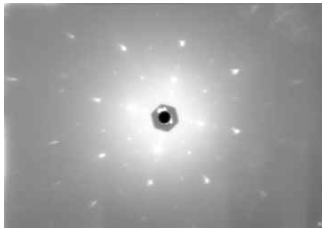
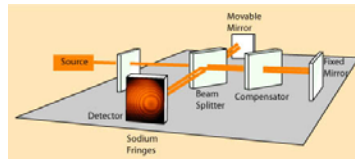


Welcome to

Phys 774: Principles of Spectroscopy



Fall 2007



Lecture 2

Andrei Sirenko, NJIT

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Instructor:

Andrei Sirenko

Associate Professor at the Dept. of Physics, NJIT

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476 Tiernan

Office hours: After the classes on We.'s or by appointment
973-596-5342

Class Schedule:

Wednesday 11:30am - 12:55pm | FMH 106

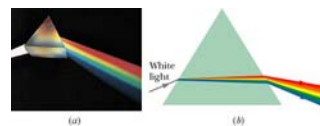
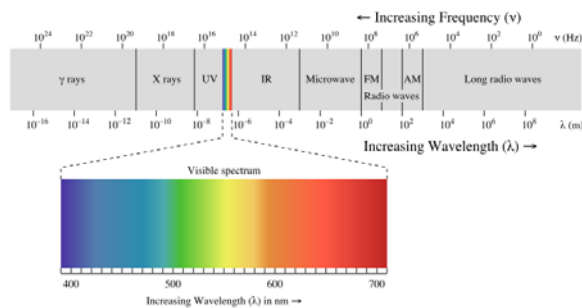
Friday 1:00pm - 2:25pm | FMH 203

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Spectrum of Electromagnetic Radiation and Light

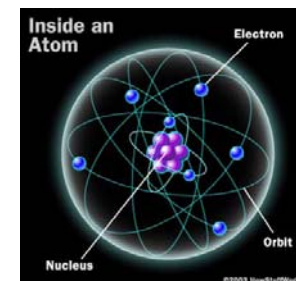
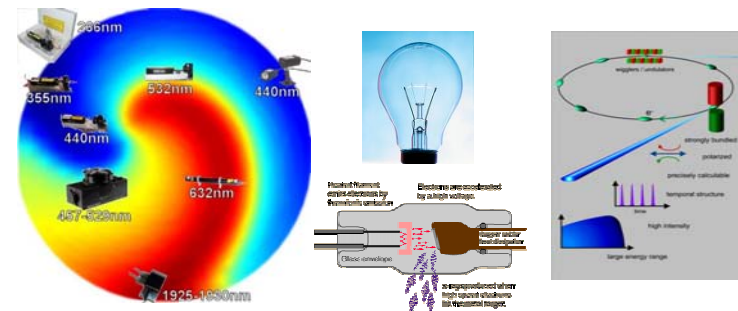


CLASS	FREQUENCY	WAVELENGTH	ENERGY
γ	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
SX	3 EHz	100 pm	12.4 keV
EUV	300 PHz	1 nm	124 eV
NIR	3 PHz	100 nm	12.4 eV
MIR	30 THz	1 μ m	1.24 eV
FIR	3 THz	100 μ m	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μ eV
UHF	3 GHz	1 dm	12.4 μ eV
VHF	300 MHz	1 m	1.24 μ eV
HF	30 MHz	1 dam	124 neV
MF	3 MHz	1 hm	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	12.4 peV
VF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV

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How can we produce EM waves ?



Lecture 2

4

How can we analyze EM waves ?

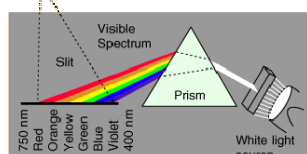
Diffraction Grating

When there is a need to separate light of different wavelengths with high resolution, then a diffraction grating is most often the tool of choice. This "super prism" aspect of the diffraction grating leads to application for measuring atomic spectra in both laboratory instruments and telescopes. A large number of parallel, closely spaced slits constitutes a diffraction grating. The condition for maximum intensity is the same as that for the double slit or multiple slits, but with a large number of slits the intensity maximum is very sharp and narrow, providing the high resolution for spectroscopic applications. The peak intensities are also much higher for the grating than for the double slit.



When light of a single wavelength, like the 632.8 nm red light from a helium-neon laser at left, strikes a diffraction grating it is diffracted to each side in multiple orders. Orders 1 and 2 are shown to each side of the direct beam. Different wavelengths are diffracted at different angles, according to the grating relationship.

Radio	Far IR, Micro-wave	IR	UV	X-ray	γ-ray
-------	--------------------	----	----	-------	-------



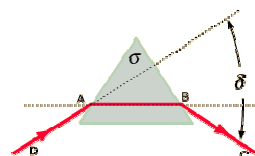
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Position → Wavelength

Resolution

Prism vs. Grating



$$\frac{n_{\text{prism}}}{n_0} = \frac{\sin \frac{1}{2}(\sigma + \delta)}{\sin \frac{1}{2}\sigma}$$

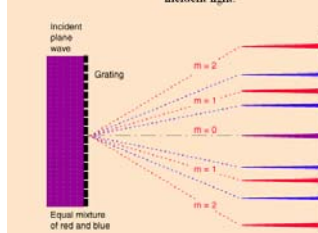
5

How can we analyze EM waves ?

Resolution depends on
 d - number of grooves per mm

Diffraction Grating

A diffraction grating is the tool of choice for separating the colors in incident light.



The condition for maximum intensity is the same as that for a double slit. However, angular separation of the maxima is generally much greater because the slit spacing is so small for a diffraction grating.

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Assumption of infinite source distance gives plane wave at slit so that all amplitude elements are in phase.

For $D \gg a$ this approaches a right angle and $\theta \approx \theta$
 $a = \text{slit width}$

$$\tan \theta = \frac{y}{D}$$

$$\tan \theta \approx \sin \theta \approx \theta \approx \frac{y}{D}$$

$$\text{Condition for maximum } d \sin \theta = m\lambda$$

$$y \approx \frac{m\lambda D}{d}$$

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How can we analyze EM waves ?

Energy Dispersive Principle

<http://www4.nau.edu/microanalysis/Microprobe/Course%20Overview.html>

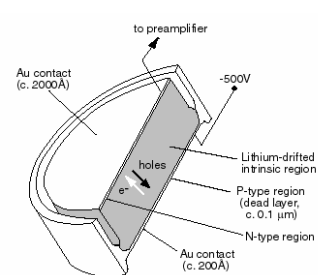


Figure 3.6.1a. Cross section of a typical lithium-drifted silicon detector. X-rays create electron-hole pairs in the intrinsic region of the semiconductor; these charge carriers then migrate to the electrodes under the influence of an applied bias voltage (after Kevex Corporation 1983).

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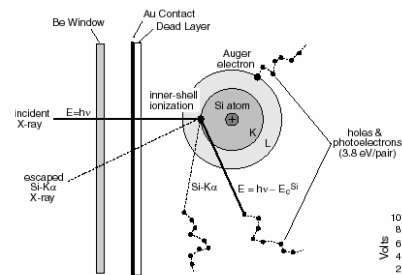


Figure 3.6.1b. The X-ray detection process in the Si(Li) detector. Incident X-rays may cause ionization in the Si of the detector. The resulting characteristic X-rays may escape or be absorbed within the detector. The incident X-ray loses energy equivalent to E_K for Si. The energy of incident X-ray can also be absorbed by production of Auger electrons and by electron-hole pairs. (after Goldstein et al. 1981).

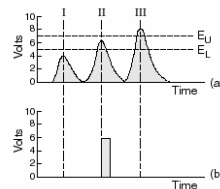
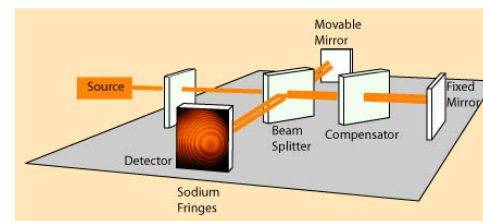


Figure 3.5.6. Schematic representation of pulse height analyzer behavior. (a) Main amplifier output, (b) single channel analyzer output with $E_L = 5$ V and $E_U = 7$ V. Pulses I and III are rejected (after Goldstein et al. 1981).

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How can we analyze EM waves ?

Interferometry Principle



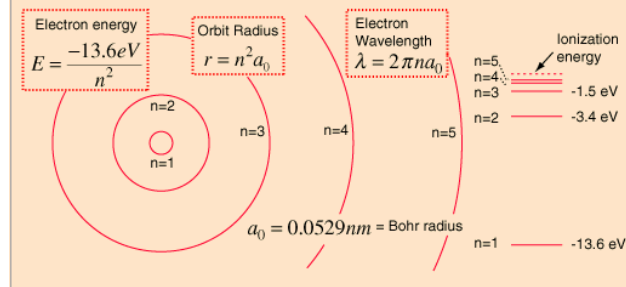
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Light and Atoms

Hydrogen Energy Levels

The basic hydrogen energy level structure is in agreement with the [Bohr model](#). Common pictures are those of a shell structure with each main shell associated with a value of the principal quantum number n .



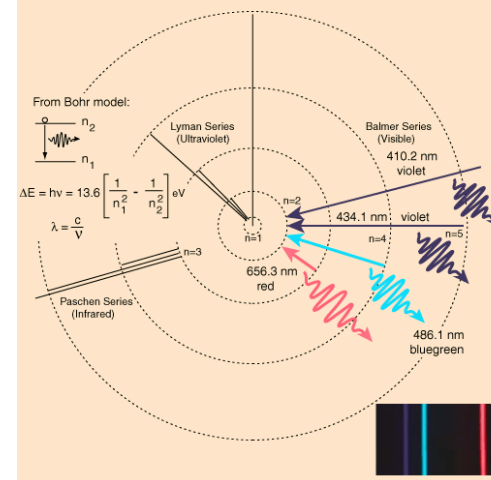
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Light and Atoms

Hydrogen Spectrum



Measured Hydrogen Spectrum

The measured lines of the [Balmer series](#) of hydrogen in the nominal visible region are:

Wavelength (nm)	Relative Intensity	Transition	Color
383.5384	5	$9 \rightarrow 2$	Violet
388.9049	6	$8 \rightarrow 2$	Violet
397.0072	8	$7 \rightarrow 2$	Violet
410.174	15	$6 \rightarrow 2$	Violet
434.047	30	$5 \rightarrow 2$	Violet
486.133	80	$4 \rightarrow 2$	Bluegreen (cyan)
656.272	120	$3 \rightarrow 2$	Red
656.2852	180	$3 \rightarrow 2$	Red

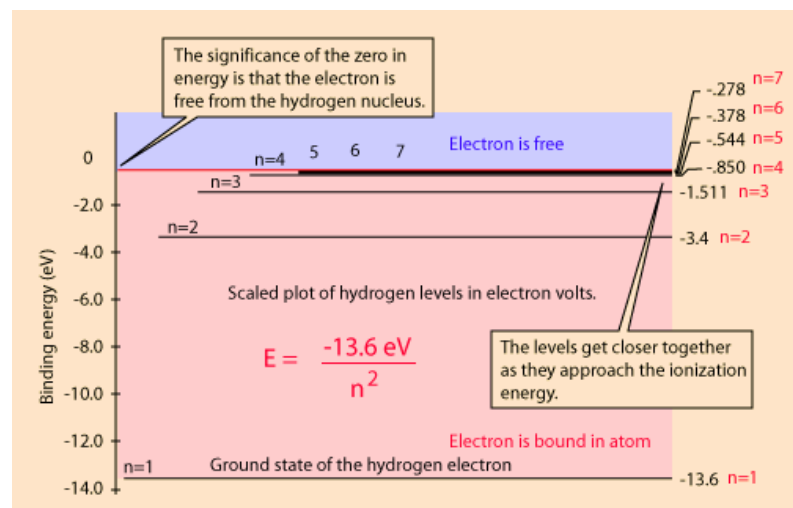
The red line of deuterium is measurably different at 656 1065 (1787 nm difference).

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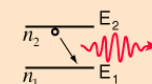
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Light and Atoms

Electron Transitions

The [Bohr model](#) for an electron transition in hydrogen between [quantized energy levels](#) with different quantum numbers n yields a photon by [emission](#) with [quantum energy](#):



A downward transition involves emission of a photon of energy:

$$E_{\text{photon}} = h\nu = E_2 - E_1$$

Given the expression for the energies of the hydrogen electron states:

$$h\nu = \frac{2\pi^2 me^4}{h^2} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] = -13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ eV}$$

This is often expressed in terms of the inverse wavelength or "wave number" as follows:

$$\frac{1}{\lambda} = R_H \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{ where } R_H = \frac{2\pi^2 me^4}{h^2} \text{ is called the Rydberg constant.}$$

$$R_H = 1.0973731 \times 10^7 \text{ m}^{-1}$$

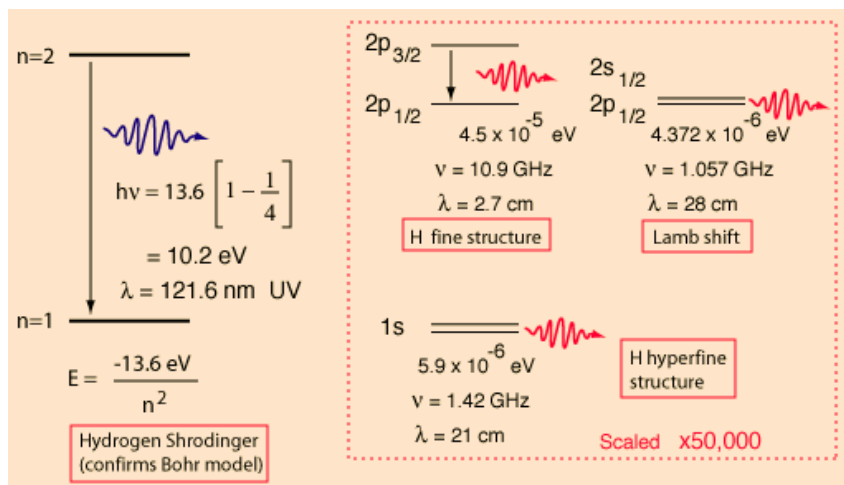
Lyman Series			
93.782	...	$6 \rightarrow 1$	UV
94.976	...	$5 \rightarrow 1$	UV
97.254	...	$4 \rightarrow 1$	UV
102.583	...	$3 \rightarrow 1$	UV
121.566	...	$2 \rightarrow 1$	UV
Balmer Series			
383.5384	5	$9 \rightarrow 2$	Violet
388.9049	6	$8 \rightarrow 2$	Violet
397.0072	8	$7 \rightarrow 2$	Violet
410.174	15	$6 \rightarrow 2$	Violet
434.047	30	$5 \rightarrow 2$	Violet
486.133	80	$4 \rightarrow 2$	Bluegreen (cyan)
656.272	120	$3 \rightarrow 2$	Red
656.2852	180	$3 \rightarrow 2$	Red
Paschen Series			
954.62	...	$8 \rightarrow 3$	IR
1004.98	...	$7 \rightarrow 3$	IR
1093.8	...	$6 \rightarrow 3$	IR
1281.81	...	$5 \rightarrow 3$	IR
1875.01	...	$4 \rightarrow 3$	IR
Brackett Series			
2630	...	$6 \rightarrow 4$	IR
4050	...	$5 \rightarrow 4$	IR
Pfund Series			
7400	...	$6 \rightarrow 5$	IR

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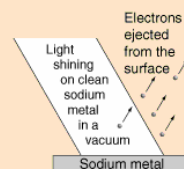
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Light and Atoms



Light and Atoms: Photoelectric Effect

The Photoelectric Effect



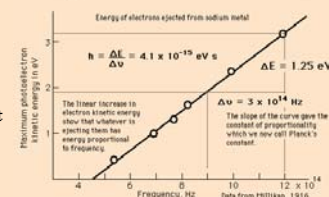
The remarkable aspects of the photoelectric effect when it was first observed were:

1. The electrons were emitted immediately - no time lag!
2. Increasing the intensity of the light increased the number of photoelectrons, but not their maximum kinetic energy!
3. Red light will not cause the ejection of electrons, no matter what the intensity!
4. A weak violet light will eject only a few electrons, but their maximum kinetic energies are greater than those for intense light of longer wavelengths!

The details of the photoelectric effect were in direct contradiction to the expectations of very well developed classical physics.

The explanation marked one of the major steps toward quantum theory.

Early Photoelectric Effect Data



The Planck Hypothesis

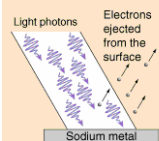
In order to explain the frequency distribution of radiation from a hot cavity (blackbody radiation) Planck proposed the ad hoc assumption that the radiant energy could exist only as discrete quanta which were proportional to the frequency. This would imply that higher modes would be less populated and avoid the ultraviolet catastrophe of the Rayleigh-Jeans Law.

$$E = h\nu$$

h = Planck's constant = $6.626 \times 10^{-34} \text{ Joule-sec} = 4.136 \times 10^{-15} \text{ eV-s}$

Light and Atoms: Photoelectric Effect

Photoelectric Effect

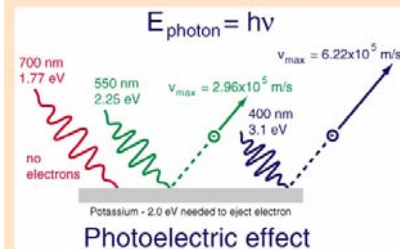


Photon energy $E = h\nu$ explains the experiment and shows that light behaves like particles.

$$E = h\nu$$

Analysis of data from the photoelectric experiment showed that the energy of the ejected electrons was proportional to the frequency of the illuminating light. This showed that whatever was knocking the electrons out had an energy proportional to light frequency. The remarkable fact that the ejection energy was independent of the total energy of illumination showed that the interaction must be like that of a particle which gave all of its energy to the electron! This fit in well with Planck's hypothesis that light in the blackbody radiation experiment could exist only in discrete bundles with energy

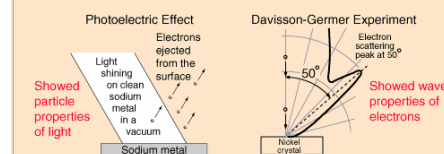
Photoelectric Effect



Most commonly observed phenomena with light can be explained by waves. But the photoelectric effect suggested a particle nature for light.

Wave-Particle Duality

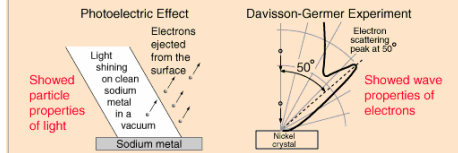
Publicized early in the debate about whether light was composed of particles or waves, a wave-particle dual nature soon was found to be characteristic of electrons as well. The evidence for the description of light as waves was well established at the turn of the century when the photoelectric effect introduced firm evidence of a particle nature as well. On the other hand, the particle properties of electrons was well documented when the DeBroglie hypothesis and the subsequent experiments by Davisson and Germer established the wave nature of the electron.



Phenomenon	Can be explained in terms of waves.	Can be explained in terms of particles.
Reflection	✓	✓
Refraction	✓	✓
Interference	✓	✗
Diffraction	✓	✗
Polarization	✓	✗
Photoelectric effect	✗	✓

Wave-Particle Duality

Publicized early in the debate about whether light was composed of particles or waves, a wave-particle dual nature soon was found to be characteristic of electrons as well. The evidence for the description of light as waves was well established at the turn of the century when the photoelectric effect introduced firm evidence of a particle nature as well. On the other hand, the particle properties of electrons was well documented when the DeBroglie hypothesis and the subsequent experiments by Davisson and Gerner established the wave nature of the electron.



Wave Nature of Electron

As a young student at the University of Paris, Louis DeBroglie had been impacted by relativity and the photoelectric effect, both of which had been introduced in his lifetime. The photoelectric effect pointed to the particle properties of light, which had been considered to be a wave phenomenon. He wondered if electrons and other "particles" might exhibit wave properties. The application of these two new ideas to light pointed to an interesting possibility.

Relativity
 $E = mc^2 = \sqrt{p^2 c^2 + m_0^2 c^4}$
 Kinetic energy term: $\frac{1}{2}mv^2$
 Rest mass energy term: $m_0 c^2$
 Momentum of a photon: $p = \frac{E}{c}$
 The de Broglie Hypothesis: $\lambda = \frac{h}{p}$
 for photon: $\lambda = \frac{h}{p}$
 for electron: $\lambda = \frac{h}{mv}$
 Photoelectric effect: $E = hf = \frac{hc}{\lambda}$

DeBroglie Wavelengths

The Davisson-Gerner experiment showed that electrons exhibit the DeBroglie wavelength given by:

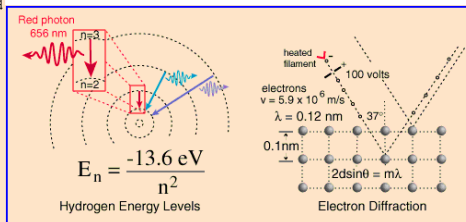
$$\lambda = \frac{h}{p}$$

Does this relationship apply to all particles? Consider a pitched baseball:

$m = 0.15 \text{ kg}$
 $v = 40 \text{ m/s} = 90 \text{ mi/hr}$
 $\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(0.15 \text{ kg})(40 \text{ m/s})} = 1.1 \times 10^{-34} \text{ m}$
 10^{-10} m Atomic diameter
 10^{-14} m Nuclear Diameter

For an electron accelerated through 100 Volts: $v = 5.9 \times 10^6 \text{ m/s}$
 $\lambda = \frac{6.626 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(5.9 \times 10^6 \text{ m/s})} = 1.2 \times 10^{-10} = 0.12 \text{ nm}$

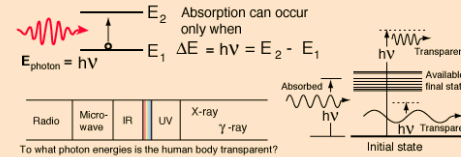
This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.



The interaction of radiation with matter

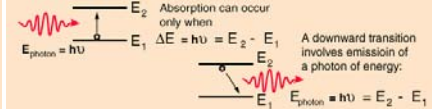
Transparency

You can see for many miles through clear air and a clear piece of glass obviously is transparent to the wavelengths of visible light. The air is fortunately not transparent to the ultraviolet rays from the sun, though increasing transparency from ozone depletion is a concern. The clear piece of glass is transparent to visible light because the available electrons in the material which could absorb the visible photons have no available energy levels above them in the range of the quantum energies of visible photons. The glass atoms do have vibrational energy modes which can absorb infrared photons, so the glass is not transparent in the infrared. This leads to the greenhouse effect. The quantum energies of the incident photons must match available energy level gaps to be absorbed.



Absorption and Emission

Taking the electron transitions associated with visible and ultraviolet interactions with matter as an example, absorption of a photon will occur only when the quantum energy of the photon precisely matches the energy gap between the initial and final states. In the interaction of radiation with matter, if there is no pair of energy states such that the photon energy can elevate the system from the lower to the upper state, then the matter will be transparent to that radiation.

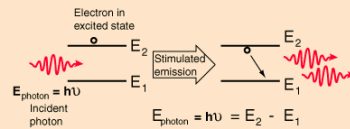


Energy levels associated with molecules, atoms and nuclei are in general discrete, quantized energy levels and transitions between those levels typically involve the absorption or emission of photons. Electron energy levels have been used as the example here, but quantized energy levels for molecular vibration and rotation also exist. Transitions between vibrational quantum states typically occur in the infrared and transitions between rotational quantum states are typically in the microwave region of the electromagnetic spectrum.

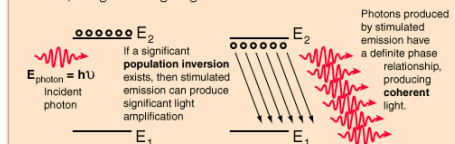
The interaction of radiation with matter

Stimulated Emission

If an electron is already in an excited state (an upper energy level, in contrast to its lowest possible level or "ground state"), then an incoming photon for which the quantum energy is equal to the energy difference between its present level and a lower level can "stimulate" a transition to that lower level, producing a second photon of the same energy.



When a sizable population of electrons resides in upper levels, this condition is called a "population inversion", and it sets the stage for stimulated emission of multiple photons. This is the precondition for the light amplification which occurs in a laser, and since the emitted photons have a definite time and phase relation to each other, the light has a high degree of coherence.



Like absorption and emission, stimulated emission requires that the photon energy given by the Planck relationship be equal to the energy separation of the participating pair of quantum energy states.

Application of Lasers



TELECOMMUNICATION



Types of Lasers

The laser medium can be a solid, gas, liquid or semiconductor.

Solid-state lasers have lasing material distributed in a solid matrix (such as the ruby or neodymium:yttrium-aluminum garnet "YAG" lasers). The neodymium-YAG laser emits infrared light at 1064 nanometers (nm).

Gas lasers (helium and helium-neon, HeNe, are the most common gas lasers) have a primary output of visible red light. CO2 lasers emit energy in the far-infrared, and are used for cutting hard materials.

Excimer lasers (the name is derived from the terms *excited* and *dimers*) use reactive gases, such as chlorine and fluorine, mixed with inert gases such as argon, krypton or xenon. When electrically stimulated, a pseudo molecule (dimer) is produced. When lased, the dimer produces light in the ultraviolet range.

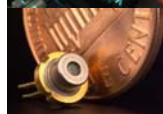
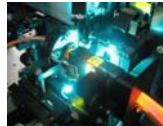
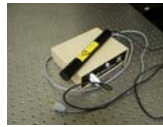
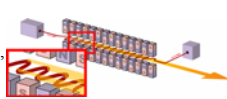
Dye lasers use complex organic dyes, such as rhodamine 6G, in liquid solution or suspension as lasing media. They are tunable over a broad range of wavelengths.

Semiconductor lasers, sometimes called diode lasers, are not solid-state lasers. These electronic devices are generally very small and use low power. They may be built into larger arrays, such as the writing source in some laser printers or CD players.

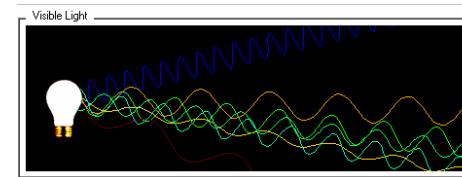
And more: Ring lasers, Disk lasers, Free electron lasers, ...

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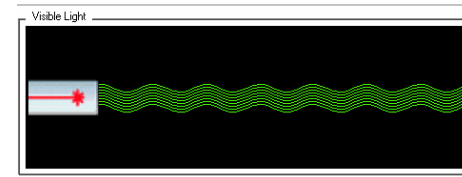
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Regular Light



Laser Light



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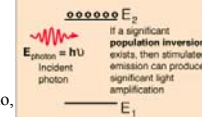
What is the difference ?

Properties of laser radiation:

- Monochromatic
- Coherent
- Directional

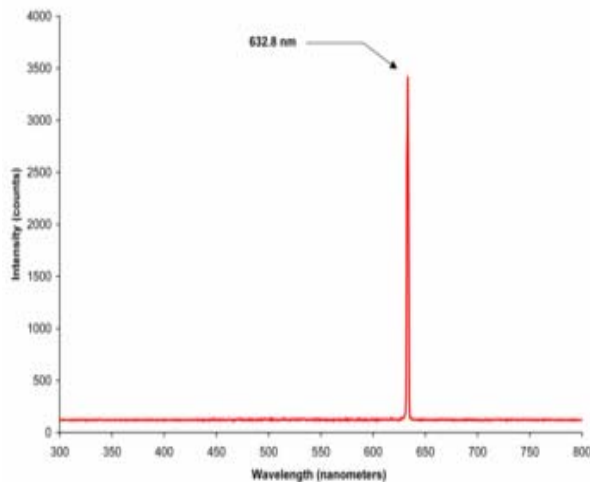
- Stimulated emission and gain

Population Inversion



The achievement of a significant population inversion in atomic or molecular energy states is a precondition for **laser** action. Electrons will normally reside in the lowest available energy state. They can be elevated to excited states by absorption, but no significant collection of electrons can be accumulated by absorption alone since both spontaneous emission and **stimulated emission** will bring them back down.

Laser Wavelength

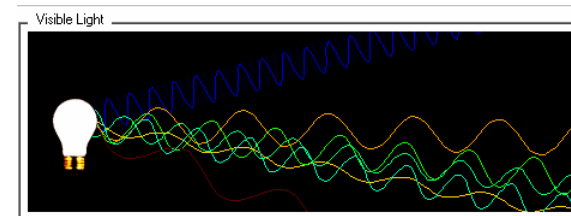


Properties of laser radiation:

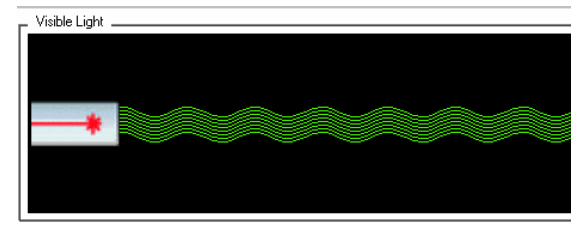
- Monochromatic
- Coherent
- Directional
- Stimulated emission and gain

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Regular Light



Laser Light



Like rain

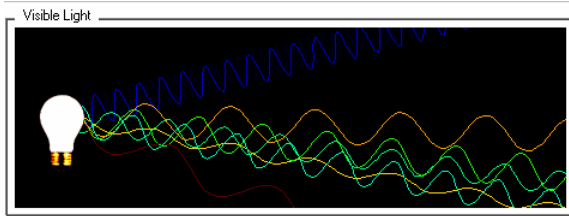


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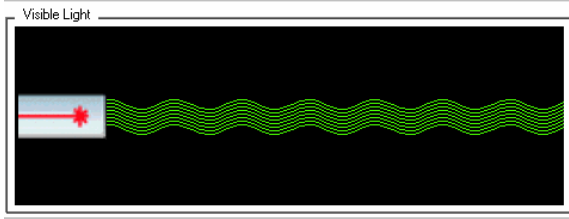
Regular Light

COHERENCE:

Like rain



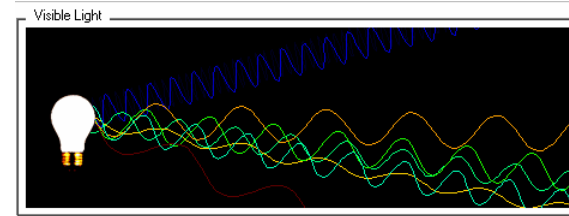
Laser Light



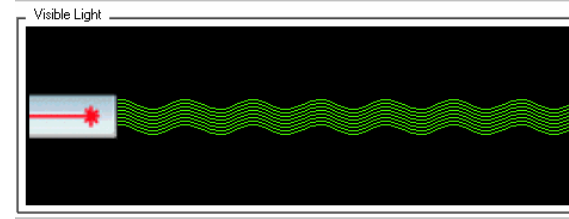
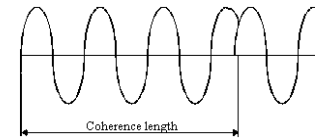
25

Regular Light

COHERENCE:



Laser Light



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Visible Lasers:

Red Laser

Orange Laser

Yellow Laser

Green Laser

Blue Laser

Indigo Laser

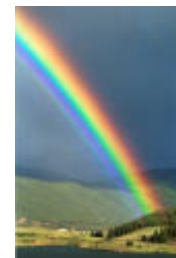
Violet Laser



Anything else ?

27

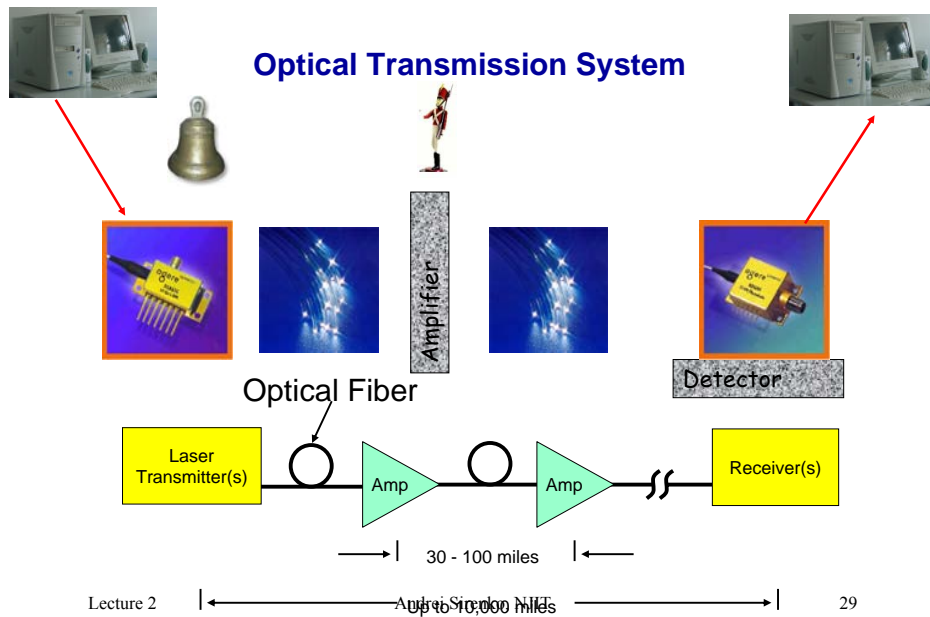
Typical Laser Wavelengths:



Laser Type	Wavelength (nm)
Argon fluoride (UV)	193
Krypton fluoride (UV)	248
Xenon chloride (UV)	308
Nitrogen (UV)	337
Argon (blue)	488
Argon (green)	514
Helium neon (green)	543
Helium neon (red)	633
Rhodamine 6G dye (tunable)	570-650
Ruby (CrAlO ₃) (red)	694
Nd:Yag (NIR)	1064
Carbon dioxide (FIR)	10600

Lecture 2

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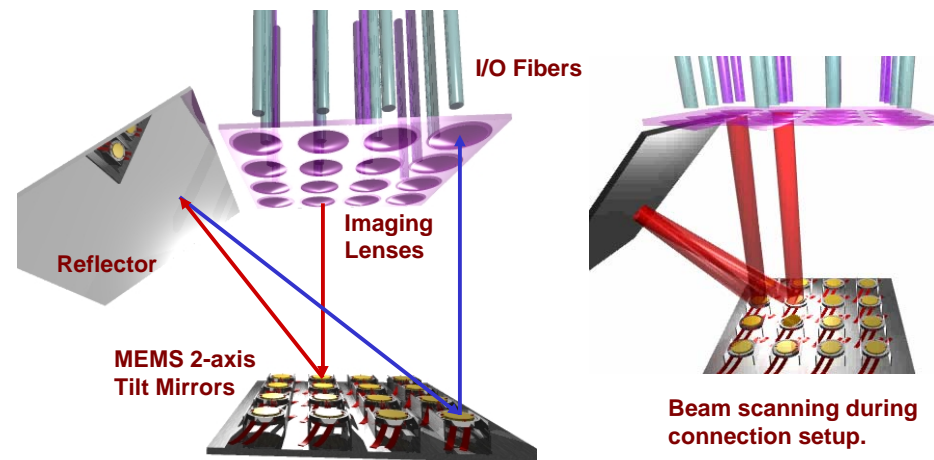


Lecture 2

August 10, 2000

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MEMS OXC-- 2N Mirror Design



2N MEMS mirrors in an NxN single-mode fiber optical crossconnect.

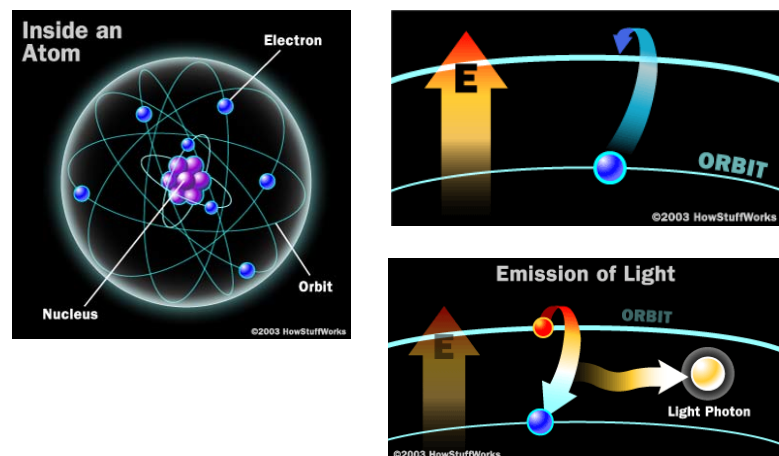
Lecture 2

Andrei Sirenko, NJIT

Lucent Technologies
Bell Labs Innovations



Principles of Laser Radiation

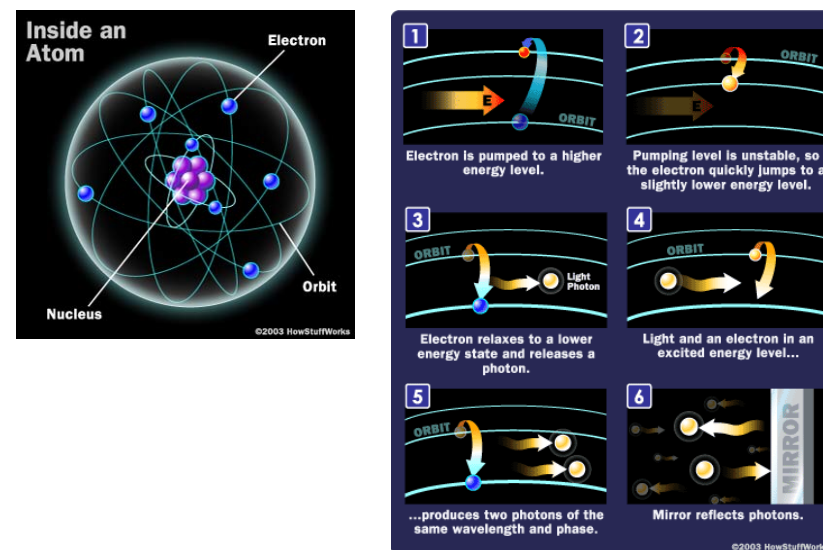


Lecture 2

Andrei Sirenko, NJIT

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Principles of 3-level laser operation

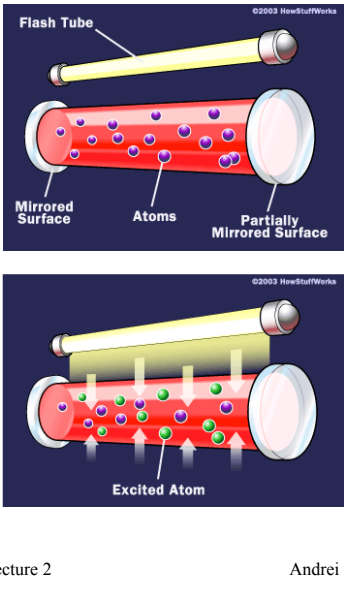


Lecture 2

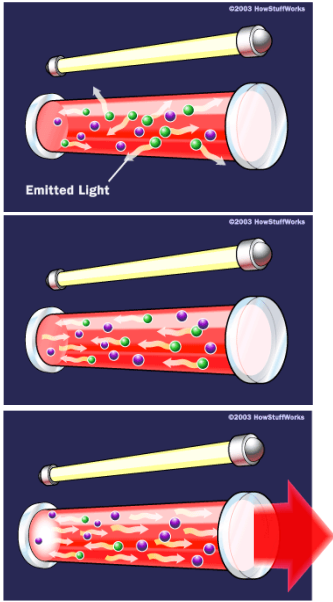
Andrei Sirenko, NJIT

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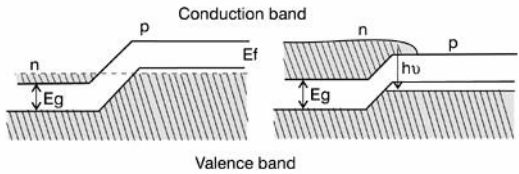
Principles of the Solid State Laser operation



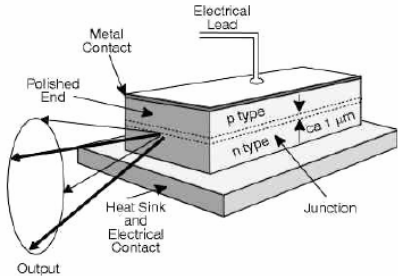
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Semiconductor Lasers

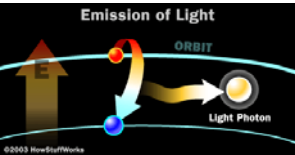
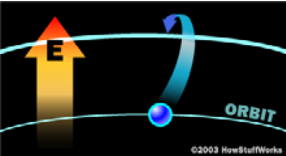


Band structure near a semiconductor p-n junction. Left: No forward-bias voltage. Right: Forward-bias voltage present



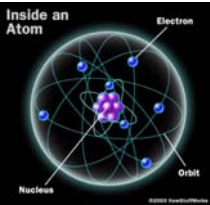
Lecture 2 34

Three level energy diagram of the He-Ne laser transition



The laser process in a HeNe laser starts with collision of electrons from the electrical discharge with the helium atoms in the gas. This excites helium from the ground state to the long-lived, metastable excited states. Collision of the excited helium atoms with the ground-state neon atoms results in transfer of energy to the neon atoms. This is due to a coincidence of energy levels between the helium and neon atoms.

This process is given by the reaction equation:
 $\text{He}^* + \text{Ne} \rightarrow \text{He} + \text{Ne}^* + \Delta E$



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Fundamentals of Laser Operation

If the atom is in the excited state, it may decay into the ground state by the process of spontaneous emission:

$$E_2 - E_1 = h\nu \quad N(t) = N(0)e^{-\frac{t}{\tau_{21}}}$$

the rate of which stimulated emission, where $\rho(\nu)$ is the radiation density of photons :

$$\frac{\partial N}{\partial t} = -B_{21}\rho(\nu)N \quad g(\nu) = \frac{1}{\pi} \frac{(\Gamma/2)}{(\nu - \nu_0)^2 + (\Gamma/2)^2} \quad g(\nu = \nu_0) = \frac{2}{\pi\Gamma}$$

Stimulated emission cross section $\sigma_{21}(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)$

Optical amplification $\Delta N_{21} = \left(N_2 - \frac{g_2}{g_1} N_1 \right)$

where g_1 and g_2 are the degeneracies of energy levels 1 and 2, respectively

General gain equation $\frac{dI}{dz} = \frac{\gamma_0(\nu)}{1 + \bar{g}(\nu)\frac{I(z)}{I_S}} \cdot I(z) \quad \gamma_0(\nu) = \sigma_{21}(\nu) \cdot \Delta N_{21} \quad I_S = \frac{h\nu}{\sigma(\nu) \cdot \tau_S}$

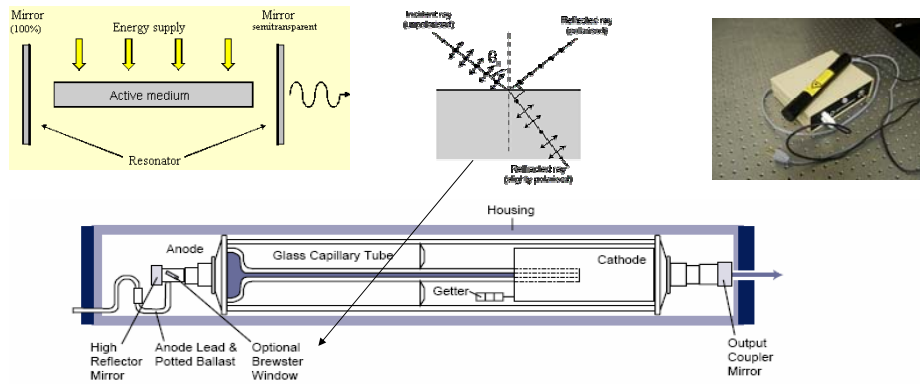
$$\ln \left(\frac{I(z)}{I_{in}} \right) + \bar{g}(\nu) \frac{I_{in}}{I_S} \left(\frac{I(z)}{I_{in}} - 1 \right) = \gamma_0(\nu) \cdot z$$

Gain: $G = G(z) = \frac{I(z)}{I_{in}} \quad \ln(G) + \bar{g}(\nu) \frac{I_{in}}{I_S} (G - 1) = \gamma_0(\nu) \cdot z \quad \text{Large signal: } G \rightarrow 1$

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 Saleh, Bahaa E. A. and Teich, Malvin Carl (1991). *Fundamentals of Photonics*.
 New York: John Wiley & Sons. ISBN 0-471-83965-5.

$$I(z) = I_{in} + \frac{\gamma_0(\nu) \cdot z}{\bar{g}(\nu)} I_S \quad 36$$

The intensity of the stimulated emission [W/m²]



General description of a HeNe laser

The typical HeNe laser is basically an optical cavity that consists of a glass capillary tube with a mirror at each end. The tube contains a helium and neon gas mixture that, when excited, utilizes the mirrors at each end of the tube to transform the spontaneous emission into a stimulated laser light emission. One mirror (called the high reflector mirror) reflects virtually 100% of the light, while the other (called the output coupler mirror) reflects approximately 99%. Therefore, about 1% of the light will exit the laser at the desired wavelength.

Some HeNe lasers do not incorporate internal mirrors but, rather, include a special glass window, called a Brewster window. This window is mounted at a precise angle (Brewster angle) to allow light to pass through and become linearly polarized. The output coupler mirror is positioned outside of the HeNe tube. The light is almost completely transmitted, virtually cutting out reflection and resulting in a minimal loss of output power. This intense, clearly visible light is ideal for applications requiring the observation of extremely tiny particles, such as dust.

Examples of transverse Gaussian laser modes

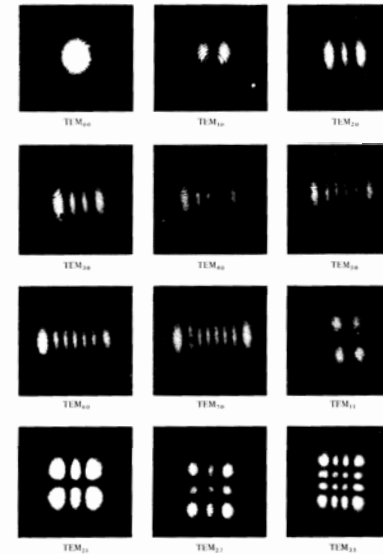
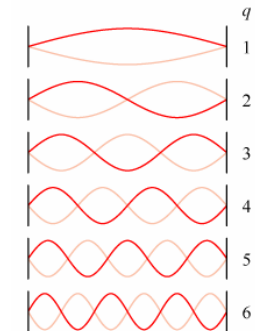


Figure 2-8 Intensity photographs of some low-order Gaussian beam modes. (After [14])

Examples of longitudinal laser modes



$$L = q \frac{\lambda}{2}, \quad \Delta\nu = \frac{c}{2L}$$

$$\Delta\nu = \sum_i \frac{c}{2n_i L_i} = \frac{c}{2} \left[\frac{1}{n_1 L_1} + \frac{1}{n_2 L_2} + \frac{1}{n_3 L_3} + \dots \right]$$