# Lecture 11

**Center of Mass**

**Linear Momentum and Momentum Conservation**

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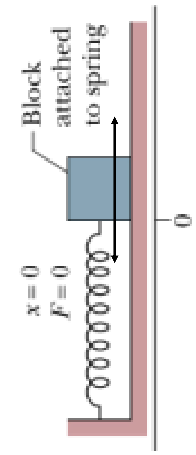
Physics 105 Fall 2009



# Examples for Energy Conservation




Kinetic Energy changes $K = \frac{1}{2}mv^2$

\+ Gravitational Potential Energy $U = mgy$

\+ Elastic Potential Energy $U = \frac{1}{2}kx^2$

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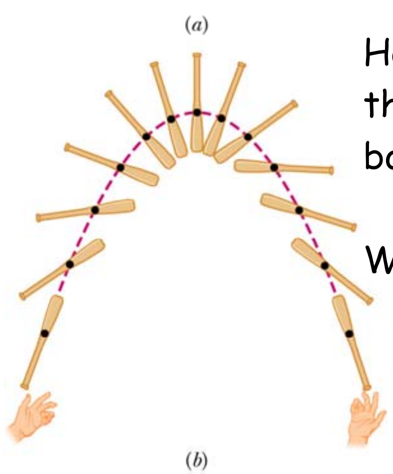
Total Mechanical Energy = *Const.*






How should we define the position of the moving body ?

What is *h* for *U = mgh*

Take the average Position of mass !

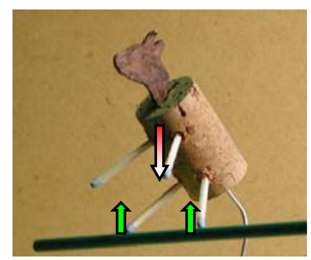
Call "Center of Mass" (com)



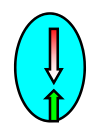
# Equilibrium for fun

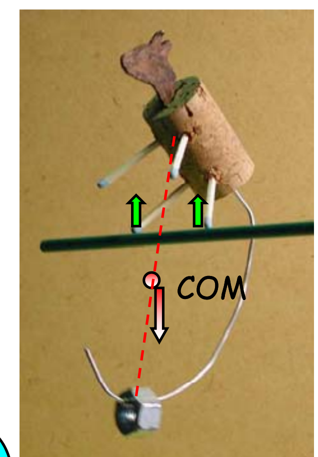
$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

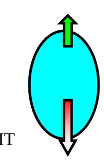
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$



Unstable Equil.

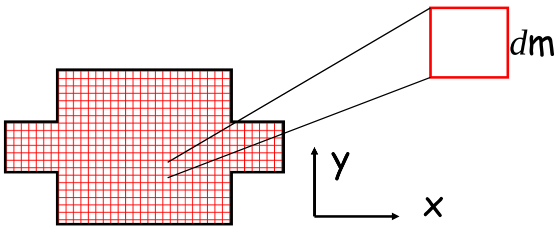




Stable Equil.

## Center of Mass of a rigid body



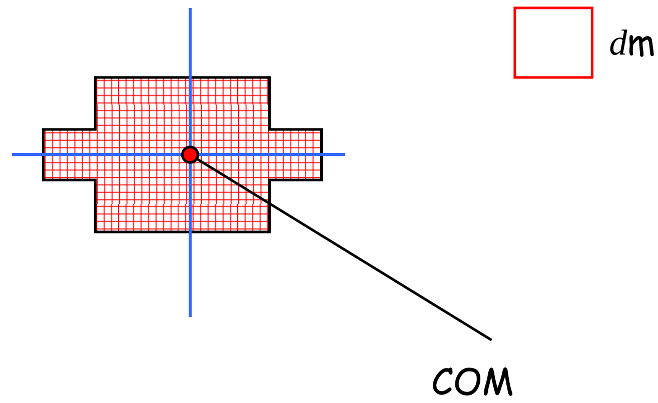

For real extended object we should
Divide the total Mass into differential mass element $dm$

$$x_{com} = \frac{1}{M}\int x\,dm, \quad y_{com} = \frac{1}{M}\int y\,dm,$$

Symmetry should help us !

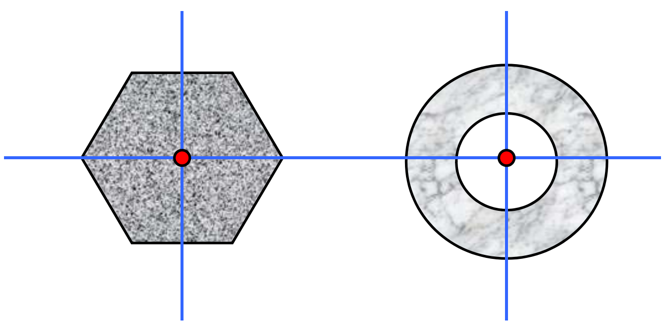


## Symmetry should help us !






## More Examples:



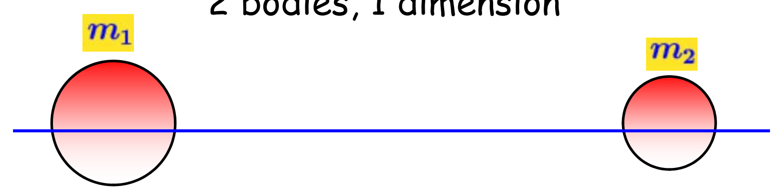
COM is inside the body

COM is outside the body



## Center of Mass for a system of particles

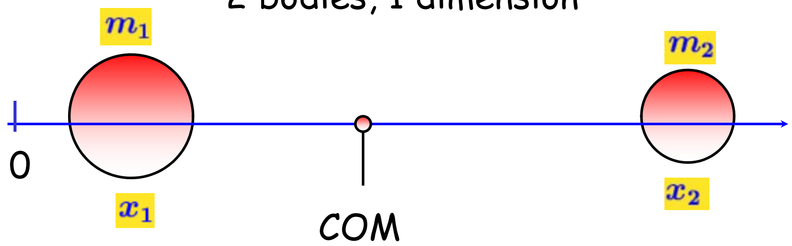
2 bodies, 1 dimension




COM should be on this line

## Center of Mass for a system of particles

2 bodies, 1 dimension

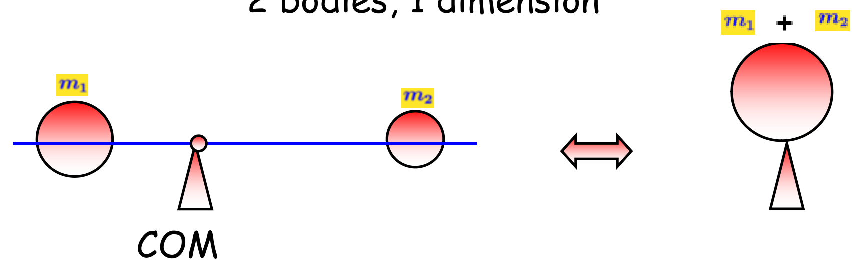



$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



## Center of Mass for a system of particles

2 bodies, 1 dimension




$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

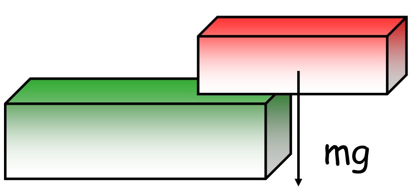


## Why do we want Center of Mass?

Can treat extended objects or groups of objects as points

$\vec{a}_{CM} = F_{tot} / M_{tot}$

Gravity pulls at the COM




Block falls if COM is not supported



## Center of Mass for a System of Particles

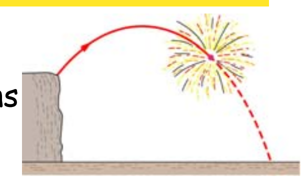
The center of mass of a body or a system of bodies moves as though all of the mass were concentrated there and all external forces were applied there.

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$ 2 bodies, 1 dimension

General case: *n* bodies, 3 dimensions

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

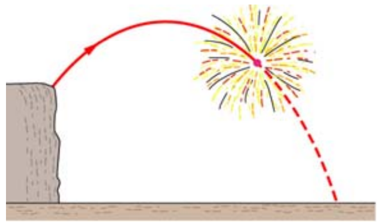
$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$ *n* bodies, 3 dimensions

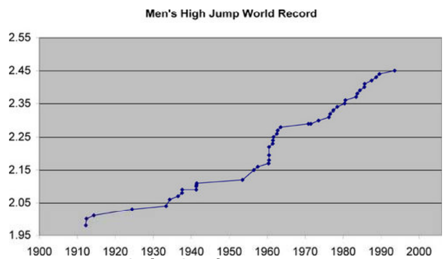


## Newton's 2nd Law for a System of Particles

System of particles A firework rocket explodes

$$\vec{F}_{\text{net}} = M\vec{a}_{\text{com}}$$








## Linear Momentum

New fundamental quantity (like force,energy,…)

Particle: $\vec{p} = m\vec{v}$

System of Particles: $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots$

Extended objects: $\vec{P} = M\vec{v}_{\text{com}}$

Relation to Force: $\vec{F}_{tot} = m\vec{a}$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$$



## Conservation of Linear Momentum

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

If $F_{tot} = 0$, then momentum is constant

For an isolated system (no external forces):

$$\vec{P} = \text{const.} \quad \Rightarrow \quad \vec{P}_i = \vec{P}_f$$

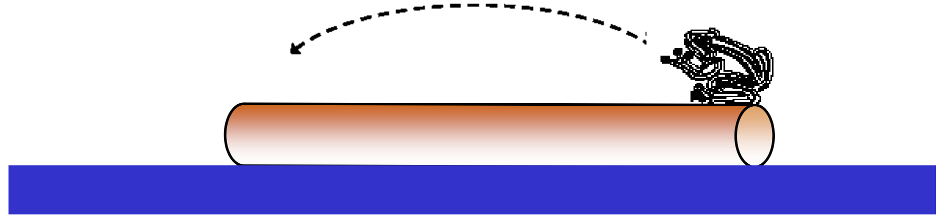
Even if there are internal forces inside the system

If no net external force acts on a system of particles, the total linear momentum ***P*** of the system cannot change

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum along that axis cannot change



## Linear Momentum Conservation



100 g frog jumps to the other end of a motionless floating 200 g log (3 m long).
How far does the log move due to the jump ?

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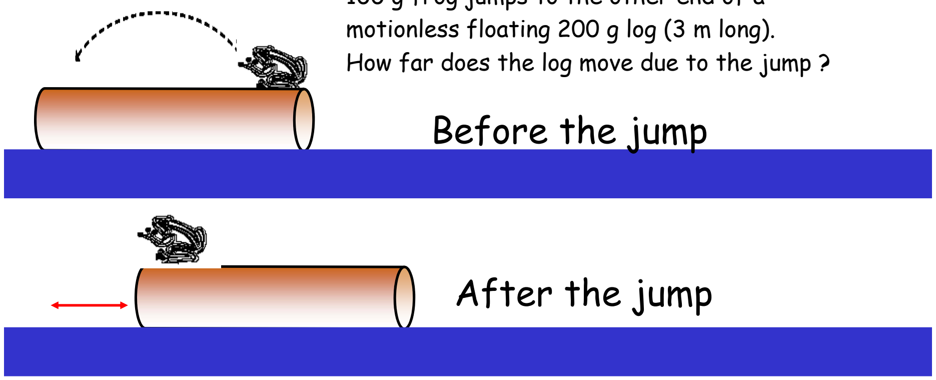


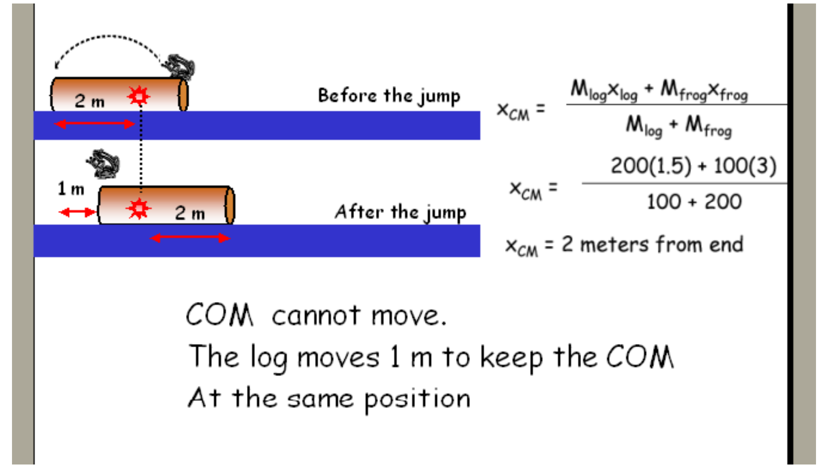

Consider frog and log together as a system of objects

No external forces → system momentum unchanged

$p_{initial} = p_{final} = 0$ →

Center of mass of system does not move





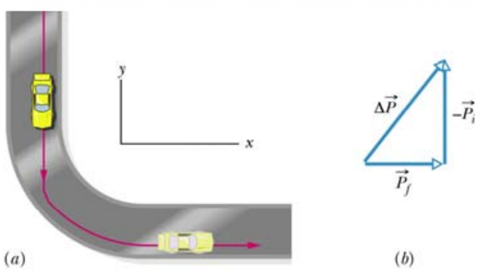

COM cannot move.

The log moves 1 m to keep the COM

At the same position



# Linear Momentum

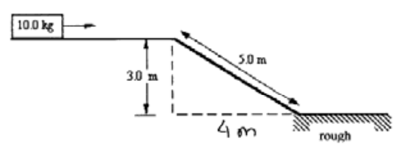



Sample Problem 9-4: The figure shows a 2.0 kg toy car before and after taking a turn on a track. Its speed is 0.50 km/s before the turn and 0.40 km/s after the turn. What is the change ***ΔP*** in the linear momentum of the car due to the turn?



# Problems:

A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.




$K_i = \frac{m v_i^2}{2}$;

$U_i = mg\Delta y$ | $\Delta y = 3m$

--- ∅ Pot. Energy

(a) [2 points] What is the speed of the crate when it arrives at the lower surface?

$E_i = K_i + U_i = \frac{m v_i^2}{2} + mg\Delta y$

$E_f = K_f + \varnothing = \frac{m v_f^2}{2}$

$v_f = \sqrt{\frac{2}{m}} \cdot \sqrt{\frac{m v_i^2}{2} + mg\Delta y} = \sqrt{v_i^2 + 2g\Delta y} =$

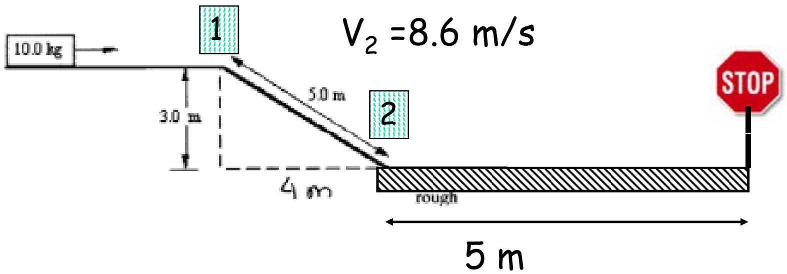
$= \sqrt{(4\,m/s)^2 + 2 \cdot 9.8\,\frac{m}{s^2} \cdot 3\,m} = 8.6\,\frac{m}{s}$

# QZ # 11

A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.

$V_1$ =4.0 m/s

$V_2$ =8.6 m/s

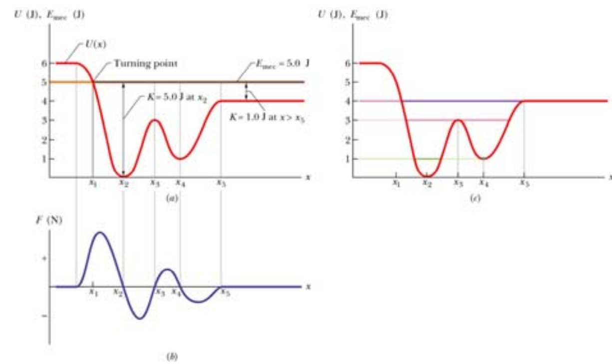



What minimum coefficient of kinetic friction

$\mu_k$ is required to bring the crate to a stop over a distance of 10 m along the lower surface ?



# Potential Energy Curve




A plot of U(x), the potential energy function of a system containing a particle confined to move along the x axis. There is no friction, so mechanical energy is conserved.

1D Motion

$$F(x) = -\frac{dU(x)}{dx}$$

Turning Points

Equilibrium Points

- Neutral Equilibrium
- Unstable Equilibrium
- Stable Equilibrium