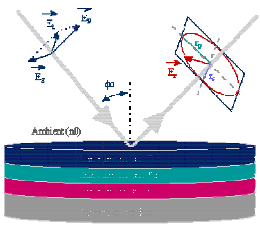
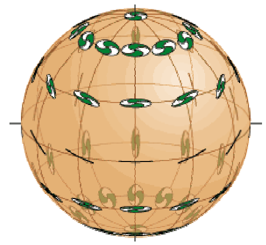
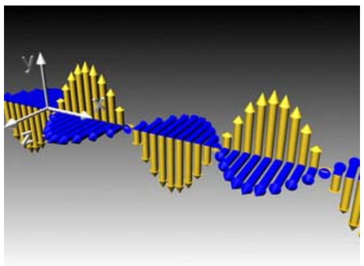
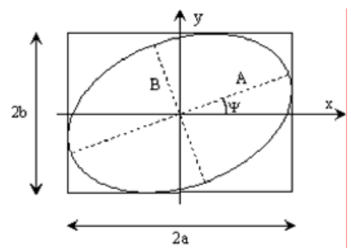




Phys 774: Polarization of Light

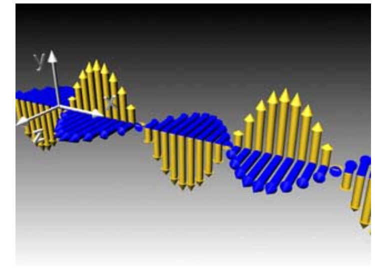


Fall 2007

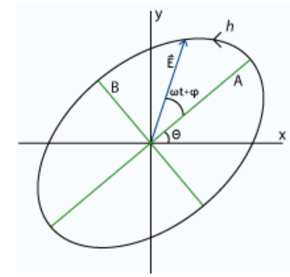


Polarization of Electromagnetic Wave

- General consideration of polarization
- Jones Formalism
- How Polarizers work
- Muller matrices
- Stokes parameters
- Poincare sphere



$$\vec{E}(r,t) = \vec{E}_0 \exp[i(\vec{k}\vec{r} - \omega t)] = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$



Where is the polarization information hidden ?

Polarization of Electromagnetic Wave

- Jones Formalism / Lecture QZ

```
Editor - C:\Program Files\MATLAB71\work\Jones.m
File Edit Text Cell Tools Debug Desktop Window Help
clear all
%%%%%%%%%%%vectors of polarization
LH=[1;0];
LV=[0;1];
CR=[1;i]/sqrt(2);
CL=[1;-i]/sqrt(2);

New=LH+CR
Absol=sqrt(New(1)*conj(New(1))+New(2)*conj(New(2)))

Answer=New/Absol
```

```
>>
New =

    1.7071
         0 + 0.7071i

Absol =

    1.8478

Answer =

    0.9239
         0 + 0.3827i
```

Elliptically polarized light

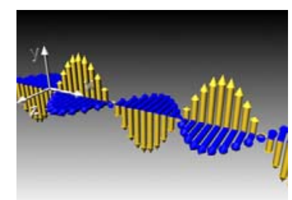
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} = 1.848 \cdot \begin{bmatrix} 0.924 \\ 0.383 \cdot i \end{bmatrix}$$

$$\vec{E}^{Jones} = E_0 \cdot \begin{bmatrix} |r_x| \cdot e^{i\delta x} \\ |r_y| \cdot e^{i\delta y} \end{bmatrix}; (\sqrt{r_x^2 + r_y^2} = 1)$$

Polarization of Electromagnetic Wave

- Jones Formalism

$$\vec{E}(r,t) = \vec{E}_0 \exp[i(\vec{k}\vec{r} - \omega t)] = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

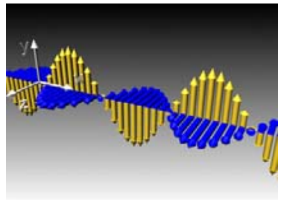


For Polarized light state of polarization is represented by 2x1 matrix of complex amplitudes:

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} E_{0x} \exp[i(\vec{k}\vec{r} - \omega t)] \\ E_{0y} \exp[i(\vec{k}\vec{r} - \omega t)] \\ 0 \quad (\text{for } \vec{k} \parallel \vec{z}) \end{bmatrix}$$

$$\vec{E}^{Jones} = \begin{bmatrix} E_{0x} \\ E_{0y} \end{bmatrix}$$

Polarization of Electromagnetic Wave



- General consideration of polarization

$$\vec{E}(r,t) = \vec{E}_0 \exp[i(\vec{k}\vec{r} - \omega t)] = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\vec{D} = \varepsilon_0 \hat{\varepsilon} \vec{E}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \cdot \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

6 independent components

$$\varepsilon(\omega) = 1 + \chi(\omega)$$

In general situation:

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$

In isotropic media:

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix} = \varepsilon$$

Jones representation for Polarization

- Jones Formalism for polarization; E-matrices

Light polarized along x

$$\vec{E}_x^{\,Jones} = \begin{bmatrix} E_{0x} \\ 0 \end{bmatrix} = E_{0x} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Light polarized along y

$$\vec{E}_y^{\,Jones} = \begin{bmatrix} 0 \\ E_{0y} \end{bmatrix} = E_{0y} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Light polarized at the angle θ with respect to x

$$\vec{E}^{Jones} = E_0 \cdot \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

Right-hand (or σ+) polarized light

$$\vec{E}^{Jones} = E_0 \cdot \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 \\ i \end{bmatrix}$$

Left-hand (or σ-) polarized light

$$\vec{E}^{Jones} = E_0 \cdot \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 \\ -i \end{bmatrix}$$

Elliptically polarized light

$$\vec{E}^{Jones} = E_0 \cdot \begin{bmatrix} |r_x| \cdot e^{i\delta x} \\ |r_y| \cdot e^{i\delta y} \end{bmatrix}; (\sqrt{r_x^2 + r_y^2} = 1)$$

Jones representation for Polarization

- Jones Matrices for optical elements

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \cdot \begin{bmatrix} E_{0x} \\ E_{0y} \end{bmatrix}$$

Optical element	Corresponding Jones matrix
Linear polarizer with axis of transmission horizontal	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Linear polarizer with axis of transmission vertical	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Linear polarizer with axis of transmission at 45°	$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
Linear polarizer with axis of transmission at -45°	$\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
Linear polarizer with axis of transmission at angle φ	$\begin{pmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \sin\varphi\cos\varphi & \sin^2\varphi \end{pmatrix}$
Left circular polarizer	$\frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$
Right circular polarizer	$\frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$
Half-wave plate with fast axis in the horizontal direction	$\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$
Quarter-wave plate with fast axis in the horizontal direction.	$e^{i\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

Non ideal polarizer

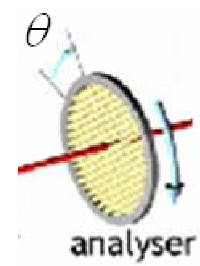
$$\begin{bmatrix} 1 & -i\gamma \\ +i\gamma & 0 \end{bmatrix}$$

Sample in Ellipsometry:

$$\begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix}$$

Change of Polarization of Electromagnetic Wave

- Jones Matrices for rotating optical elements



If an optical element is rotated about the optical axis by angle θ, the Jones matrix for the rotated element, $M(\theta)$, is constructed from the matrix for the unrotated element, M, by the transformation

$$M(\theta) = R(-\theta)\, M\, R(\theta),$$

where $R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

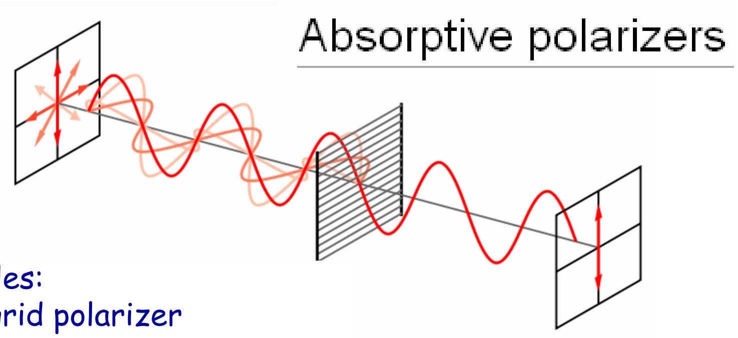
Note that Jones calculus is only applicable to light that is already fully polarized. Light which is unpolarized, partially polarized, or incoherent must be treated using Mueller calculus.

How Polarizers work ?

$$\vec{D} = \varepsilon_0 \hat{\varepsilon} \vec{E}$$

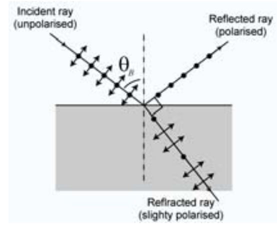
$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \varepsilon_0 \cdot \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}$$



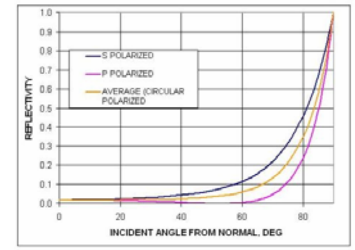
Examples:
Wire-grid polarizer
Electro-absorption modulator

How Polarizers work ?

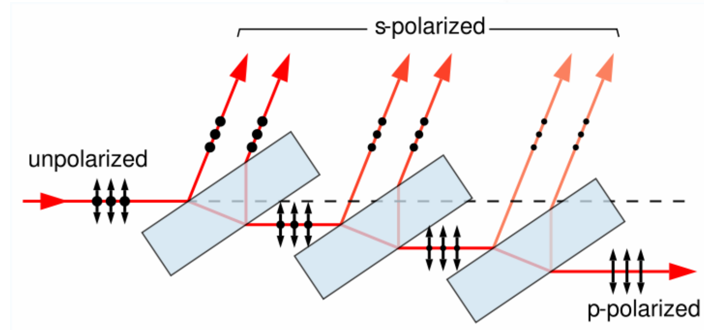


$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right)$$

Water reflectivity



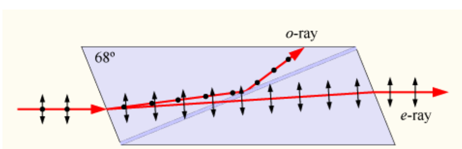
Brewster polarizer



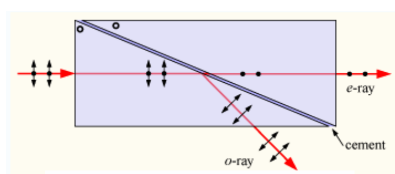
How Polarizers work ?

Birefringent polarizers

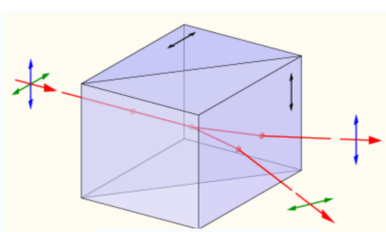
$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} n^2_e & 0 & 0 \\ 0 & n^2_O & 0 \\ 0 & 0 & n^2_O \end{bmatrix} \qquad \Delta n = n_e - n_o$$



Nicol-prism



Glan-Thompson prism

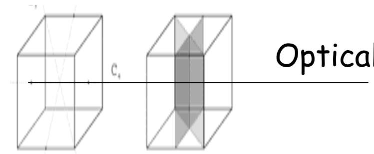


Wollaston-prism

polarizer consisting of two triangular calcite prisms with orthogonal crystal axes that are cemented together. At the internal interface, an unpolarized beam splits into two linearly polarized rays which leave the prism at a divergence angle of 15°-45°.

How Polarizers work ?

Thin-film polarizers (narrow band)



Optical axis Z

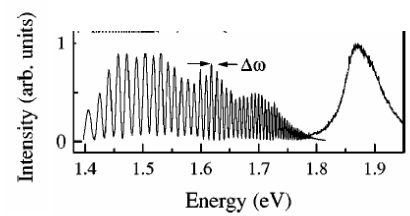
$$\vec{D} = \hat{\varepsilon}(\omega)\vec{E}$$

$$\varepsilon_{zz}(\omega) \neq \varepsilon_{xx}(\omega) = \varepsilon_{yy}(\omega)$$

$$\Delta n = (n_\perp - n_\parallel).$$

$$\Delta n(\lambda) d = M\lambda,$$

$$I(\omega) \sim \sin^2\left[\frac{d}{2c}\Delta n(\omega)\omega\right].$$



Stokes Parameters for Polarization of Electromagnetic Wave

- Stokes Parameters, Intensity, and Poincare

$I_0 \sim E_x^* E_x + E_y E_y^*$

$S_0 = I_0$

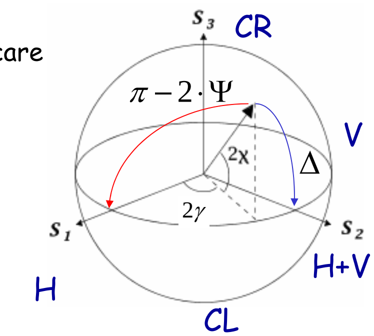
$S_1 = I_x - I_y$

$S_2 = I_{+\pi/4} - I_{-\pi/4}$

$S_3 = I_{\sigma+} - I_{\sigma-}$

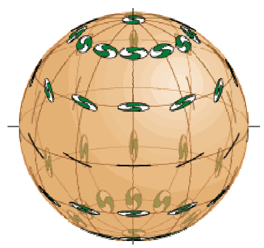
Stokes vector:

$$\vec{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$



$$S_0^{\,2} \geq \sum_{j=1}^{3} S_j^{\,2}$$

$$P = \frac{\sqrt{\sum_{j=1}^{3} S_j^{\,2}}}{S_0} \quad [P = 1 \quad (?); P = 0 \quad (?)]$$

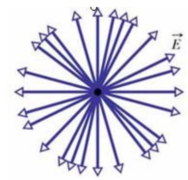


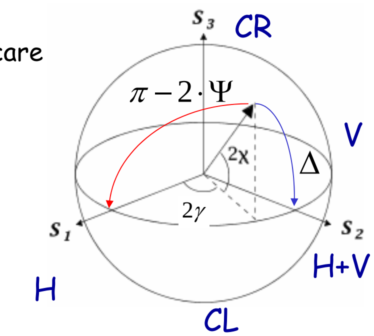
Stokes Parameters for Polarization of Electromagnetic Wave

- Stokes Parameters, Intensity, and Poincare

Unpolarized light

$$\vec{S} = S_0 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$





Linearly and circularly polarized light

$$\vec{S}_l = S_0 \cdot \begin{bmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{S}_c = S_0 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \\ \pm 1 \end{bmatrix}$$

Elliptically polarized light

$$\vec{S} = S_0 \cdot \begin{bmatrix} 1 \\ -\cos 2\psi \\ \sin 2\psi \cdot \cos \Delta \\ -\sin 2\psi \cdot \sin \Delta \end{bmatrix}$$

Change of Polarization of Electromagnetic Wave

- Optical elements, like polarizers and returders, are equivalent to rotation of the Stokes vector around their characteristic vectors by a certain angle.

$$\frac{d\hat{s}}{dz} = \vec{\beta} \times \hat{s}.$$

$$\vec{S}_2 = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \qquad \vec{S}_1 = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

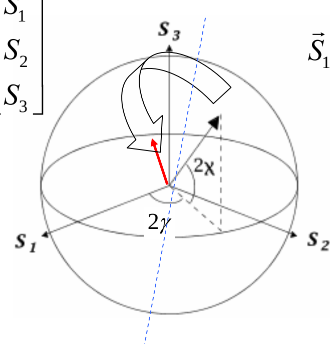


Table 1. Elementary rotations

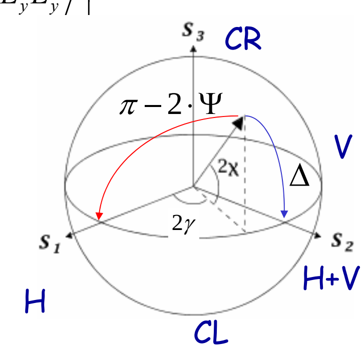
Rotation axis	Jones matrix	Stokes space rotation
1	$U_1 = \begin{pmatrix} e^{-j\varphi/2} & 0 \\ 0 & e^{j\varphi/2} \end{pmatrix}$	$R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{pmatrix}$
2	$U_2 = \begin{pmatrix} \cos\varphi/2 & -j\sin\varphi/2 \\ -j\sin\varphi/2 & \cos\varphi/2 \end{pmatrix}$	$R_2 = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix}$
3	$U_3 = \begin{pmatrix} \cos\varphi/2 & -\sin\varphi/2 \\ \sin\varphi/2 & \cos\varphi/2 \end{pmatrix}$	$R_3 = \begin{pmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Relationship between different representations of the light Polarization

Stokes parameters and J-Jones matrix elements for time-averaged quantities:

$$\vec{S} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} \qquad \hat{J} = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{xx} & J_{xy} \end{bmatrix} = \begin{bmatrix} \langle E_x E_x^* \rangle & \langle E_x E_y^* \rangle \\ \langle E_y E_x^* \rangle & \langle E_y E_y^* \rangle \end{bmatrix}$$

$$\begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -i & i & 0 \end{bmatrix} \cdot \begin{bmatrix} J_{xx} \\ J_{xy} \\ J_{yx} \\ J_{yy} \end{bmatrix}$$



Muller Matrices for Polarization of Electromagnetic Wave

- Muller Matrices

Mueller calculus is a matrix method for manipulating Stokes vectors, which represent the polarization of incoherent light. It was developed in 1943 by Hans Mueller, then a professor of physics at the Massachusetts Institute of Technology. Light which is unpolarized or partially polarized must be treated using Mueller calculus, while fully polarized light can be treated with either Mueller calculus or the simpler Jones calculus. Coherent light generally must be treated with Jones calculus because the latter works with amplitude rather than intensity of light. The effect of a particular optical element is represented by a Mueller matrix; which is a 4×4 matrix and a generalization of the Jones matrix.

Any fully polarized, partially polarized, or unpolarized state of light can be represented by a Stokes vector ($\vec{S}$). Any optical element can be represented by a Mueller matrix (M).

If a beam of light is initially in the state $\vec{S}_i$ and then passes through an optical element M and comes out in a state $\vec{S}_o$, then it is written

$$\vec{S}_o = \mathrm{M}\vec{S}_i \ .$$

If a beam of light passes through optical element M_1 followed by M_2 then M_3 it is written

$$\vec{S}_o = \left(\mathrm{M}_3\left(\mathrm{M}_2(\mathrm{M}_1\vec{S}_i\ )\right)\right) .$$

given that matrix multiplication is associative it can be written

$$\vec{S}_o = \mathrm{M}_3\mathrm{M}_2\mathrm{M}_1\vec{S}_i \ .$$

Beware, matrix multiplication is *not* commutative, so in general

$$\mathrm{M}_3\mathrm{M}_2\mathrm{M}_1\vec{S}_i \neq \mathrm{M}_1\mathrm{M}_2\mathrm{M}_3\vec{S}_i \ .$$

MM and Polarization

- Muller Matrices for some ideal common optical elements:

$$\frac{1}{2}\begin{pmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$ Linear polarizer (Horizontal Transmission)

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{pmatrix}$$ Quarter wave plate (fast-axis vertical)

$$\frac{1}{2}\begin{pmatrix}1&-1&0&0\\-1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$ Linear polarizer (Vertical Transmission)

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}$$ Quarter wave plate (fast-axis horizontal)

$$\frac{1}{2}\begin{pmatrix}1&0&1&0\\0&0&0&0\\1&0&1&0\\0&0&0&0\end{pmatrix}$$ Linear polarizer (+45° Transmission)

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$ Half wave plate (fast-axis vertical)

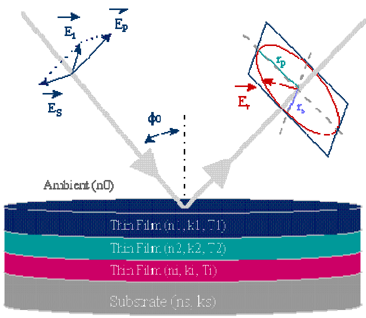
$$\frac{1}{2}\begin{pmatrix}1&0&-1&0\\0&0&0&0\\-1&0&1&0\\0&0&0&0\end{pmatrix}$$ Linear polarizer (-45° Transmission)

$$\frac{1}{4}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$ Attenuating filter (25% Transmission)

Polarization of Electromagnetic Wave
Another view at Ellipsometry

- Muller Matrices for sample measured in Ellipsometry:

$$S^M = \begin{bmatrix}1 & -\cos 2\psi & 0 & 0\\ -\cos 2\psi & 1 & 0 & 0\\ 0 & 0 & \sin 2\psi\cdot\cos\Delta & \sin 2\psi\cdot\cos\Delta\\ 0 & 0 & \sin 2\psi\cdot\cos\Delta & \sin 2\psi\cdot\cos\Delta\end{bmatrix}$$



$$\rho = \frac{r_p}{r_s} = \tan\Psi\cdot e^{i\Delta}$$

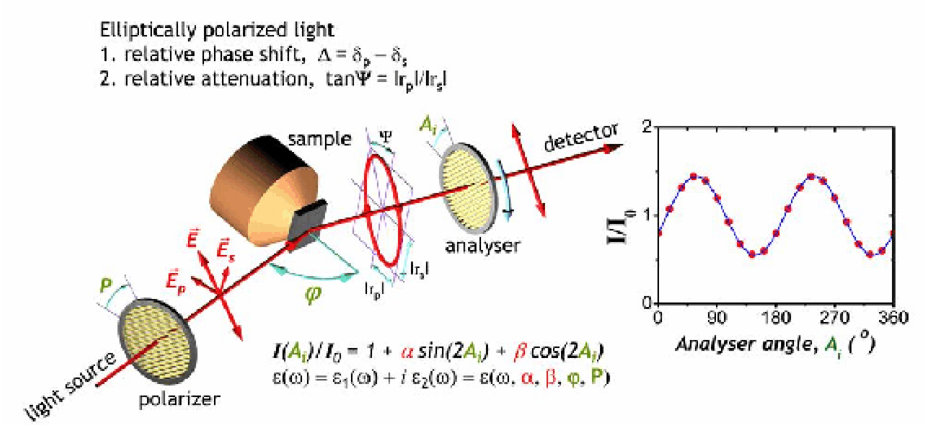
$$\phi_0 \text{ variable, } \tan\Psi = \left|\frac{r_p}{r_s}\right|,\ \Delta = \delta_p - \delta_s$$

Description of Polarization changes

- Muller Matrix for rotation of the coordinate system:

$$S^R = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos 2\theta & \sin 2\theta & 0\\ 0 & \text{-}\sin 2\theta & \cos 2\theta & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

What is far-IR Ellipsometry?



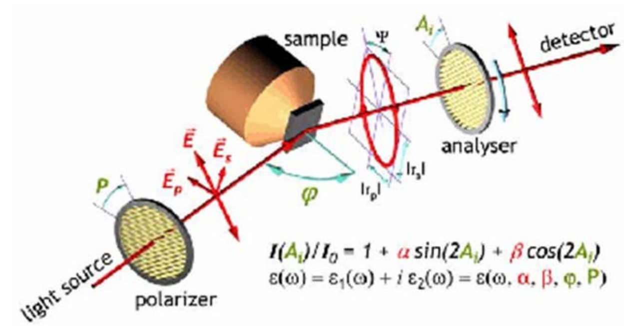
Far-Infrared Ellipsometry is a technique which allows one to measure very accurately and with high reproducibility the complex dielectric function $\varepsilon(\omega) = \varepsilon(\omega)_1 + i\,\varepsilon(\omega)_2$ of oxide thin films and single crystals. It measures the change in polarization of Infrared light upon non-normal reflection on the surface of a sample to be studied. To extend the Ellipsometry technique to the Far-Infrared part of the electromagnetic spectrum, we are going to carry out these experiments at Brookhaven National Laboratory, National Synchrotron Light Source. Synchrotron light provides three orders of magnitude more brilliant light in the Far-Infrared as compared to conventionally available light sources, like mercury arc lamps.

Jones matrices and Ellipsometry

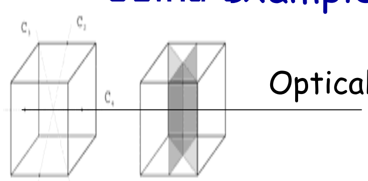
❖ Jones matrix representation for Ellipsometry measurement:

$$E^{Jones}{}_{out} = \hat{J} \cdot E^{Jons}{}_{in}$$

$$E^{Jones}{}_{out} = \hat{J}_{n.-i\ polarizer} \cdot \hat{J}_{rotate(A-A_s)} \cdot \hat{J}_{sample} \cdot \hat{J}_{rotate(-(P-P_s))} \cdot \hat{J}_{n.-i\ polarizer} \cdot E^{Jons}{}_{in}$$

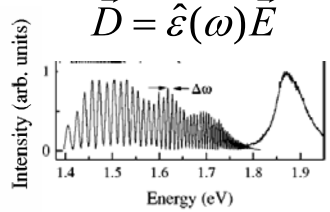


Limitations of the previous formalism Usina example of birefringence

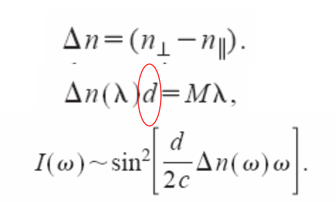


Optical axis Z

$$\vec{D} = \hat{\varepsilon}(\omega)\vec{E}$$



$$\varepsilon_{zz}(\omega) \neq \varepsilon_{xx}(\omega) = \varepsilon_{yy}(\omega)$$



$$\Delta n = (n_\perp - n_\parallel).$$

$$\Delta n(\lambda) d = M\lambda,$$

$$I(\omega) \sim \sin^2\left[\frac{d}{2c}\Delta n(\omega)\,\omega\right].$$

What if we have simultaneous birefringence and optical activity?

Answer:
we will use Jones N– matrices for non-local effects