Energy

Work and Kinetic Energy

Kinetic and Potential Energy

ENERGY

Energy is a property of the state of an object: hard to define precisely

Energy is a scalar quantity. It does not have a direction associated with it

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule = kg·m²/s²

Kinetic Energy

Kinetic Energy = Energy of motion

\[ K = \frac{1}{2}mv^2 \]  for object moving with velocity \( v \)

\[ J = \text{kg m}^2/\text{s}^2 \]
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Kinetic Energy: Orders of Magnitude

\[ K = \frac{1}{2}mv^2 \]

for object moving with velocity \( v \)

Earth orbiting sun: \( 2 \times 10^{29} \) J

Car at 60 mph: \( 100,000 \) J

Nolan Ryan pitch: \( 300 \) J

Professor walking: \( 40 \) J

Angry bee: \( 0.005 \) J

Why \( K = \frac{1}{2}mv^2 \) ?

It’s good that \( K = \frac{1}{2} \ldots \)

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Energy and Work

Kinetic energy

\[ K = \frac{1}{2}mv^2 \quad [J = \text{kg} \cdot \text{m}^2/\text{s}^2] \]

Units of Work and Energy: Joule

Work done by a constant force

\[ W = F \cdot d = F d \cos \theta \]

Work-kinetic energy theorem

\[ \Delta K = K_f - K_i = W \]
**Work**

Work = Energy transferred by a force

Work done on an object is the energy transferred to/from it

\[ W > 0 \rightarrow \text{energy added} \]
\[ W < 0 \rightarrow \text{energy taken away} \]

\[ W = \vec{F} \cdot \vec{r} \equiv \text{Work done on an object by a constant force } \vec{F} \text{ while moving through a displacement } \vec{r} \]

**Dot Product: Physical Meaning**

\[ \vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z \]

\[ \theta = 0 \rightarrow \vec{A} \cdot \vec{B} = AB \]
\[ \theta = 90^\circ \rightarrow \vec{A} \cdot \vec{B} = 0 \]

Dot product measures how much vectors are along each other

**What does \( W = \vec{F} \cdot \vec{r} \) mean?**

- \( W > 0 \) if \( \theta < 90^\circ \) → force is adding energy to object
- \( W < 0 \) if \( \theta > 90^\circ \) → force is reducing energy of object

\[ W = 0 \] if \( \vec{F} = 0 \) or \( \vec{r} = 0 \) or \( \vec{F} \perp \vec{r} \)

**Work Examples**

- Push on a wall: \( W = 0 \) since wall does not move (\( \vec{r} = 0 \))
- A weightlifter does work when lifting a weight: \( W = mgh \) (\( h \) is the vertical drop)
Work Done by a Gravitational Force

Work done by gravitational force

\[ W_g = mgd \cos \theta \]

Tomato thrown upward

\[ W_g < 0 \quad W_g > 0 \]

Lifting/lowering an object

\[ \Delta K = K_f - K_i = W_a + W_g \]

Restoring Force

Equilibrium - no force
Stretched - force towards equilibrium point

\[ F = -kx \]

Hooke’s Law

Work Done by a Spring Force

Hooke’s law:

\[ F = -kx \]

Work done by a spring force:

\[ W_s = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_f^2 \]
Sample Problem 7-8

A block of mass $m = 0.40 \, \text{kg}$ slides across a horizontal frictionless counter with a speed of $v = 0.50 \, \text{m/s}$. It runs into and compresses a spring of spring constant $k = 750 \, \text{N/m}$. When the block is momentarily stopped by the spring, by what distance $d$ is the spring compressed?

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Work Done by a General Variable Force

Work: variable force

$$W = \int_{x_i}^{x_f} F(x) \, dx$$

- Calculus
- Divide area under curve
- Add increments of $W$ (numerically)
- Analytical form?
- Integration!!!

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Quiz 9

1. As a sled is pulled by dogs across a flat, snow-covered field at a constant velocity, net work done on the sled is __________.
2. ... and work done by the air resistance and friction is __________.
3. ... and the work done by dogs is __________.
   a) Positive
   b) Zero
   c) Negative

4. A 200-kg sled is moving along a flat road with initial velocity $5 \, \text{m/s}$. A 10-N friction force is acting on the sled (dogs take some rest). The sled has traveled 100 m. What is its final kinetic energy?

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Power

Work doesn't depend on the time interval

Work to climb a flight of stairs: $3000 \, \text{J}$

10 s

1 min

1 hour

Power is work done per unit time

Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

Instantaneous Power

$$P = \frac{dW}{dt} = F \frac{dx}{dt} = Fv$$

Units

$$\frac{\text{Work}}{\text{time}}$$

$$\frac{1 \, \text{J}}{1 \, \text{s}} = 1 \, \text{Watt}$$

1 hp = 746 W

$$P = \frac{1}{2} \times 50 \, \text{kg} \times (5 \, \text{m/s})^2 / 1 \, \text{s}$$
Sample Problem 7-10: Two constant forces $F_1$ and $F_2$ acting on a box as the box slides rightward across a frictionless floor. Force $F_1$ is horizontal, with magnitude 2.0 N, force $F_2$ is angled upward by 60º to the floor and has a magnitude of 4.0 N. The speed $v$ of the box at a certain instant is 3.0 m/s.

a) What is the power due to each force acting on the box? Is the net power changing at that instant?

b) If the magnitude $F_2$ is, instead, 6.0 N, what is now the net power, and is it changing?
Path Independence of Conservative Forces

Sample Problem 8-1: A 2.0 kg block slides along a frictionless track from point a to point b. The block travels through a total distance of 2.0 m, and a net vertical distance of 0.8 m. How much work is done on the block by the gravitational force?

\[ U = mgy \]

Conservation of Mechanical Energy

Mechanical Energy

\[ E_{mec} = K + U \]

Conservation of Mechanical Energy

\[ K_2 + U_2 = K_1 + U_1 \]

In an isolated system where only conservative forces cause energy changes, the kinetic and potential energy can change, but their sum, the mechanical energy \( E_{mec} \) of the system, cannot change.

Kinetic Energy: \[ K = \frac{1}{2}mv^2 \]

Potential Energy:

- Gravitation:
  \[ U = mgy \]

- Elastic (due to spring force):
  \[ U = \frac{1}{2}kx^2 \]

Conservation of Mechanical Energy

\[ K_2 + U_2 = K_1 + U_1 \]
NYC – SF by train. On a regular schedule
It takes 4 days for 1 way trip
One train per day: starts at 1 pm in NYC
and arrives in 4 days at 1 pm to SF.
Quickly unload – load and go back.

(a) Calculate the work of the engine when the train of the mass 10,000 kg
accelerates to \( v = 72 \text{ km/h} \) from zero at the departure from NYC.

(b) Calculate the work done by the breaks (friction force) when the train slows
down from \( v = 72 \text{ km/h} \) to \( v = 0 \) arriving to SF.

(c) How many other trains will our train meet during one way trip?

- The total energy of a system can change only by amounts of energy that are transferred to or from the system.
- The total energy \( E \) of an isolated system cannot change.

\[ W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} \]

Potential Energy Curve

A plot of \( U(x) \), the potential energy function of a system containing a particle confined to move along the \( x \) axis. There is no friction, so mechanical energy is conserved.

Sample Problem 8-4

A 61.0 kg bungee-cord jumper is on a 45.0 m bridge above a river. The elastic bungee cord has a relaxed length of \( L = 25.0 \text{ m} \). Assume that the cord obeys Hooke’s law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height \( h \) of her feet above the water at her lowest point?
Sample Problem 8-8
A circus beagle of mass \( m = 6.0 \text{ kg} \) runs onto the left end of a curved ramp with speed \( v_0 = 7.8 \text{ m/s} \) at height \( y_0 = 8.5 \text{ m} \) above the floor. It then slides to the right and comes to a momentary stop when it reaches a height \( y = 11.1 \text{ m} \) from the floor. The ramp is not frictionless. What is the increase \( \Delta E_{\text{th}} \) in the thermal energy of the beagle and the ramp because of the sliding?

Sample Problem 7-2
Two industrial spies sliding an initially stationary 225 kg floor safe a displacement \( d \) of magnitude 8.50 m, straight toward their truck. The push \( F_1 \) of spy 001 is 12.0 N, directed at an angle of 30° downward from the horizontal; the pull \( F_2 \) of spy 002 is 10.0 N, directed at 40° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

a) What is the net work done on the safe by the forces \( F_1 \) and \( F_2 \) during the displacement \( d \)?

b) During the displacement, what is the work \( W \) done on the safe by the gravitational force \( F_g \) and what is the work done on the safe by the normal force \( N \) from the floor?

c) The safe is initially stationary. What is its speed \( v_f \) at the end of the 8.50 m displacement?