Combination of Forces: Net Force
Dealing with Multiple Forces

If multiple forces are acting on the same object, the net force determines the acceleration.

\[
\vec{F}_{net} = \vec{F}_1 + \vec{F}_2
\]

\[
F_{net} = ma
\]

Use a free body diagram to keep track of the forces on one object.

Uniform Circular Motion
Centripetal Force

Centripetal acceleration

\[
a = \frac{v^2}{r}
\]

Period

\[
T = \frac{2\pi r}{v}
\]

Centripetal force: 

\[
F = ma
\]
Centripetal Force is a combination of:

- **Gravitational Force:** down to the ground
- **Tension Force:** along the string
- **Normal Force:** perpendicular to the support
- **Static Friction Force:** maximum value $F_{fr}^{max} = \mu_{st}N$

\[ \text{Normal Force: } N = m - mg \]
\[ \text{ma} = \frac{mv^2}{R} \]

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**Centripetal Force and Tension Force:**

\[ \text{ma} = T \]
\[ \text{ma} = \frac{mv^2}{R} = T \]

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**Centripetal Force and Kinetic Friction Force:**

\[ \text{ma}_c = T \]
\[ \text{ma}_c = \frac{mv^2}{R} = T \]

Kinetic friction does not affect Centripetal acceleration directly

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**Net Force and Centripetal Force**

How fast can we turn?

for a flat road: $\frac{v^2}{r} = f_s$

max speed: $\frac{v_{max}^2}{r} = f_{fr}^{max} = N\mu_s = mg\mu_s$; $v_{max} = \sqrt{\mu_sgr}$

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Net Force and Centripetal Force

What is the origin of the centripetal force?

\[ F_{\text{centripetal}} = m \frac{v^2}{r} \]

What is the direction of centripetal force?

Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

What is her centripetal acceleration?

\[ a = \frac{v^2}{r} \]

Period:

\[ T = \frac{2\pi r}{v} \]

Problem #1

\[ m = 5 \text{ kg} \]
\[ L = 5 \text{ m} \]
\[ R = 2 \text{ m} \]

Find \( v \), \( T \), and \( a \)

\[ v = 100 \text{ m} / 12 \text{ s} = 8.33 \text{ s} \]
\[ R = \frac{100}{\pi} = 31.8 \text{ m} \]
\[ a = (8.33)^2 / 31.8 \text{ m/s}^2 = 2.2 \text{ m/s}^2 \]
Problem solving tactics: m = 5 kg
L = 5 m
R = 2 m
Find v, T, and a

\[
\sin \theta = \frac{R}{L} = 0.4; \quad \tan \theta = \frac{(R/L)}{(1-(R/L)^2)}^{\frac{1}{2}} = 0.44
\]

\[
X: \quad ma = T \times \sin \theta
\]
\[
Y: \quad ma = 0 = -mg + T \times \cos \theta
\]

Centripetal Force originates from the tension force!

Circular motion:
\[
ma = \frac{mv^2}{R}
\]
\[
a = \frac{v^2}{R}
\]
\[
T = \frac{mg}{\cos \theta} \quad a = g \times \tan \theta
\]
\[
v = (aR)^{\frac{1}{2}}
\]

Problem is solved:

\[
\sin \theta = \frac{R}{L} = 0.4; \quad \tan \theta = \frac{(R/L)}{(1-(R/L)^2)}^{\frac{1}{2}} = 0.44
\]

\[
X: \quad ma = T \times \sin \theta
\]
\[
Y: \quad ma = 0 = -mg + T \times \cos \theta
\]

Circular motion:
\[
ma = \frac{mv^2}{R}
\]
\[
a = \frac{v^2}{R}
\]
\[
T = \frac{mg}{\cos \theta} \quad a = g \times \tan \theta
\]
\[
v = (aR)^{\frac{1}{2}}
\]

\[
T = 5 \text{ kg} \times 9.8 \text{ m/s}^2 / (1-(2m/5m)^2) = 53 \text{ N}
\]
\[
a = 4.3 \text{ m/s}^2; \quad ma = 5 \text{ kg} \times 4.3 \text{ m/s}^2 = 21 \text{ N};
\]
\[
v = (4.3 \times 2)^{\frac{1}{2}} \text{ m/s} = 2.9 \text{ m/s}
\]
Problem #2

$R = 20\, \text{m}; \mu_{st} = 0.5$

Angle $10^\circ$

Find $v_{\text{max}}$

\[\begin{align*}
X: \quad ma &= N \sin \theta + \mu_{st} N \cos \theta \\
Y: \quad 0 &= N \cos \theta - mg - \mu_{st} N \sin \theta
\end{align*}\]

$ma = \frac{mv^2}{R}$
Problem #2

R = 20 m; \( \mu_{st} = 0.5 \)

Angle 10°

Find \( v_{\text{max}} \)

\[
ma = N \cdot (\sin \theta + \mu_{st} \cos \theta)
\]

\[
N = \frac{mg}{(\cos \theta - \mu_{st} \sin \theta)}
\]

\[
ma = m \cdot \frac{v_{\text{max}}^2}{R}
\]

\[
m \cdot \frac{v_{\text{max}}^2}{R} = mg \cdot \frac{(\sin \theta + \mu_{st} \cos \theta)}{(\cos \theta - \mu_{st} \sin \theta)}
\]

Mass m disappeared !!!

\[
v_{\text{max}} \approx 12.8 \text{ m/s} \approx 29 \text{ ml/h}
\]

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QZ # 7 Analyze the previous problem

1. R = 20 m; \( \mu_{st} = 0.5 \); Angle 10°

What is going to happen to the static friction force for the case when the velocity of the track is doubled: \( v = v_{\text{max}} \times 2 \)

What is going to happen to the truck? (describe)

2. R = 20 m; \( \mu_{st} = 0.5 \), \( v=5 \text{ m/s} \), \( m=3000 \text{ kg} \), and \( \theta = 0 \)

What is the value and direction of the static friction force?