
# Lecture 7

## Centripetal Motion

http://web.njit.edu/~sirenko/

Physics 105; Fall 2009



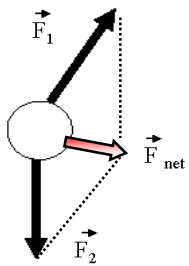
# Combination of Forces: Net Force

## Dealing with Multiple Forces

If multiple forces are acting on the same object, the net force determines the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$F_{net} = ma$$




Use a free body diagram to keep track of the forces on *one object*.



# Uniform Circular Motion

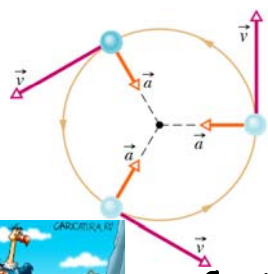
## Centripetal Force

Centripetal acceleration

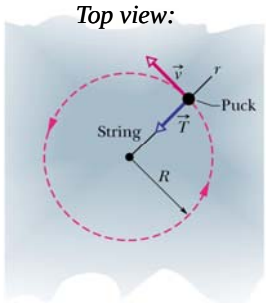
$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$



*Top view:*



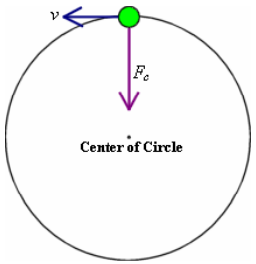


Centripetal force : F = ma

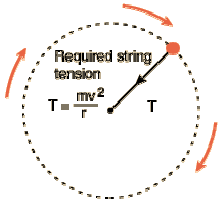
$$F = m\frac{v^2}{r}$$



# Net Force and Centripetal Force

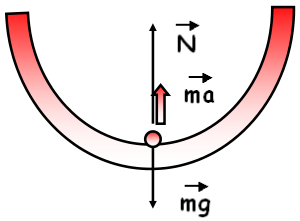



$$F = m\frac{v^2}{r}$$




# Centripetal Force is a combination of:

- Gravitational Force: $\vec{mg}$
  down to the ground
- Tension Force: $\vec{T}$
  along the string
- Normal Force: $\vec{N}$
  perpendicular to the support
- Static Friction Force
  maximum value $F_{fr}^{max} = \mu_{st} N$

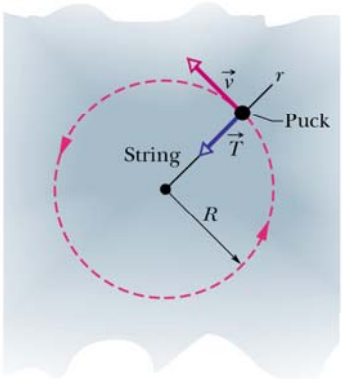



$ma = N - mg$

$ma = mv^2/R$



# Centripetal Force and Tension Force:

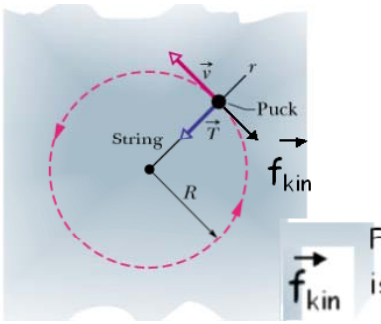



$ma = T$

$ma = mv^2/R = T$



# Centripetal Force and Kinetic Friction Force:




$ma_c = T$

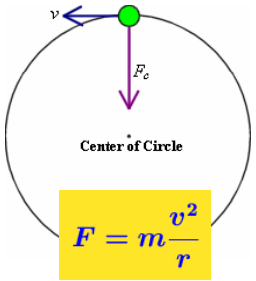
$ma_c = mv^2/R = T$

For uniform circular motion $\vec{f}_{kin}$ is perpendicular to $\vec{ma}_c$

*Kinetic friction does not affect Centripetal acceleration directly*



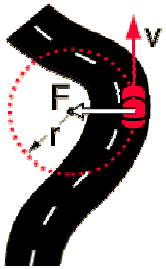
# Net Force and Centripetal Force

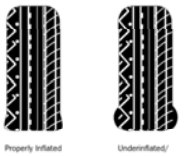



$$F = m\frac{v^2}{r}$$

$$F_{centripetal} = m\frac{v^2}{r}$$

$\frac{v^2}{r}$ is the centripetal acceleration



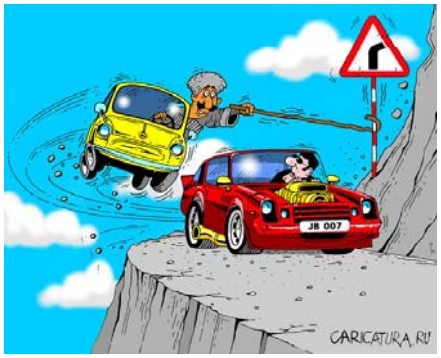

## How fast can we turn?

for a flat road: $m\frac{v^2}{r} = f_{st}$

max speed: $m\frac{v_{max}^2}{r} = f^{max}_{st} = N\mu_{st} = mg\mu_{st}$; $\boxed{v_{max} = \sqrt{\mu_{st} gr}}$

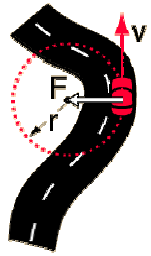
# Net Force and Centripetal Force

What is the origin of the centripetal force ?

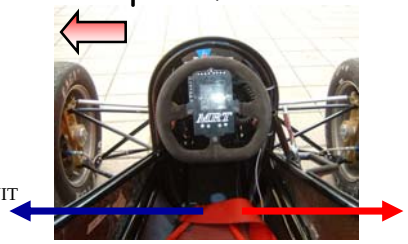



$$F_{centripetal} = m\frac{v^2}{r}$$

$\frac{v^2}{r}$ is the centripetal acceleration




What is the direction of centripetal force ?





# Uniform Circular Motion

## Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

What is her centripetal acceleration?

Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$



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Centripetal acceleration

$$a = \frac{v^2}{r}$$

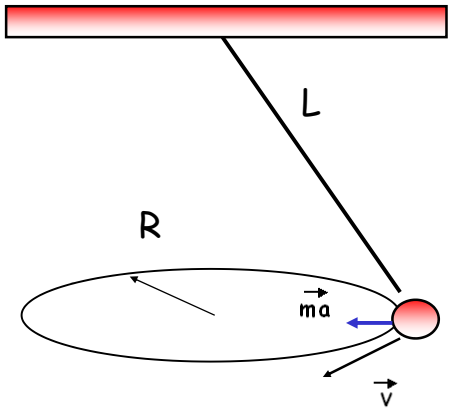
Period: $$T = \frac{2\pi r}{v}$$

v = 100 m /12 s = 8.33 s; R = 100/π = 31.8 m

a = $(8.33)^2$/31.8 m/s $^2$ = 2.2 m/s $^2$



## Problem #1



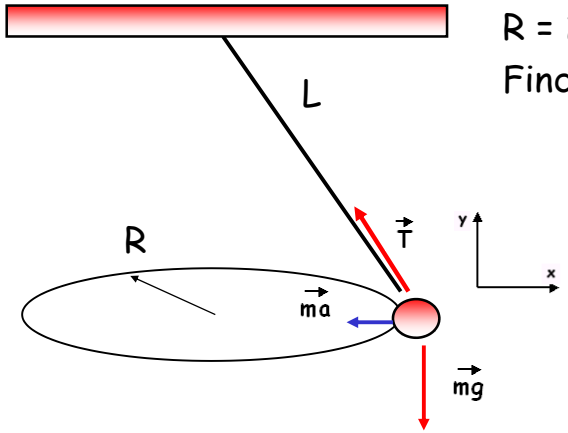

m = 5 kg

L = 5 m

R = 2 m

Find v, T, and a

## Problem solving tactics:




m = 5 kg
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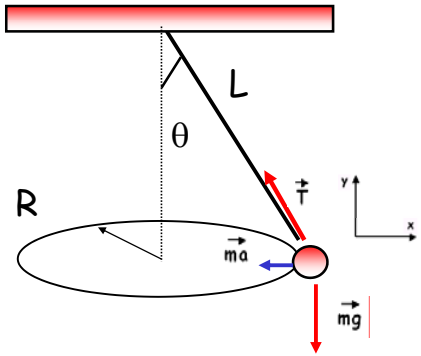
## Problem solving tactics:




m = 5 kg
L = 5 m
R = 2 m
Find v,T, and a

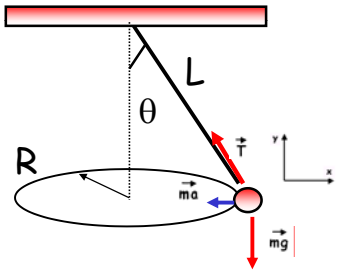
$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/(1-(R/L)^2)^{\frac{1}{2}} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = -mg + T * \cos\theta$



## Problem solving tactics:




m = 5 kg
L = 5 m
R = 2 m
Find v,T, and a

Centripetal Force originates from the tension force!

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/(1-(R/L)^2)^{\frac{1}{2}} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = -mg + T * \cos\theta$

$ma = mg*\sin\theta / \cos\theta = mg*\tan\theta$

$T = mg/\cos\theta$ $a = g*\tan\theta$

Circular motion:

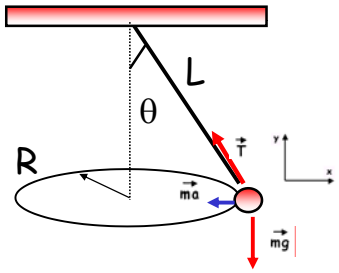
$ma = mv^2/R$

$a = v^2/R$

$v = (aR)^{\frac{1}{2}}$



## Problem is solved:

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Find v,T, and a

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/(1-(R/L)^2)^{\frac{1}{2}} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = -mg + T * \cos\theta$

$ma = mg*\sin\theta / \cos\theta = mg*\tan\theta$

$T = mg/\cos\theta$ $a = g*\tan\theta$

$T = 5\text{ kg} * 9.8\text{ m/s}^2 / (1-(2\text{m}/5\text{m})^2)^{\frac{1}{2}} = 53\text{ N}$

$a = 4.3\text{ m/s}^2$; $ma = 5\text{ kg} * 4.3\text{ m/s}^2 = 21\text{ N}$;

Circular motion:

$ma = mv^2/R$

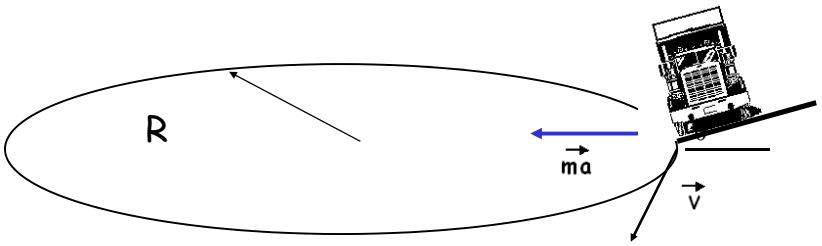
$a = v^2/R$

$v = (aR)^{\frac{1}{2}}$

$v = (4.3*2)^{\frac{1}{2}}\text{ m/s} = 2.9\text{ m/s}$

## Problem #2

R = 20 m; $\mu_{st}$ = **0.5**
Angle 10°
Find $v_{max}$



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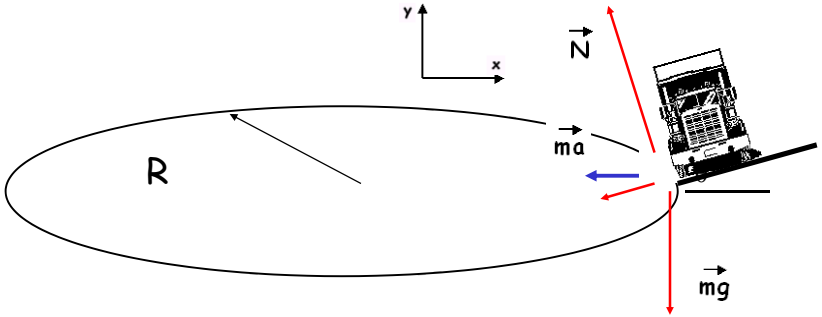


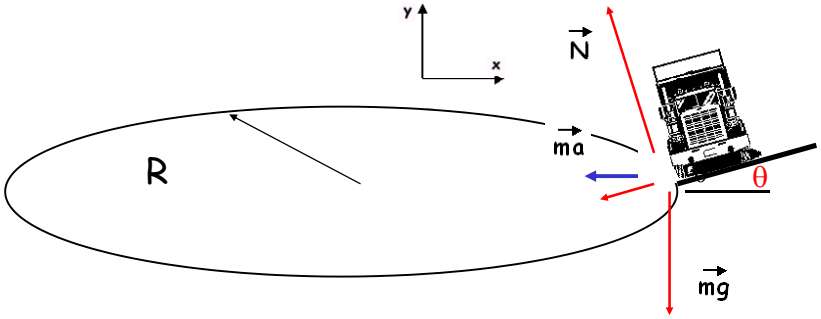



## Problem #2

R = 20 m; $\mu_{st}$ = **0.5**
Angle 10°
Find $v_{max}$

X: $ma = N \sin\theta + \underline{\mu_{st} N \cos\theta}$

Y: $0 = N \cos\theta - mg - \underline{\mu_{st} N \sin\theta}$





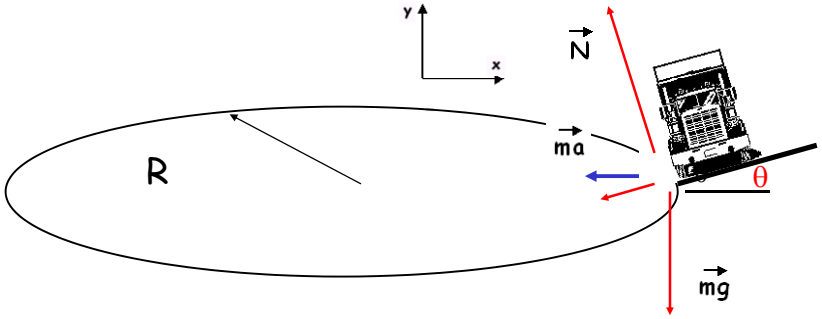

## Problem #2

R = 20 m; $\mu_{st}$ = **0.5**
Angle 10°
Find $v_{max}$

X: $ma = N*(\sin\theta + \mu_{st} \cos\theta)$

Y: $N = mg / (\cos\theta - \mu_{st} \sin\theta)$

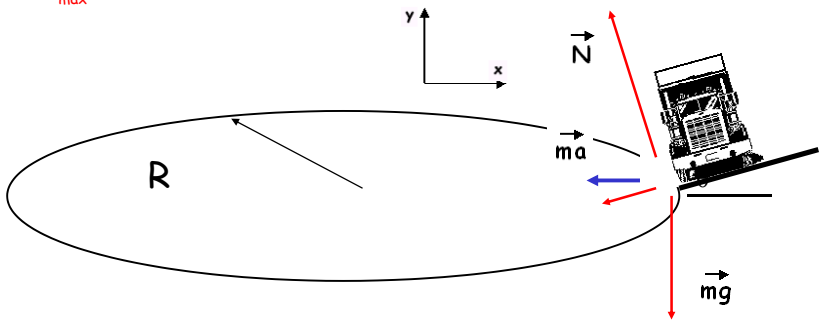
$ma = mv^2/R$



## Problem #2

R = 20 m; $\mu_{st}$= **0.5**
Angle 10°
Find $v_{max}$

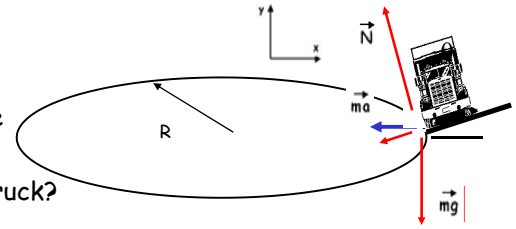
$ma = N*(\sin\theta + \mu_{st}\cos\theta)$
$N = mg / (\cos\theta - \mu_{st}\sin\theta)$
$ma = m\, v_{max}^2 / R$
$m\, v_{max}^2 / R = mg*(\sin\theta + \mu_{st}\cos\theta)/(\cos\theta - \mu_{st}\sin\theta)$
$v_{max}^2 = g*R*(\sin\theta + \mu_{st}\cos\theta)/(\cos\theta - \mu_{st}\sin\theta)$
Mass m disappeared !!!
$v_{max} \approx 12.8$ m/s $\approx$ 29 ml/h






## QZ # 7 Analyze the previous problem

1. R = 20 m; $\mu_{st}$= **0.5**; Angle 10°
What is going to happen to the <u>static friction force</u> for the case when the velocity of the track is doubled: $v = v_{max} * 2$
What is going to happen to the truck? (describe)




2. R = 20 m; $\mu_{st}$= **0.5**, v=5 m/s; m=3000 kg,
and $\theta = 0$
What is the value and direction of the static friction force ?