for an energy gap – heat capacity

Specific heat of normal metal has form:

\[ \text{AT} + \text{BT}^3 \]

Electronic vibrations

Exponential \( T \)-dependence

\[ C \propto e^{-b/kT} \]

Characteristic of thermal behavior of a system whose excited levels are separated from the ground state by an energy \( 2\Delta \)

From Kittel-Phillips
**Experimental evidence for an energy gap – tunneling**

As $T$ increases towards $T_c$, threshold voltage decreases
⇒ energy gap decreases with increasing $T$

No current flows below $eV = \Delta$

**Theory**

- **Phenomenological:**
  - F & H. London (1935)
  - Ginzburg & Landau (1950)

- **Quantum:**
  - Fröhlich (1950)
  - Bardeen, Cooper & Schrieffer, BCS (1957)

**London model**

- Using two fluid model of Gorter and Casimir:
  Assume only a fraction of electrons $n_s(T)/n$ participate in supercurrent
  - $n_s(T)$ is the density of superconducting electrons:
    $n_s \sim n$ at $T << T_c$, $n_s \to 0$ at $T \to T_c$
  - $n - n_s$ electrons exhibit normal dissipation
  - Current and supercurrent flow in parallel ⇒ superconducting electrons carry all current, normal current is inert and can be ignored

**London equations**

In an electric field $E$, S/C electrons will accelerate without dissipation, so we can relate the mean velocity $v_s$ to the current density $j$:

$$ m \frac{dv_s}{dt} = -eE \quad \text{using} \quad j = -eV_s n_s \quad \text{get} \quad \frac{d}{dt} j = - \frac{n_s e^2}{m} E $$

1st London equation

In a steady state, $j = \text{const} \Rightarrow E = 0$ Electric field inside a S/C vanishes

Maxwell’s equation: $\nabla \times E = -\frac{\partial B}{\partial t} \quad \Rightarrow \quad \frac{\partial B}{\partial t} = 0 \Rightarrow B = \text{const}$

These equations describe the magnetic fields and current densities within a perfect conductor, but they are incompatible with the Meissner effect.

From the above, have

$$ \frac{\partial B}{\partial t} = - \frac{m}{n_s e^2} \nabla \times j $$

2nd London equation

London assumed that

$$ B = - \frac{m}{n_s e^2} \nabla \times j $$

i.e. to successfully predict the Meissner effect the constant of integration must be chosen to be zero

Combining equation $B = -\frac{m}{n_s e^2} \nabla \times j$ and $\nabla \times B = \mu_0 J$

and using $\nabla \times \nabla \times B = \nabla (\nabla \cdot B) - (\nabla \cdot \nabla) B = -\nabla^2 B$

get $\nabla^2 B = \frac{\mu_0 n_s e^2}{m} B$ and $\nabla^2 j = \frac{\mu_0 n_s e^2}{m} j$

Solution (one-dimensional case):

$$ B(x) = B_0 e^{-x/\lambda} $$

- the London penetration depth
  i.e. the Meissner effect is predicted

Solution for $j$ gives a surface current – exponentially decaying into a S/C
BCS theory

- Fröhlich (1950): e-e attraction via phonons (.... Isotope effect)
- Cooper (1956): electrons just above the Fermi surface form bound pairs
- Most stable when center of mass is at rest and total spin = 0, So, \( +k\uparrow \) and \( -k\downarrow \)
- Attractive interaction is provided by lattice vibrations – phonons
- First electron deforms the lattice and second electron is then attracted by the deformation (i.e. the changed positive charge distribution)

\[ e^- \quad \text{i}ons \quad e^- \]

\[ v_c \sim 10^{-8} \text{cm/s} \]

\[ \text{region of positive charge attracts a second electron} \]

- Time-scales: electron motion \( \sim 10^{-16} \text{s} \); lattice deformed for \( \sim 10^{-13} \text{s} \)
In this time, first electron has traveled \( \sim v_F t \sim 10^5 \text{m/s} \times 10^{-13} \text{s} \sim 1000 \text{Å} \)

Lattice deformation attracts 2nd electron without it feeling the Coulomb repulsion of the 1st

- Cooper calculation: solve Schrödinger eq. for 2 interacting electrons in the presence of a Fermi sphere of non-interacting electrons. Only effect of \( N-2 \) electrons - restrict \( k \) values of e-e pair to be \( > k_F \), i.e. outside the Fermi sphere
- Cooper pair – boson.
- A single, coherent wave function extending over entire system.
  Can’t change momentum of a pair without changing all pairs
- Bardeen, Cooper and Schrieffer (BCS) → extend Cooper’s theory, construct a ground state where all electrons form bound pairs
- Each electron now has 2 roles:
  - Provide restriction on allowed wavevectors via Pauli principle
  - Participate in bound pair (called a Cooper pair)
- Electron-phonon interactions: responsible for resistance of metals and superconductivity
- Superconductors are generally poor conductors in normal state

Summary

- When a superconductor is cooled below the critical temperature \( (T_C) \), it enters a new state, in which its resistance vanishes.
- Superconductors expel magnetic field completely when in superconducting phase – the Meissner effect
- When a magnetic field higher than a certain value called the critical field \( (B_c) \) is applied to a superconductor, it reverts to a normal state
- Type I and type II superconductors are distinguished by their behavior in a magnetic field. In a type II S/C there are 2 critical fields. At intermediate fields, the material has both superconducting and normal regions
- Electrodynamics of superconductors is described by phenomenological London equations
- BCS theory – microscopic mechanism for superconductivity through the formation of e-e Cooper pairs via electron-phonon interaction.
  A Cooper pair has a lower energy than 2 individual electrons. The energy difference is \( 2\Delta \) - energy gap.