

# Lecture 9

## Energy Work and Kinetic Energy Kinetic and Potential Energy

Physics 105 Fall 2009



## ENERGY

Energy is a property of the state of an object: hard to define precisely

Energy is a scalar quantity. It does not have a direction associated with it

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule =  $\text{kg} \cdot \text{m}^2/\text{s}^2$

## Kinetic Energy

Kinetic Energy  $\equiv$  Energy of motion

$K = \frac{1}{2}mv^2$  for object moving with velocity  $v$

$$K = \frac{1}{2}mv^2 \quad \left[ J = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

## Kinetic Energy: Orders of Magnitude

$$K = \frac{1}{2}mv^2 \quad \text{for object moving with velocity } v$$



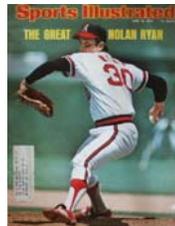
Earth orbiting sun:  $2 \times 10^{29}$  J

Car at 60 mph: 100,000 J

Nolan Ryan pitch: 300 J

Professor walking: 40 J

Angry bee: 0.005 J



Why  $K = \frac{1}{2}mv^2$  ?



Why  $K = \frac{1}{2}mv^2$  ?

## Special case: Constant Acceleration

Remember result eliminating  $t$ :

$$v^2 - v_0^2 = 2a(x - x_0)$$

Multiply by  $\frac{1}{2}m$ :

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0) = ma\Delta x$$

But  $F=ma!$

$$\Delta\left(\frac{1}{2}mv^2\right) = F\Delta x$$

## Energy and Work

### Kinetic energy

$$K = \frac{1}{2}mv^2 \quad \left[ J = \text{kg} \frac{\text{m}^2}{\text{s}^2} \right]$$

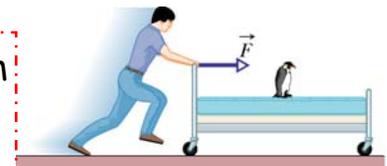
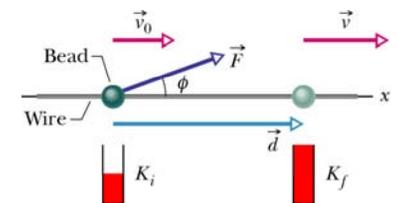
Units of Work and Energy: Joule

### Work done by a constant force

$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

### Work-kinetic energy theorem

$$\Delta K = K_f - K_i = W$$



## Work

**Work**  $\equiv$  Energy transferred by a force

Work done on an object is the energy transferred to/from it

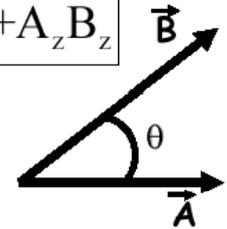
$W > 0 \rightarrow$  energy added

$W < 0 \rightarrow$  energy taken away

$W = \vec{F} \cdot \vec{r} \equiv$  Work done on an object by a constant force  $\vec{F}$  while moving through a displacement  $\vec{r}$

## Dot Product: Physical Meaning

$$\vec{A} \cdot \vec{B} = \underline{AB\cos\theta} = A_x B_x + A_y B_y + A_z B_z$$

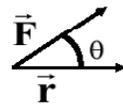
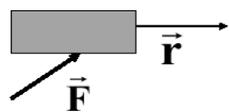


$$\theta = 0 \rightarrow \vec{A} \cdot \vec{B} = \underline{AB}$$

$$\theta = 90^\circ \rightarrow \vec{A} \cdot \vec{B} = \underline{0}$$

Dot product measures how much vectors are along each other

What does  $W = \vec{F} \cdot \vec{r}$  mean?



$$\begin{aligned} W &= \vec{F} \cdot \vec{r} \\ &= F_x r_x + F_y r_y \\ &= Fr\cos\theta \end{aligned}$$

$W > 0$  if  $\theta < 90^\circ \rightarrow$  force is adding energy to object

$W < 0$  if  $\theta > 90^\circ \rightarrow$  force is reducing energy of object



$W = 0$  if  $r = 0$  or  $F = 0$  or  $\vec{F} \perp \vec{r}$

### Work Examples

Push on a wall

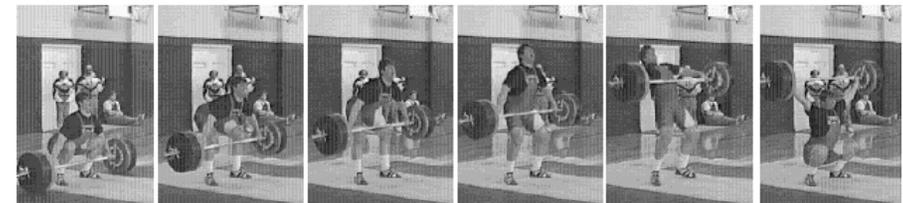
$W = 0$  since wall does not move ( $\vec{r} = 0$ )

## Work due to Gravity

A weightlifter does work when lifting a weight

$$W = mgh$$

(h is the vertical drop)

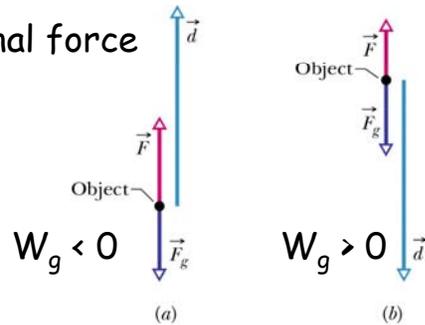
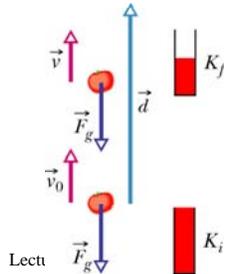


# Work Done by a Gravitational Force

Work done by gravitational force

$$W_g = mgd \cos \theta$$

Tomato thrown upward

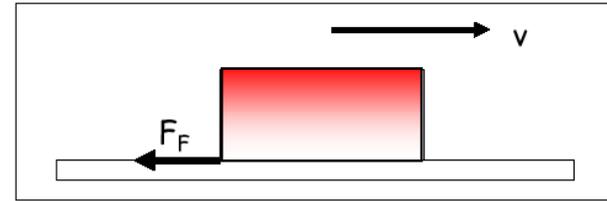


Lifting/lowering an object

Change in kinetic energy:

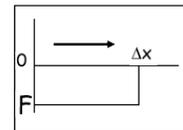
$$\Delta K = K_f - K_i = W_a + W_g$$

# Work due to Friction



The frictional force always opposes the motion

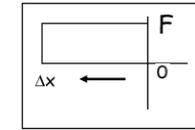
Moving to the right



$$W = -|F| \Delta x$$

$$\Delta x > 0$$

Moving to the left

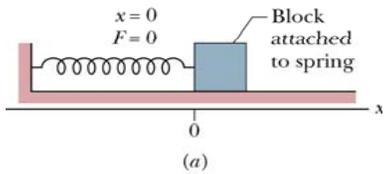


$$W = +|F| \Delta x$$

$$\Delta x < 0$$

W negative in both cases

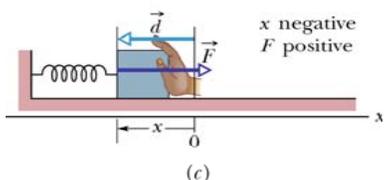
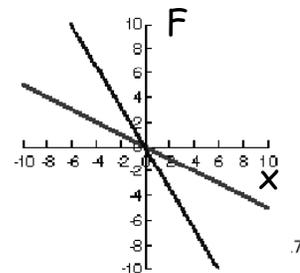
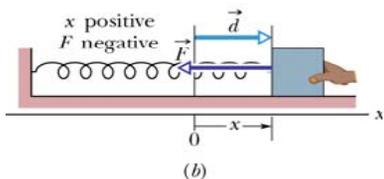
# Restoring Force



Equilibrium - no force  
Stretched - force towards equilibrium point

$$F = -kx$$

Hooke's Law

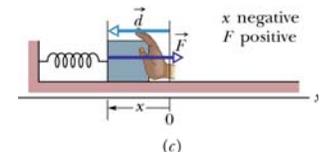
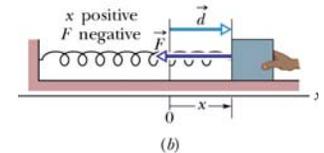
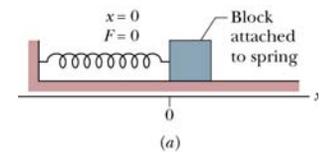


# Work Done by a Spring Force

Hooke's law:  $\vec{F} = -k\vec{d}$

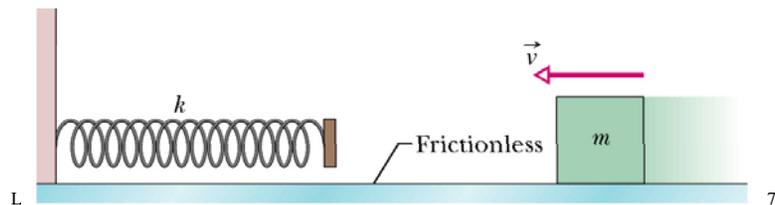
Work done by a spring force:

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$



# Sample Problem 7-8

A block of mass  $m = 0.40 \text{ kg}$  slides across a horizontal frictionless counter with a speed of  $v = 0.50 \text{ m/s}$ . It runs into and compresses a spring of spring constant  $k = 750 \text{ N/m}$ . When the block is momentarily stopped by the spring, by what distance  $d$  is the spring compressed?

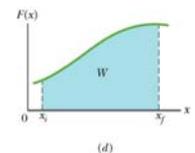
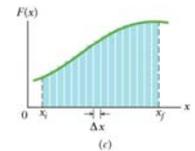
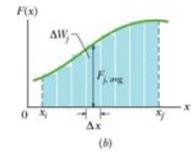
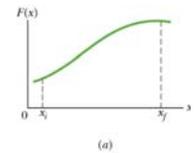


# Work Done by a General Variable Force

Work: variable force

$$W = \int_{x_i}^{x_f} F(x) dx$$

- Calculus
- Divide area under curve
- Add increments of  $W$  (numerically)
- Analytical form?
- Integration!!!



Lecture 9

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# Power

Work doesn't depend on the time interval

Work to climb a flight of stairs ~ 3000 J  
 10 s  
 1 min  
 1 hour

Power is work done per unit time

Average Power  $P_{avg} = \frac{W}{\Delta t}$

Instantaneous Power  $P = dW/dt = F dx/dt = Fv$

Units  $\frac{\text{Work}}{\text{time}} = \frac{1 \text{ J}}{1 \text{ s}} = 1 \text{ Watt} \quad 1 \text{ hp} = 746 \text{ W}$

$$P = \frac{1}{2} * 50 \text{ kg} * (5 \text{ m/s})^2 / 1 \text{ s}$$

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# Power

Average Power

$$P_{avg} = \frac{W}{\Delta t}$$

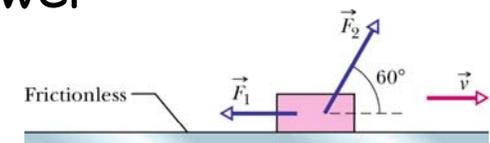
Units: Watts

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = \frac{dE}{dt}$$



**Sample Problem 7-10:** Two constant forces  $F_1$  and  $F_2$  acting on a box as the box slides rightward across a frictionless floor. Force  $F_1$  is horizontal, with magnitude  $2.0 \text{ N}$ , force  $F_2$  is angled upward by  $60^\circ$  to the floor and has a magnitude of  $4.0 \text{ N}$ . The speed  $v$  of the box at a certain instant is  $3.0 \text{ m/s}$ .

- What is the power due to each force acting on the box? Is the net power changing at that instant?
- If the magnitude  $F_2$  is, instead,  $6.0 \text{ N}$ , what is now the net power, and is it changing?

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### Quiz 9

1. As a sled is pulled by dogs across a flat, snow-covered field at a constant velocity, net work done on the sled is \_\_\_\_\_,

2. ...and work done by the air resistance and friction is \_\_\_\_\_,

3. ... and the work done by dogs is \_\_\_\_\_.

- a) Positive
- b) Zero
- c) Negative



4. A 200-kg sled is moving along a flat road with initial velocity 5 m/s. A 10-N friction force is acting on the sled (dogs take some rest). The sled has traveled 100 m. What is its final kinetic energy?