Energy

Work and Kinetic Energy

Kinetic and Potential Energy

ENERGY

Energy is a property of the state of an object: hard to define precisely.

Energy is a scalar quantity. It does not have a direction associated with it.

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule = kg·m²/s²

Kinetic Energy

Kinetic Energy = Energy of motion

\[ K = \frac{1}{2}mv^2 \text{ for object moving with velocity } v \]

\[ J = \text{kg}\frac{m^2}{s^2} \]
Kinetic Energy: Orders of Magnitude

\[ K = \frac{1}{2}mv^2 \] for object moving with velocity \( v \)

Earth orbiting sun: \( 2 \times 10^{29} \text{ J} \)
Car at 60 mph: \( 100,000 \text{ J} \)
Nolan Ryan pitch: \( 300 \text{ J} \)
Professor walking: \( 40 \text{ J} \)
Angry bee: \( 0.005 \text{ J} \)

Why \( K = \frac{1}{2}mv^2 \)?

It's good that \( K = \frac{1}{2} ... \)

Energy and Work

Kinetic energy

\[ K = \frac{1}{2}mv^2 \] \[ J = \text{kg m}^2\text{s}^{-2} \]

Work done by a constant force

\[ W = F \cdot d = Fd \cos \theta \]

Work-kinetic energy theorem

\[ \Delta K = K_f - K_i = W \]
Work

Work = Energy transferred by a force

Work done on an object is the energy transferred to/from it

\[ W > 0 \rightarrow \text{energy added} \]
\[ W < 0 \rightarrow \text{energy taken away} \]

\[ W = \vec{F} \cdot \vec{r} \equiv \text{Work done on an object by a constant force } \vec{F} \text{ while moving through a displacement } \vec{r} \]

Dot Product: Physical Meaning

\[ \vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z \]

\[ \theta = 0 \rightarrow \vec{A} \cdot \vec{B} = AB \]
\[ \theta = 90^\circ \rightarrow \vec{A} \cdot \vec{B} = 0 \]

Dot product measures how much vectors are along each other

What does \( W = \vec{F} \cdot \vec{r} \) mean?

\[ W = \vec{F} \cdot \vec{r} = F_x x + F_y y = F \cos \theta \]

\[ W > 0 \text{ if } \theta < 90^\circ \rightarrow \text{force is adding energy to object} \]

\[ W < 0 \text{ if } \theta > 90^\circ \rightarrow \text{force is reducing energy of object} \]

\[ W = 0 \text{ if } \vec{F} = 0 \text{ or } \vec{r} = 0 \text{ or } \vec{F} \perp \vec{r} \]

Work Examples

A weightlifter does work when lifting a weight \( W = mgh \) (\( h \) is the vertical drop)

Push on a wall

\( W = 0 \) since wall does not move (\( \vec{r} = 0 \))
Work Done by a Gravitational Force

Work done by gravitational force

\[ W_g = mgd \cos \theta \]

Tomato thrown upward

\[ W_g < 0 \quad \text{and} \quad W_g > 0 \]

Lifting/lowering an object

\[ \Delta K = K_f - K_i = W_a + W_g \]

Change in kinetic energy:

Work Done by a Spring Force

Equilibrium - no force

Stretched - force towards equilibrium point

\[ F = -kx \]

Hooke’s Law

\[ W_s = \frac{1}{2} kx_i^2 - \frac{1}{2} kx_f^2 \]
Sample Problem 7-8

A block of mass \( m = 0.40 \text{ kg} \) slides across a horizontal frictionless counter with a speed of \( v = 0.50 \text{ m/s} \). It runs into and compresses a spring of spring constant \( k = 750 \text{ N/m} \). When the block is momentarily stopped by the spring, by what distance \( d \) is the spring compressed?

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Work Done by a General Variable Force

Work: variable force

\[ W = \int_{x_i}^{x_f} F(x) \, dx \]

- Calculus
- Divide area under curve
- Add increments of \( W \) (numerically)
- Analytical form?
- Integration!!

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Power

Work doesn’t depend on the time interval

Work to climb a flight of stairs: \( 3000 \text{ J} \)
- 10 s
- 1 min
- 1 hour

Power is work done per unit time

Average Power

\[ P_{\text{avg}} = \frac{W}{\Delta t} \]

Instantaneous Power

\[ P = \frac{dW}{dt} = F \frac{dx}{dt} = Fv \]

Units

\[
\begin{align*}
\text{Work} & : \frac{1 \text{ J}}{1 \text{ s}} = 1 \text{ Watt} \\
\text{Power} & : \frac{\text{J/s}}{\text{s}} = \text{Watt} \\
1 \text{ hp} & = 746 \text{ W}
\end{align*}
\]

\[
P = \frac{1}{2} \times 50 \text{kg} \times (5 \text{m/s})^2 / 1 \text{ s}
\]
Quiz 9

1. As a sled is pulled by dogs across a flat, snow-covered field at a constant velocity, net work done on the sled is ________.

2. ...and work done by the air resistance and friction is ________.

3. ... and the work done by dogs is ________.
   a) Positive
   b) Zero
   c) Negative

4. A 200-kg sled is moving along a flat road with initial velocity 5 m/s. A 10-N friction force is acting on the sled (dogs take some rest). The sled has traveled 100 m. What is its final kinetic energy?