Common QZ includes:

Unit Conversion

Vectors (addition, subtraction, multiplication, angle between vectors)

Motion along the Straight line with constant acceleration

Projectile Motion

Unit Conversions

Multiply quantities and units:

\[
60 \text{ min} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot 0.0254 \text{ m} \cdot \frac{1 \text{ hr}}{3600 \text{ s}}
\]

\[
26.8 \frac{m}{s}
\]

EXAMPLE:

What is the volume of the book in cm\(^3\).
(hint: 1 inch = 2.54 cm)

Solution:

\[
V = (2 \times 2.54)(8 \times 2.54)(11 \times 2.54) \text{ cm}^3 = 2.9 \times 10^3 \text{ cm}^3
\]
Vectors:
(variables with magnitude and direction)

Displacement:

Components of Vectors:
- aligned along axis
- add to give vector
- are vectors

Components of a vector are still vectors

Unit Vectors
Components of a vector are still vectors

Unit Vectors
Components of a vector are still vectors

Vector Addition
Consider Two Vectors

\[ \vec{A} = A_x \hat{i} + A_y \hat{j} \]

\[ \vec{B} = B_x \hat{i} + B_y \hat{j} \]

\[ \vec{A} + \vec{B} = (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) \]

\[ = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} \]
Just add components.
Example

\[ \vec{A} = 12 \text{m} \cdot \hat{i} + 5 \text{m} \cdot \hat{j} \]
\[ \vec{B} = 2 \text{m} \cdot \hat{i} - 5 \text{m} \cdot \hat{j} \]

\[ \vec{C} = \vec{A} + \vec{B} \]
\[ = (12 \text{m} \cdot \hat{i} + 5 \text{m} \cdot \hat{j}) + (2 \text{m} \cdot \hat{i} - 5 \text{m} \cdot \hat{j}) \]
\[ = 14 \text{m} \cdot \hat{i} \]

Scalar product

\[ \vec{A} \cdot \vec{B} = \text{AB}\cos\theta = A_x B_x + A_y B_y \]
\[ \theta \text{ is the angle between the vectors if you put their tails together.} \]
\[ \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \]
\[ \text{since } \cos(\theta) = \cos(-\theta) \]

Motion Along a Straight Line

Position is a function of time: \( x = x(t) \)

Velocity is the rate of change of the position

\[ v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \]

Acceleration is the rate of change of the velocity

\[ a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \]
### Table 2.1: Equations for Motion with Constant Acceleration

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<td>( v = v_0 + at )</td>
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<td>2-15</td>
<td>( x - x_0 = v_0 t + \frac{1}{2} at^2 )</td>
<td>( v )</td>
</tr>
<tr>
<td>2-16</td>
<td>( v^2 = v_0^2 + 2a(x - x_0) )</td>
<td>( t )</td>
</tr>
<tr>
<td>2-17</td>
<td>( x - x_0 = \frac{1}{2} (v_0 + v)t )</td>
<td>( a )</td>
</tr>
<tr>
<td>2-18</td>
<td>( x - x_0 = vt - \frac{1}{2} at^2 )</td>
<td>( v_0 )</td>
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</table>

*Make sure that the acceleration is indeed constant before using the equations in this table.*

---

**Example:**

A car starts at rest and accelerates for 10 seconds with \( a = +5 \text{ m/s}^2 \). Then the driver pushes the breaks and comes to a complete stop with accelerates of \( a = -3 \text{ m/s}^2 \). What is the total traveled distance?

\[ a = +5 \text{ m/s}^2 \]
\[ x = 0 \]
\[ v = 0 \]
\[ x_0 = x(t=0); \text{ position at t=0; do not mix with the origin!} \]
\[ a = 0 \]
\[ v(t) = v_0 + at; \]
\[ x(t) - x_0 = v_0 t + \frac{1}{2} at^2 \]

\[ \begin{align*}
  x_1 &= v_0 t + \frac{1}{2} at^2/2 \quad \text{5 m/s}^2 \times 100 \text{ s}^2/2 = 250 \text{ m} \\
  v &= at = 5 \text{ m/s}^2 \times 10 \text{ s} = 50 \text{ m/s} \quad \text{(ticket ?)} \\
  x_2 &= \frac{(v(t)^2 - v_0^2)}{2a} = (0-50^2 \text{ m}^2/\text{s}^2)/(-2\times3 \text{ m/s}^2) = 417 \text{ m} \\
  x_1 + x_2 &= 250 \text{ m} + 417 \text{ m} = 667 \text{ m}
\end{align*} \]

---

**What does zero mean?**

- \( t = 0 \): beginning of the process
- \( x = 0 \): (origin) is arbitrary; can set where you want it
- \( x_0 = x(t=0); \text{ position at t=0; do not mix with the origin!} \)
- \( v(t) = 0 \): \( x \) does not change
- \( v_0 = 0 \): \( v(t) = at \);
- \( a = 0 \): \( v(t) = v_0 \);

\[ \begin{align*}
  x(t) - x_0 &= 0 \\
  x(t) - x_0 &= at^2/2 \\
  x(t) - x_0 &= v_0 t
\end{align*} \]

\[ \begin{align*}
  a \neq 0 &\quad v(t) = v_0 + at; \\
  &\quad x(t) - x_0 = v_0 t + at^2/2 \\
  \text{help:} &\quad t = (v - v_0)/a \\
  &\quad x - x_0 = \frac{1}{2} (v^2 - v_0^2)/a \\
  &\quad a = (v - v_0)/t \\
  &\quad x - x_0 = \frac{1}{2} (v + v_0)t
\end{align*} \]

> Acceleration and velocity are positive in the same direction as displacement is positive.
**Free Fall Motion**

As learned in an earlier unit, free fall is a special type of motion in which the only force acting upon an object is gravity. Objects which are said to be undergoing free-fall are not encountering a significant force of air resistance; they are falling under the sole influence of gravity. Under such conditions, all objects will fall with the same rate of acceleration, regardless of their mass. But why? Consider the free-falling motion of a 1000-kg baby elephant and a 1-kg overgrown mouse.

\[ a = -g \quad \text{where} \quad g = 9.8 \text{ m/s}^2 \]

(Defining "up" as the positive direction)

**More general case of the Free Fall Motion**

\[ a = -g \quad \text{where} \quad g = 9.8 \text{ m/s}^2 \]

(Defining "up" as the positive direction)

**Projectile Motion**

Horizontal motion + Vertical motion

"Free fall with horizontal motion"

- \[ x = \text{horizontal} \]
- \[ y = \text{vertical (take positive direction as "up")} \]
- \[ z = \text{not relevant} \]

\[ \vec{a} \text{ is only in the vertical direction: } \vec{a} = -g \hat{j} \]

\[ a_y = -g \quad a_x = 0 \]

**Horizontal motion**

\[ \vec{a}_x = 0 \]

**Vertical motion**

\[ \vec{a}_y = -g \]

In both directions the acceleration is constant

\[ \vec{v}_x = \vec{v}_{0x} = \text{constant} \]

\[ x = x_0 + v_{0x}t \]

\[ \vec{v}_y = \vec{v}_{0y} - gt \]

\[ y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \]

\[ x = x_0 + v_{0x}t \]

\[ v_x = v_{0x} - gt \]

\[ v_y = v_{0y} - gt \]

\[ v_y^2 = (v_{0y} - gt)^2 - 2g(y - y_0) \]
Projectile Motion: General Case
Trajectory and horizontal range

\[ y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \]

\[ R = \frac{v_0^2}{g} \sin 2\theta_0 \]

Example: Projectile Motion

\( V_0 = 20 \text{ m/s} \)

Angle 35°

1. Find the magnitude of the final velocity and find the velocity components when it touches the ground

2. What is the horizontal distance \( x \)?

Example: Projectile Motion

\( V_0 = 20 \text{ m/s} \)

Angle 35°

1. \( v_x = \text{Constant} \)
2. \( v_x = v_0 \cos 35^\circ = 16.4 \text{ m/s} \)
3. \( v_y = v_0 \sin 35^\circ - gt \)
4. Find time of flight using

\[ 0 = h + v_0 \sin 35^\circ t - \frac{1}{2}gt^2 \]

Plug the numbers in:

\[ 0 = 10 \text{m} + 11.5 \text{m/s} \cdot t - 9.8 \text{m/s}^2 \cdot \frac{t^2}{2} \]

4.9 \( t^2 - 11.5 \text{m/s} \cdot t - 10 \text{m} = 0 \)

Solve it for \( t \) and find two roots:

\( t_1 = -0.7 \text{ s} \) (what does it mean?)

\( t_2 = 3.0 \text{ s} \)

\( v_y = 11.5 \text{m/s} - 9.8 \text{m/s}^2 \cdot 3\text{s} = -18 \text{m/s} \)

\( v = (v_x^2 + v_y^2)^{\frac{1}{2}} = 24 \text{ m/s} \)

\( x = 16.4 \text{ m/s} \cdot 3\text{s} = 49 \text{ m} \)

Example for the Horizontal range:

A football is thrown toward a receiver with an initial speed of 20 m/s at an angle of 25° above the horizontal. At what horizontal distance the receiver should be to catch the football at the level at which it was thrown?

\[ R = \frac{v_0^2}{g} \sin 2\theta_0 \]

\[ \theta = 25^\circ \]

\[ R = \frac{(20 \text{m/s})^2}{9.8 \text{m/s}^2} \cdot \sin 50^\circ = 31 \text{ m} \]
Newton’s Laws

I. If no net force acts on a body, then the body’s velocity cannot change.

II. The net force on a body is equal to the product of the body’s mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction ($F_{12} = -F_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration:

\[ \sum F = 0 \iff \vec{a} = 0 \text{ (constant velocity)} \]

\[ \vec{F}_{\text{tot}} = m\vec{a} \text{ for any object} \]

\[ F_{\text{tot},x} = ma_x \quad F_{\text{tot},y} = ma_y \quad F_{\text{tot},z} = ma_z \]

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### Equations for Motion with Constant Acceleration\(^6\)

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\(^6\) Make sure that the acceleration is indeed constant before using the equations in this table.

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**Problem 1:** Please mark the version of the exam you are taking

A) YOU ARE TAKING VERSION A
B) 
C) 
D) 
E) 

**Problem 2:** Find the mass of an object whose initial speed of 4 m/s is reduced to zero with a constant 4 N force in 2 seconds.

- A) 0.5 kg
- B) 2 kg
- C) 4 kg
- D) 8 kg
- E) 16 kg

**Solution:**

\[ F = ma \]
\[ a = \frac{F}{m} \]
\[ m = \frac{F}{a} \]
\[ v = v_0 - at \]
\[ \alpha = \frac{\Delta v}{\Delta t} \]
\[ m = \frac{F}{a} \]
\[ v = v_0 - a \cdot t \]
\[ v = a \cdot t \]
\[ m = \frac{F}{a} \]
\[ v = v_0 - a \cdot t \]

**Problem 3:** Two forces acting on an object of mass 5.0 kg give rise to an acceleration $a = (2.0 \text{ m/s}^2 \hat{i} + 3.0 \text{ m/s}^2 \hat{j})$.

One of the forces is $F_1 = (10 \text{ N} \hat{i} + 15 \text{ N} \hat{j})$. The other must be

- A) $F_2 = (10 \text{ N} \hat{i} + 15 \text{ N} \hat{j})$
- B) $F_2 = (20 \text{ N} \hat{i} + 11 \text{ N} \hat{j})$
- C) $F_2 = (10 \text{ N} \hat{i} + 15 \text{ N} \hat{j})$
- D) $F_2 = (12 \text{ N} \hat{i} - 1 \text{ N} \hat{j})$
- E) $F_2 = (19 \text{ N} \hat{i} - 13 \text{ N} \hat{j})$

\[ F_1 + F_2 = ma \]
\[ F_1 = \frac{m \alpha}{1} \]
\[ F_2 = \frac{m \alpha}{1} - F_1 \]
\[ F_2 = 10 \hat{i} + 15 \hat{j} \]
\[ \alpha = 2 \hat{i} + 3 \hat{j} \]
\[ m = \frac{F_2}{\alpha} \]

**Solution:**

\[ F_2 = (19 \text{ N} \hat{i} - 13 \text{ N} \hat{j}) \]
**Problem 4:** A 5 kg lamp is suspended by a string from the ceiling inside an elevator moving up with decreasing speed. If the magnitude of the elevator’s acceleration is 3 m/s², what is the tension in the string?

A) 64 N  
B) 49 N  
C) 34 N  
D) 15 N  
E) 60 N

- \( T = m\ddot{a} = ma \)
- \( N - F \sin \theta = mg \)
- \( N = - F \sin \theta + mg \)

\( T = \Sigma \left( 9.8 - \frac{3}{2} \right) \cdot \frac{g}{2} = 56 \text{ N} \)

\[ v_f^2 - v_i^2 = -2ax \]
\[ \alpha = \frac{v_f^2}{2x} \]
\[ F = mg \sin \theta \]
\[ \mu = \frac{25 \text{ kg}}{19.6 \text{ N} \cdot 20 \text{ m}} = 0.064 \]

**Problem 6:** A block initially moving at 4 m/s upwards on an incline comes to rest after traveling 5 m up the incline. What is the angle between the incline and the horizontal in degrees?

A) 0°  
B) 1°  
C) 45°  
D) 53°  
E) 6.7°

\[ \alpha = \frac{v_f^2 - v_i^2}{2x} \]
\[ F = mg \sin \theta \]
\[ \alpha = g \sin \theta \]
\[ \sin \theta = \frac{v_i^2}{(2gx)} \]
\[ \theta = \sin^{-1} (0.16) = 9.4° \]

\( ma = F - mg \sin \theta - f \)
\( ma = F - 42N - f \)
\( f \) is directed as “3”

**Problem 10:** As shown in the figure below, a sled is pulled up a snow-covered hill by a force \( F \). The angle of the slope is 25 degrees. The weight of the sled is 100N. Which of the labeled arrows below indicate the DIRECTION of the frictional force?

A) Arrow 1  
B) Arrow 2  
C) Arrow 3  
D) Arrow 4  
E) None of the above

**Problem 11:** Referring to the sled problem above, the coefficient of static friction is 0.25 and the coefficient of kinetic friction is 0.15. What value of \( F \) is required such that the sled moves at a constant velocity?

A) 56 N  
B) 65 N  
C) 42 N  
D) 91 N  
E) 100 N

\[ y: \quad mg \cos \theta = N; \quad f = \mu N \]
\[ f = \mu mg \cos \theta \]

\( v_0 = 5 \text{ m/s; } h = 2 \text{ m} \)

**Problem 7:** The tension in the string on the right of the right block is 36 N. Each block has a mass of 2 kg. The surface is frictionless. What is the tension in the string between the blocks?

A) 9 N  
B) 30 N  
C) 18 N  
D) 12 N  
E) 27 N

**Problem 8:** A 2000 kg car slides on the ice and stops in 20 m due to the frictional force between the car and the ice. If the initial speed of the car is 5 m/s, the coefficient of kinetic friction between the ice and car is:

A) 0.6  
B) 0.064  
C) 0.13  
D) 1.0  
E) 9.8

\[ \mu = \frac{25 \text{ kg} \cdot \text{m/s}^2}{19.6 \text{ N} \cdot 20 \text{ m}} = 0.064 \]

**Problem 9:** A block of mass 5 kg is pulled along a horizontal floor by a force of 20 N as shown in the figure. The coefficient of dynamic friction is 0.2. The magnitude of the acceleration of the block is:

A) The block does not accelerate. The 20N force is not strong enough.  
B) The acceleration is zero, but the block moves at constant velocity.  
C) 2.04 m/s²  
D) 0.24 m/s²  
E) 9.8 m/s²

\[ F > F_{fs} \quad (20N > 19.6N) \text{ or } F \approx F_{fs} \quad (20N \approx 19.6N) \]
\[ a = (F - F_{fs})/m = (20 - 19.6)/5 = 0.24 \text{ m/s}^2 \text{ (too many significant figures.)} \]

**QZ#7:** A ball rolls off a table of height \( h \). The ball has horizontal velocity \( v_0 \) when it leaves the table.

How far away does it strike the ground?
How long does it take to reach the ground?
\[ \Delta x = ??? \]

\[ v_{0x} = v_0; \quad x_0 = 0 \]
\[ v_{0y} = 0; \quad y_0 = h \]

For x direction: \( t = \Delta x / v_0 \)
For y direction: \( y(t) = 0 \)
\[ y(t) - h = v_{0y} t - gt^2 / 2 \]
\[ \Delta x = v_0 \sqrt{2h/g} \]