Lecture 11

Center of Mass
Linear Momentum and Momentum Conservation

http://web.njit.edu/~sirenko/

Physics 105  Fall 2009

Examples for Energy Conservation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$K = \frac{1}{2}mv^2$</td>
<td>$U = mgy$</td>
<td>$U = \frac{1}{2}kx^2$</td>
</tr>
</tbody>
</table>

Total Mechanical Energy = Const.

Equilibrium for fun

How should we define the position of the moving body?

What is $h$ for $U = mgh$?

Take the average Position of mass!

Call “Center of Mass” (com)

Unstable Equil.

Stable Equil.
Center of Mass of a rigid body

For real extended object we should divide the total mass into differential mass element \( dm \)

\[
x_{\text{com}} = \frac{1}{M} \int x \, dm, \quad y_{\text{com}} = \frac{1}{M} \int y \, dm,
\]

Symmetry should help us!

More Examples:

Center of Mass for a system of particles

\( \text{COM} \text{ is inside the body} \)
\( \text{COM is outside the body} \)
Center of Mass for a system of particles

2 bodies, 1 dimension

\[
x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}
\]

Why do we want Center of Mass?

Can treat extended objects or groups of objects as points

\[
\ddot{x}_{\text{CM}} = \frac{F_{\text{tot}}}{M_{\text{tot}}}
\]

Gravity pulls at the COM

Block falls if COM is not supported

Center of Mass for a System of Particles

The center of mass of a body or a system of bodies moves as though all of the mass were concentrated there and all external forces were applied there.

\[
x_{\text{com}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}
\]

General case: \( n \) bodies, 3 dimensions

\[
x_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i, \quad y_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i, \quad z_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i z_i
\]

\[
\vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i
\]
Newton’s 2nd Law for a System of Particles

System of particles
A firework rocket explodes

\[ \vec{F}_{net} = M \vec{a}_{com} \]

Linear Momentum

New fundamental quantity (like force, energy, ...)

Particle:

\[ \vec{p} = m \vec{v} \]

System of Particles:

\[ \vec{P} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + ... \]

Extended objects:

\[ \vec{P} = M \vec{v}_{com} \]

Relation to Force:

\[ \vec{F}_{net} = \frac{d \vec{P}}{dt} \]

Conservation of Linear Momentum

\[ \vec{F}_{net} = \frac{d \vec{P}}{dt} \]

If \( F_{tot} = 0 \), then momentum is constant

For an isolated system (no external forces):

\[ \vec{P} = \text{const.} \Rightarrow \vec{P}_i = \vec{P}_f \]

Even if there are internal forces inside the system

If no net external force acts on a system of particles, the total linear momentum \( P \) of the system cannot change

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum along that axis cannot change

Linear Momentum Conservation

100 g frog jumps to the other end of a motionless floating 200 g log (3 m long).

How far does the log move due to the jump?
100 g frog jumps to the other end of a motionless floating 200 g log (3 m long). How far does the log move due to the jump?

Consider frog and log together as a system of objects. No external forces → system momentum unchanged.

\[
\text{Center of mass of system does not move}
\]

The log moves 1 m to keep the COM at the same position.

**Linear Momentum**

Sample Problem 9-4: The figure shows a 2.0 kg toy car before and after taking a turn on a track. Its speed is 0.50 km/s before the turn and 0.40 km/s after the turn. What is the change \( \Delta P \) in the linear momentum of the car due to the turn?

A 10.0 kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.

\[
\begin{align*}
E_i & = K_i + U_i = \frac{mv_i^2}{2} + mg \Delta y \\
E_f & = K_f + \phi = \frac{mv_f^2}{2} \\
V_f & = \sqrt{\frac{2}{m} \left( \frac{mv_i^2}{2} + mg \Delta y \right)} = \sqrt{v_i^2 + 2g \Delta y} = \sqrt{(4 \text{ m/s})^2 + 2 \times 9.8 \times 3} = 8.6 \text{ m/s}
\end{align*}
\]
What minimum coefficient of kinetic friction $\mu_k$ is required to bring the crate to a stop over a distance of 10 m along the lower surface?

A plot of $U(x)$, the potential energy function of a system containing a particle confined to move along the x axis. There is no friction, so mechanical energy is conserved.