Lecture 3

Vectors
- Free Fall again
- Intro to the Motions in Two and Three Dimensions

http://web.njit.edu/~sirenko/

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Chapter 3: Vectors
- Vectors and Scalars
- Adding Vectors Geometrically
- Components of Vectors
- Unit Vectors
- Adding Vectors by Components
- Vectors and the Laws of Physics
- Multiplying Vectors
  - Scalar Product
  - Vector or Cross Product

Writing Vectors

We need to distinguish vectors from other quantities (scalars)

Common notation:
Bold face: \( \mathbf{c} \) or Arrow: \( \vec{c} \)
Vectors and Scalars

Displacement
Path length and Displacement

Components of Vectors:
- aligned along axis
- add to give vector
- are vectors

Length (Magnitude)

Trig Review

Unit Vectors and Coordinate Systems
2D (2 dimensions)
3D (3 dimensions)
Unit Vectors

Components of a vector are still vectors
\[ \vec{D} = D_x \hat{i} + D_y \hat{j} \]

Vectors have units (i.e. m/s)
\[ \hat{i} \rightarrow x \]
\[ \hat{j} \rightarrow y \]
\[ \hat{k} \rightarrow z \]

Unit vectors
Dimensionless
Used to specify direction

Magnitude + sign
Unit Vector

Laws of Vector Addition

\[ \vec{a} + \vec{b} = \vec{b} + \vec{a} \]
\[ \vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \]

Vector sum
Start
Finish

Example

\[ \vec{A} = 12m \cdot \hat{i} + 5m \cdot \hat{j} \]
\[ \vec{B} = 2m \cdot \hat{i} - 5m \cdot \hat{j} \]

\[ \vec{C} = \vec{A} + \vec{B} \]
\[ = (12m \cdot \hat{i} + 5m \cdot \hat{j}) + (2m \cdot \hat{i} - 5m \cdot \hat{j}) \]
\[ = 14m \cdot \hat{i} \]
Vector Multiplication

Scalar product

\[ \vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z \]

\(\theta\) is the angle between the vectors if you put their tails together

\[ \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \]

since \(\cos(\theta) = \cos(-\theta)\)

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Last Lecture:
Motion along the straight line + Vectors

**One dimension (1D)**

- Position: \(x(t) \text{ m}\)
- Velocity: \(v(t) \text{ m/s}\)
- Acceleration: \(a(t) \text{ m/s}^2\)

**Three dimension (2D)**

- Position: \(r(t) \text{ m}\)
- Velocity: \(v(t) \text{ m/s}\)
- Acceleration: \(a(t) \text{ m/s}^2\)

All are *vectors*: have direction and magnitude.

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### TABLE 2-1 Equations for Motion with Constant Acceleration

<table>
<thead>
<tr>
<th>Equation Number</th>
<th>Equation</th>
<th>Missing Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-11</td>
<td>(v = v_0 + at)</td>
<td>(x - x_0)</td>
</tr>
<tr>
<td>2-15</td>
<td>(x - x_0 = v_0 t + \frac{1}{2} at^2)</td>
<td>(v)</td>
</tr>
<tr>
<td>2-16</td>
<td>(v^2 = v_0^2 + 2a(x - x_0))</td>
<td>(a)</td>
</tr>
<tr>
<td>2-17</td>
<td>(x - x_0 = \frac{1}{2}(v_0 + v)t)</td>
<td>(a)</td>
</tr>
<tr>
<td>2-18</td>
<td>(x - x_0 = vt - \frac{1}{2} at^2)</td>
<td>(v_0)</td>
</tr>
</tbody>
</table>

*Make sure that the *acceleration* is indeed constant before using the equations in this table.*

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Relative Motion/Reference Frames

\[ \vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA} \]

\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \text{ and } \vec{v}_{BA} = \text{const.} \]

\[ \vec{a}_{PA} = \vec{a}_{PB} \]
Relative Motion/Reference Frames

Relative Velocity: Rowing a Boat
You can row a boat at $v_{\text{row}} = 3 \text{ m/s}$, and you want to go straight across a river which flows with $v_{\text{river}} = 2 \text{ m/s}$. At what angle should you row?

\[ \vec{v}_{\text{boat}} = \vec{v}_{\text{row}} + \vec{v}_{\text{river}} \]

you want $\vec{v}_{\text{boat}}$ in y-direction to go straight across

1. What is the length (or magnitude) of the vector $C$ if $C = A + B$?
   \[ |C| = \text{???} \]

2. What is the angle between vectors $A$ and $B$?
   \[ \theta = \text{???} \]

3. What is the scalar (dot) product of the same vectors $A$ and $B$:
   \[ (A \cdot B) = \text{???} \]

4. (huge extra credit) What is the magnitude of the vector (cross) product of the same vectors $A$ and $B$:
   \[ |A \times B| = \text{???} \]

   Hint: $i$ and $j$ are the unit vectors.