Lecture 6

Forces & three Newton's Laws
Friction Force, Centripetal Force.
Problem-Solving Tactics

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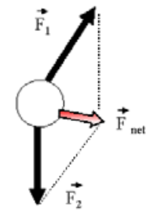
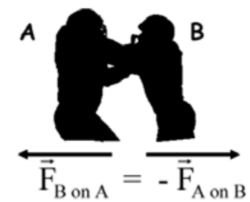
Physics 105; Fall 2009

Newton's Laws

I. If no net force acts on a body, then the body's *velocity* cannot change.
II. The net force on a body is equal to the product of the body's mass and acceleration.
III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction.

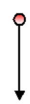
$$\vec{F}_{net} = m\vec{a}$$



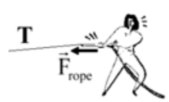


Forces:

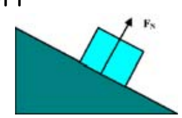
- Gravitational Force: $\vec{F}_g = m\vec{g}$ down to the ground



- Tension Force: $\vec{T}$ along the string

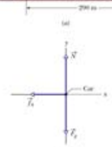


- Normal Force: $\vec{N}$ perpendicular to the support

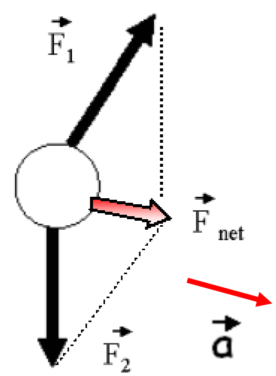


- Friction Force
  - Static; maximum value $f_s = \mu_{st}N$
    opposite to the component of other forces parallel to the support
  - Kinetic; value $f_k = \mu_{kin}N$
    opposite to the **velocity**, parallel to the support

$\mu_{st} > \mu_{kin}$

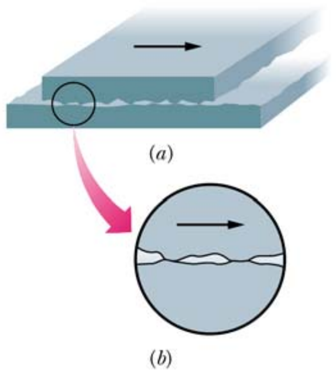


Net Force (or Total Force)



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$\vec{F}_{net} = m\vec{a}$$

Static and Kinetic Friction



Static frictional force

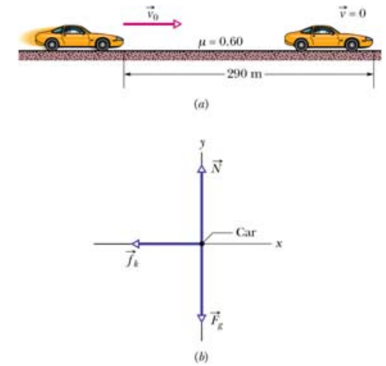
$f_{s,\max} = \mu_s N$

Kinetic frictional force

$f_{k,\max} = \mu_k N$

Kinetic Friction Force

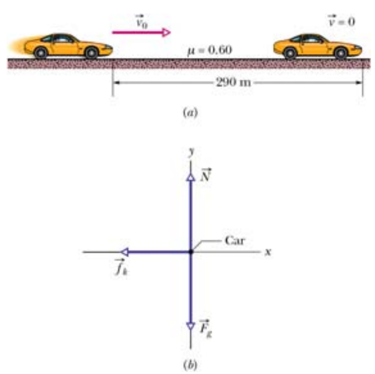
$f_{k,\max} = \mu_k N$



Skid marks are 290 m long! μ_k = 0.6 and ***a*** = const. How fast was the car going when the wheels became locked?

Kinetic Friction

$f_{k,\max} = \mu_k N$



Skid marks are 290 m long! μ_k = 0.6 and ***a*** = const. How fast was the car going when the wheels became locked?

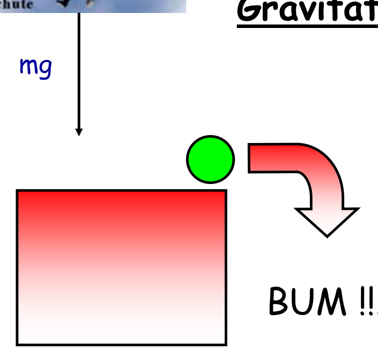
$v^2 = v_0^2 + 2a(x-x_0)$

$-f_k = ma$

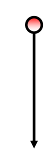
$a = F/m = -\mu mg/m = -\mu g$

$v_0 = (2\mu g(x-x_0))^{\frac{1}{2}}$; v = 58 m/s = 210 km/h= 130 ml/h

Gravitational Force

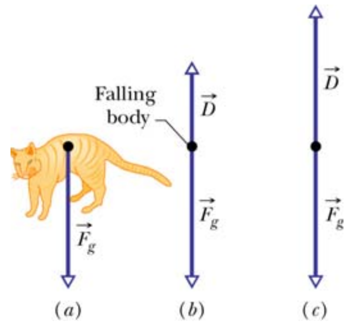


FBD



$g = 9.8\ m/s^2$

Drag Force and Terminal Speed



Drag force

$$D = \frac{1}{2} C\rho A v^2$$

Terminal speed

$$v_t = \sqrt{\frac{2F_g}{c\rho A}}$$

Drag coefficient *C*, air density ρ, and effective cross-section *A*.

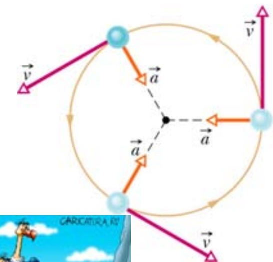
Uniform Circular Motion Centripetal Force

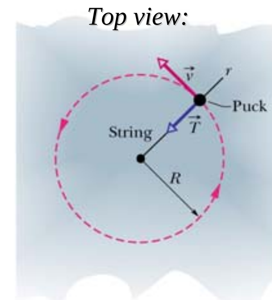
Centripetal acceleration

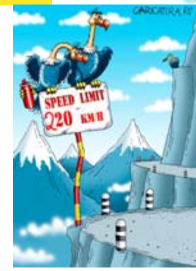
$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$







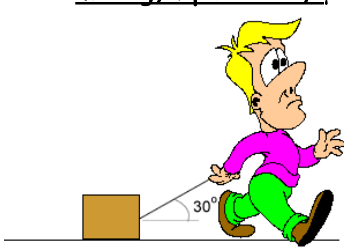
Centripetal force : F = ma

$$F = m\frac{v^2}{r}$$

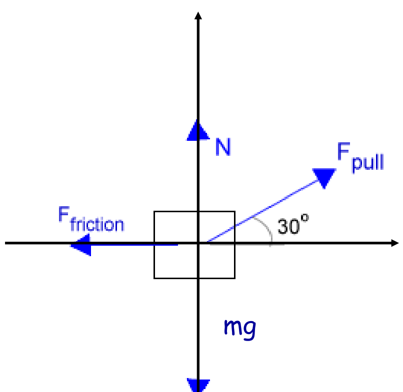
EXAMPLES of Free Body Diagrams

Picture of Situation

m=5kg, Fpull=50N, μ=0.4



FBD



X: - N *μ +Fpull*cos30⁰ = ma_x

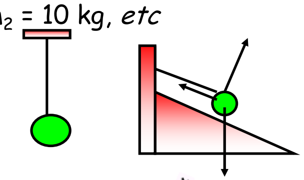
Y: N-mg+Fpull*sin30⁰ = ma_y =0

N-5*9.8+50*0.5=0; N=25[N]

-25[N]*0.4+50*0.86=ma =33[N]; a=33[N]/5kg=6.6 m/s^2

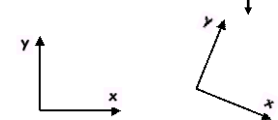
Problem-Solving Tactics :

- Identify the body / bodies
  Examples: block, puck, sphere, knot, pulley, penguin, *etc.*
- Identify the masses of the bodies: m_1 = 5 kg, m_2 = 10 kg, *etc*
- Make a sketch to visualize the Problem



- Make a choice for the coordinate system (x-y)
  Recommended:
  x – horizontal
  y – vertical, or

  x – along the plane of support and
  y – perpendicular to the plane of support



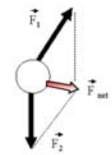
Problem-Solving Tactics (cont.):

- Identify the conditions of the body (moving or at rest)

  at rest means $\vec{F}_{net} = 0$

  if moving, then

  - moving with a constant velocity $\vec{F}_{net} = 0$
  - accelerating $\vec{F}_{net} \neq 0$ $\vec{F}_{net} = m\vec{a}$



Problem-Solving Tactics (cont.):

- Identify all Forces and their directions:

  $m\vec{g}$ down to the ground (always)

  $\vec{T}$ along the string (if any)

  $\vec{N}$ perpendicular to the support (if any)

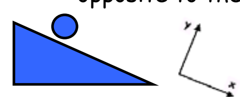
  $f_s = \mu_{st}N$ (only for the max value of the force)

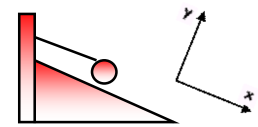
  friction force (if any)

  - Static Friction; maximum value $F_{fr} = \mu_{st}N$

    opposite to the component of other forces parallel to the support
  - Kinetic Friction; value $F_{fr} = \mu_{kin}N$

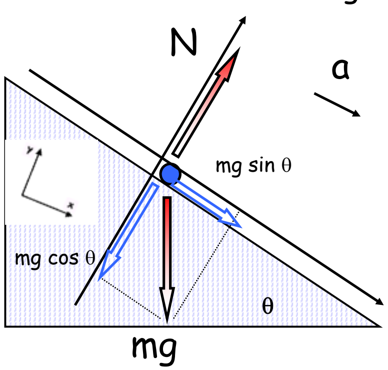
    opposite to the velocity, parallel to the support

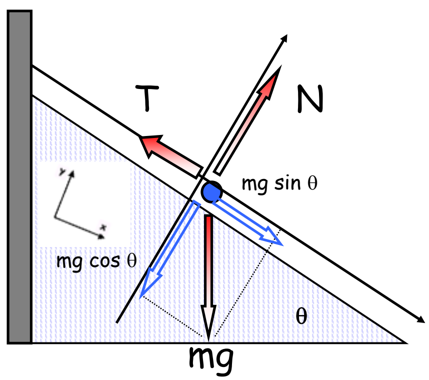




Problem-Solving Tactics (cont.):

- Do the calculations using FBD





For X: $F_{net} = mg \sin\theta$; $a = g \sin\theta$

For Y: $F_{net} = N - mg \cos\theta = 0$

$a = g \sin\theta$

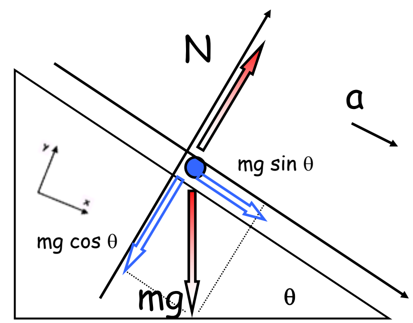
For X: $F_{net} = mg \sin\theta - T = 0$

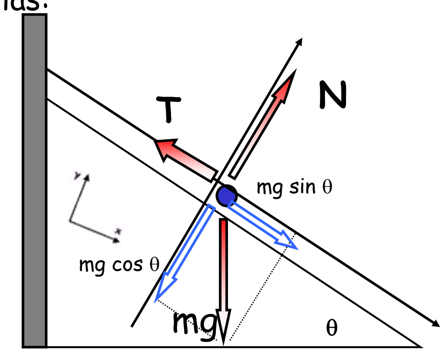
For Y: $F_{net} = N - mg \cos\theta = 0$

$ma = 0$

Problem-Solving Tactics (cont.):

- Plug the numbers in the formulas:





For X: $F_{net} = mg \sin\theta$; $a = g \sin\theta$

For Y: $F_{net} = N - mg \cos\theta = 0$

$a = g \sin\theta$

For $\theta = 30°$, $a = 9.8/2\ m/s^2 = 4.9\ m/s^2$

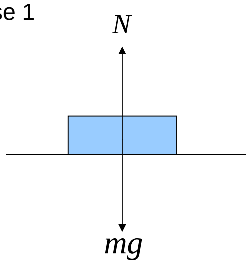
For X: $F_{net} = mg \sin\theta - T = 0$

For Y: $F_{net} = N - mg \cos\theta = 0$

$ma = 0;$

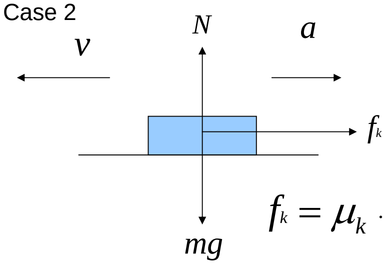
$N = mg \cos\theta$; $T = mg \sin\theta$

Case 1



$$f_{st} = mg \cdot \sin\theta = 0$$

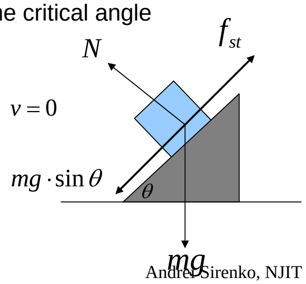
Case 2



$$f_k = \mu_k \cdot N$$

$$f_k = mg \cdot \cos\theta \cdot \mu_k = mg\mu_k$$

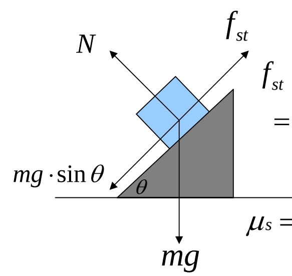
Case 3: below the critical angle



$$N = mg \cdot \cos\theta$$

$$f_{st} = mg \cdot \sin\theta$$

Case 4: At the critical angle

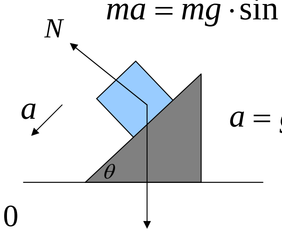


$$f_{st} = mg \cdot \sin\theta^{max} = $$
$$= mg \cdot \cos\theta^{max} \cdot \mu_{st}$$

$$\mu_s = \tan\theta^{max}$$

Case 5

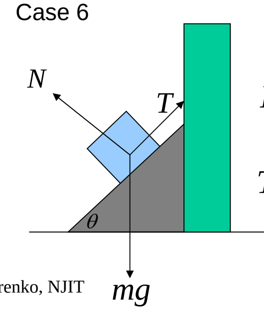
$$N = mg \cdot \cos\theta$$
$$ma = mg \cdot \sin\theta$$



$$a = g \cdot \sin\theta$$

$$f = 0$$

Case 6



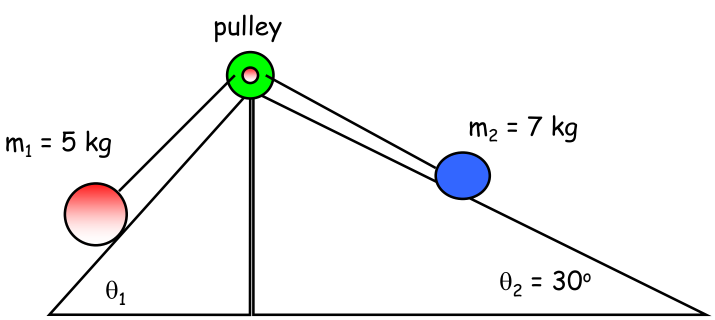
$$N = mg \cdot \cos\theta$$

$$T = mg \cdot \sin\theta$$

Problem #1

two masses are at equilibrium (no acceleration, no friction)

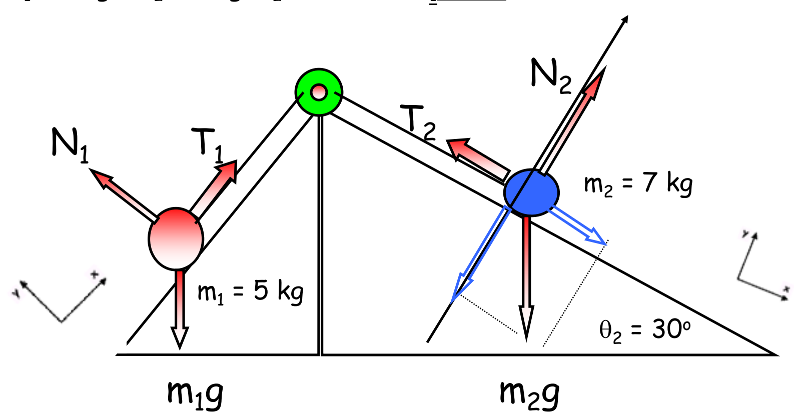
m_1 = 5 kg, m_2 = 7 kg; θ_2 = 30° <u>θ_1 = ???</u>



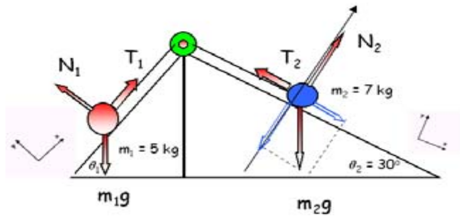
Problem #1

two masses are at equilibrium (no acceleration, no friction)

m_1 = 5 kg, m_2 = 7 kg; θ_2 = 30° <u>θ_1 = ???</u>



Problem #1



For X1: $F_{net} = -m_1 g \sin\theta_1 + T_1 = 0$

For Y1: $F_{net} = N_1 - m_1 g \cos\theta_1 = 0$

$m_1 a = 0;$

$N_1 = m_1 g \cos\theta_1$; $T_1 = m_1 g \sin\theta_1$

For X2: $F_{net} = m_2 g \sin\theta_2 - T_2 = 0$

For Y2: $F_{net} = N_2 - m_2 g \cos\theta_2 = 0$

$m_2 a = 0;$

$N_2 = m_2 g \cos\theta_2$; $T_2 = m_2 g \sin\theta_2$

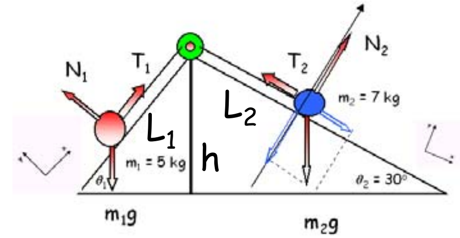
$T_1 = T_2 \Rightarrow m_1 g \sin\theta_1 = m_2 g \sin\theta_2$

$m_1/m_2 = \sin\theta_2 / \sin\theta_1$

$\sin\theta_1 = m_2 \sin\theta_2 / m_1 = 7 \text{ kg} * \sin(30°)/5 \text{ kg} = 0.7;$

$\underline{\theta_1 = 44°}$

Problem #1



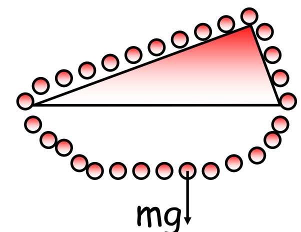
$L_1 = h / \sin\theta_1$

$L_2 = h / \sin\theta_2$

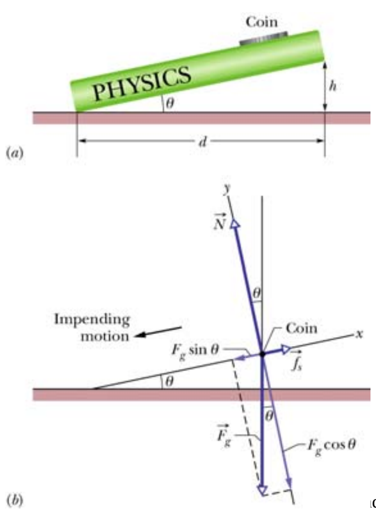
$m_1/m_2 = \sin\theta_2 / \sin\theta_1 = (h / \sin\theta_2) / (h / \sin\theta_1)$

$m_1/m_2 = L_1 / L_2$

Is the chain going to move ???

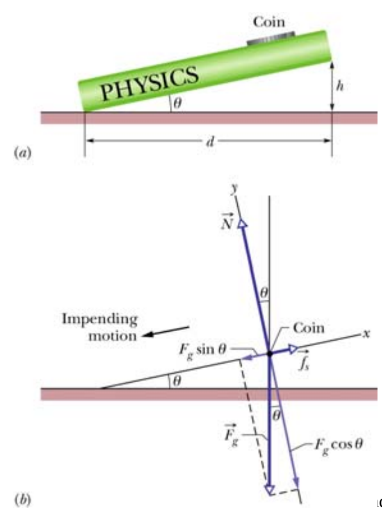


Sample Problem



A coin of mass ***m*** rests on a book that has been tilted at an angle θ with the horizontal. When θ is increased to 13°, the coin is on the verge of sliding down the book. What is the coefficient of static friction μ_s between the coin and the book?

Sample Problem



$f_{s,\max} = \mu_s N$

For X: $F_{net} = 0 = -mg \sin\theta + f_s$

For Y: $F_{net} = 0 = N - mg \cos\theta$

$N = mg \cos\theta$;

<u>ma = 0 along X direction:</u>

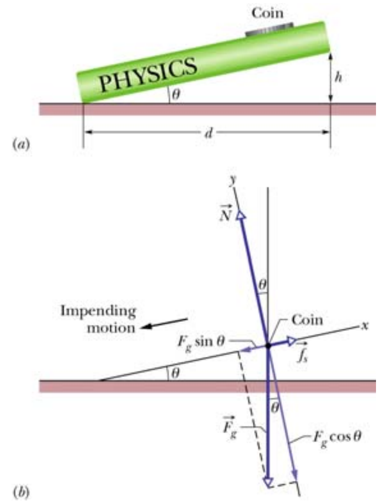
$0 = -g(\sin\theta - \mu_s \cos\theta)$

$\sin\theta - \mu_s \cos\theta = 0$

$\tan\theta = \mu_s$

$\mu_s = \tan(13°) = \underline{0.23}$

Sample Problem (cont.)



θ is increased to 20° (the max angle is 13°),

the coefficient of static friction $\mu_s = 0.23$

the coefficient of kinetic friction $\mu_k = 0.15$

What is the coin acceleration ?

<u>$ma \neq 0$:</u>

For X: $F_{net} = ma = -mg \sin\theta + f_k$

For Y: $F_{net} = N - mg \cos\theta = 0$

$N = mg \cos\theta$;

$ma = -mg \sin\theta + f_k = -mg \sin\theta + \mu_k N =$

$= -mg \sin\theta + \mu_k mg \cos\theta =$

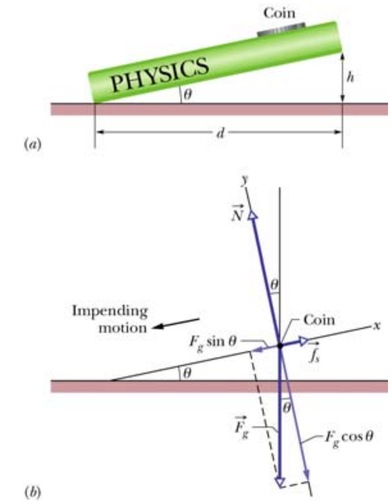
$= -mg (\sin\theta - \mu_k \cos\theta)$;

$a = -g (\sin\theta - \mu_k \cos\theta)$;

$a = -9.8*(\sin 20^\circ - 0.15* \cos 20^\circ)\ m/s^2 =$

$= -9.8*(0.34-0.14)\ m/s^2 = -2\ m/s^2$

Sample Problem (cont.)



θ is decreased to stop the coin.

At what angle it will <u>move with a constant speed</u> ?

the coefficient of static friction $\mu_s = 0.23$

the coefficient of kinetic friction $\mu_k = 0.15$

<u>$ma = 0$:</u>

$a = -g (\sin\theta - \mu_k \cos\theta)$

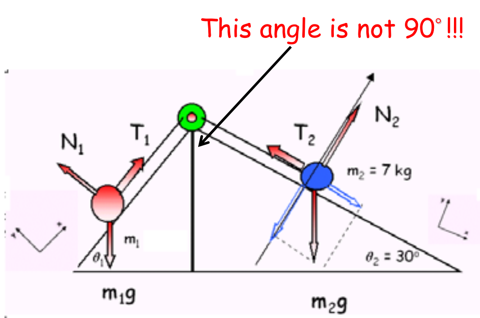
$\sin\theta - \mu_k \cos\theta = 0$

$\tan\theta = \mu_k$

$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(\mathbf{0.15}) = \underline{8.5^\circ}$

Note the difference:
13° and 8.5°

QZ #6



$m_1/m_2 = \sin\theta_2 / \sin\theta_1$

$m_2 = 7$ kg; $\theta_2 = 30^\circ$ and we can vary m_1 and θ_1; <u>Neglect friction</u>

1. What is the <u>smallest</u> mass m_1 which can balance

   $m_2 = 7$ kg; $\theta_2 = 30^\circ$ $m_1 = ???$

2. At what angle the <u>smallest</u> mass m_1 can balance

   $m_2 = 7$ kg; $\theta_2 = 30^\circ$ $\theta_1 = ???$

3. If we cut the string, which object (#1 with the mass m_1 at the angle θ_1 or object #2 with $m_2 = 7$ kg; $\theta_2 = 30^\circ$) will have a bigger magnitude of acceleration ??? (note that $a = g \sin\theta$),

4. Make a sketch and show the direction of a_1 and a_2