# Lecture 3

- **Vectors**
- **Free Fall again**
- **Intro to the Motions in Two and Three Dimensions**

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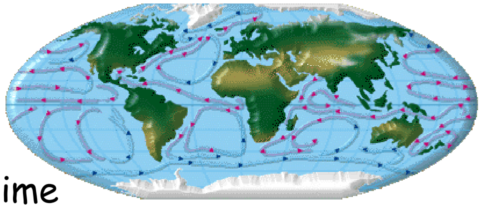
Physics 105, Fall 2009



# Velocity is a vector !



Velocity has direction !
Velocity can change with time



# Chapter 3: Vectors

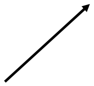
- Vectors and Scalars
- Adding Vectors Geometrically
- Components of Vectors
- Unit Vectors
- Adding Vectors by Components
- Vectors and the Laws of Physics
- Multiplying Vectors
  - Scalar Product
  - Vector or Cross Product



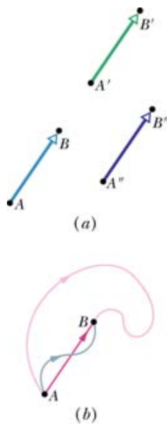
# Writing Vectors

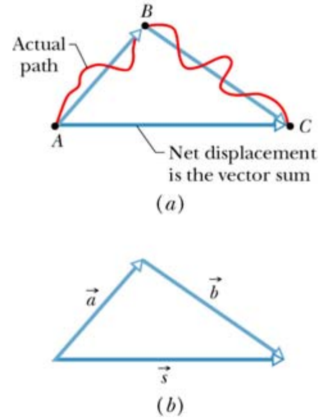
We need to distinguish vectors
From other quantities (scalars)

Common notation:
Bold face: **c** or Arrow: $\vec{c}$



# Vectors and Scalars



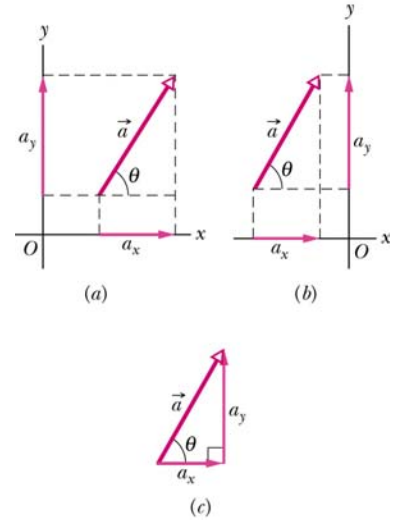

Displacement

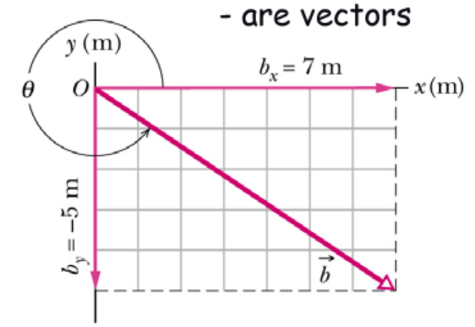
Path length and Displacement



# Components of Vectors:

- aligned along axis
- add to give vector
- are vectors



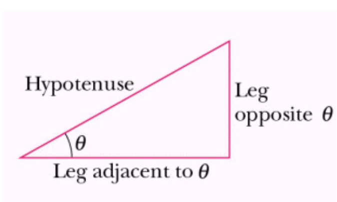

$a_x = a\cos\theta$ and $a_y = a\sin\theta$

$a = \sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \frac{a_y}{a_x}$

Length (Magnitude)



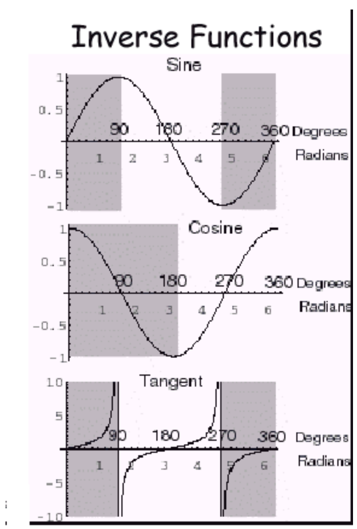
# Trig Review




$$\sin\theta = \frac{\text{leg opposite }\theta}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{leg adjacent to }\theta}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{leg opposite }\theta}{\text{leg adjacent to }\theta}$$

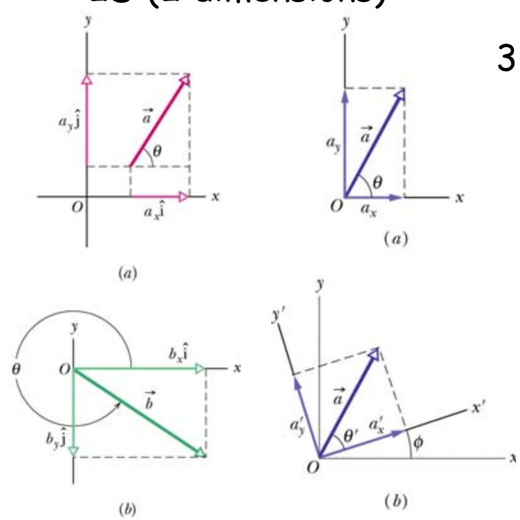


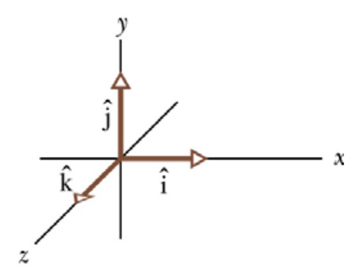



# Unit Vectors and Coordinate Systems

2D (2 dimensions)

3D (3 dimensions)



# Unit Vectors

Components of a vector are still vectors

$$\vec{D} = \vec{D}_x + \vec{D}_y$$

Vectors have units (i.e. m/s)

$\hat{i} \rightarrow x$

$\hat{j} \rightarrow y$

$\hat{k} \rightarrow z$

Unit vectors

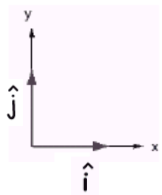
Unit Magnitude
Dimensionless

Used to specify direction

$$\vec{D} = D_x \hat{i} + D_y \hat{j}$$

Magnitude + sign

Unit Vector






# Vector Addition

## Consider Two Vectors

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j}$$

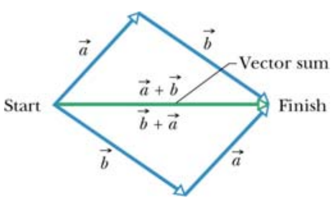
$$\vec{A} + \vec{B} = \left(A_x \hat{i} + A_y \hat{j}\right) + \left(B_x \hat{i} + B_y \hat{j}\right)$$

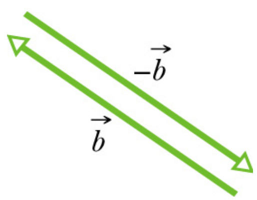
$$= \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$$

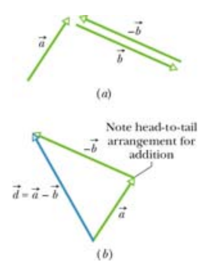
Just add components.



# Laws of Vector Addition

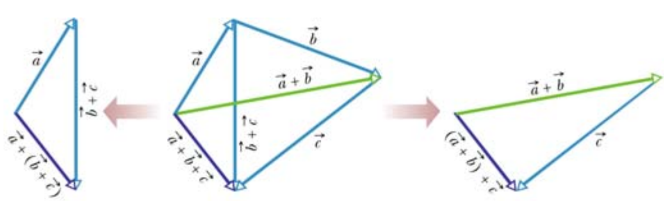



$\vec{a} + \vec{b} = \vec{b} + \vec{a}$

$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$




$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$



# Example

$$\vec{A} = 12m \cdot \hat{i} + 5m \cdot \hat{j}$$

$$\vec{B} = 2m \cdot \hat{i} - 5m \cdot \hat{j}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$= \left(12m \cdot \hat{i} + 5m \cdot \hat{j}\right) + \left(2m \cdot \hat{i} - 5m \cdot \hat{j}\right)$$

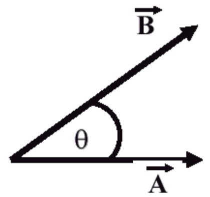
$$= 14m \cdot \hat{i}$$

# Vector Multiplication

## Scalar product

$$\vec{A} \bullet \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

θ is the angle between the vectors if you put their tails together




$$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$$

since cos(θ) =cos(-θ)



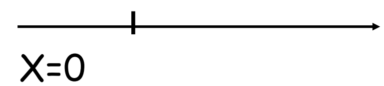
# Last Lecture:

## Motion along the straight line + Vectors

**One dimension (1D)**

| | |
|---|---|
| Position: | $x(t)$ m |
| Velocity: | $v(t)$ m/s |
| Acceleration: | $a(t)$ m/s$^2$ |

All are vectors: have direction and magnitude.




**Three dimension (2D)**

| | |
|---|---|
| Position: | $\vec{r}(t)$ m |
| Velocity: | $\vec{v}(t)$ m/s |
| Acceleration: | $\vec{a}(t)$ m/s$^2$ |

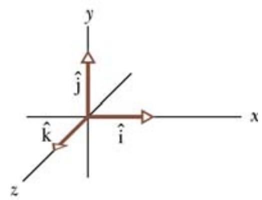




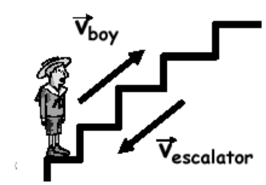

**TABLE 2-1 Equations for Motion with Constant Acceleration**[a]

| Equation Number | Equation | Missing Quantity |
|---|---|---|
| 2-11 | $v = v_0 + at$ | $x - x_0$ |
| 2-15 | $x - x_0 = v_0 t + \frac{1}{2}at^2$ | $v$ |
| 2-16 | $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ |
| 2-17 | $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ |
| 2-18 | $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ |

[a] Make sure that the acceleration is indeed constant before using the equations in this table.



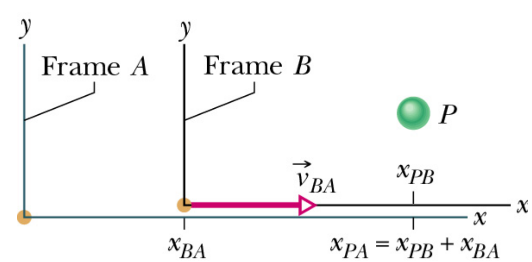
# Relative Motion/Reference Frames

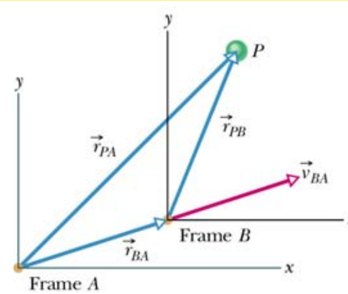



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \quad \text{and} \quad \vec{v}_{BA} = \text{const.}$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$

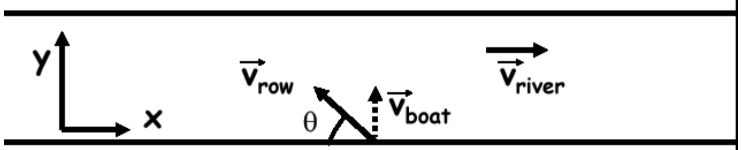


# Relative Motion/Reference Frames

## Relative Velocity: Rowing a Boat

You can row a boat at $v_{row}$ = 3 m/s, and you want to go straight across a river which flows with $v_{river}$ = 2 m/s. At what angle should you row?




$\vec{v}_{boat} = \vec{v}_{row} + \vec{v}_{river}$

you want $\vec{v}_{boat}$ in y-direction to go straight across

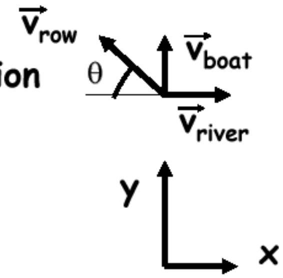


# Relative Motion/Reference Frames

## Rowing a Boat (continued)

$\vec{v}_{boat} = \vec{v}_{row} + \vec{v}_{river}$

you want $\vec{v}_{boat}$ in y direction




need $v_{row,x} = - v_{river,x}$

$v_{row} \cos\theta = v_{river}$

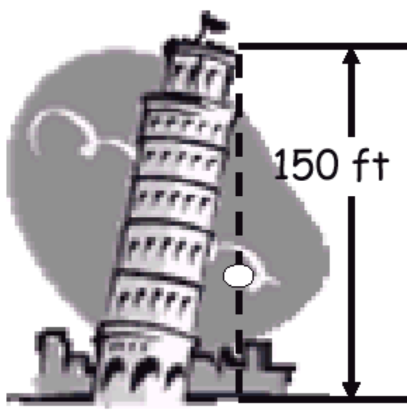


# Gravitation is universal

Most important case of constant acceleration: Free fall

$a = -g$ where $g = 9.8$ m/s$^2$

(defining "up" as the positive direction)

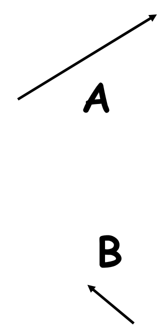



Gravitational acceleration does not depend on the nature of the material or the mass of the object.



# Lecture QZ3

**A**=(4m)·**i** + (2m)·**j** and **B** = (–1m)·**i** + (2m)·**j**




1. What is the length (or magnitude) of the vector **C** if **C**=**A**+**B**
   **│C│=???**
2. What is the angle between vectors **A** and **B**
   **θ=???**
3. What is the scalar (dot) product of the same vectors **A** and **B:**
   **(A·B)=???**
4. **(huge extra credit)** What is the magnitude of the vector (cross) product of the same vectors **A** and **B:**
   **│A×B│=???**

*Hint:* **i** and **j** are the unit vectors.