Review 3

Review for the CQZ#3

Centripetal Motion and Energy Conservation

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Examples for Energy Conservation

Kinetic Energy changes
+ Gravitational Potential Energy
+ Elastic Potential Energy

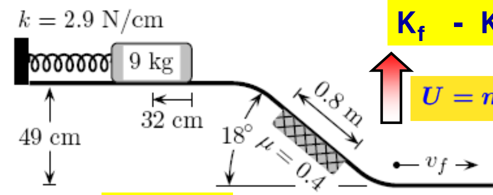
Total Mechanical Energy = *Const.*

$U = \frac{1}{2}kx^2$ → $K = \frac{1}{2}mv^2$

$K_f - K_i = W = mgy - |W_{friction}|$

$U = mgy$

$K = \frac{1}{2}mv^2$



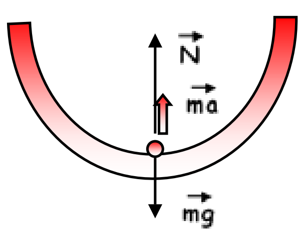
$W_{friction}$

$E_f - E_i = K_f - (K_i + mgy) = -|W_{friction}| = f_k \cdot d \cdot \cos 180° =$

$= - f_k \cdot d = -mg\,\mu \cdot d \cdot \cos 18°$

Centripetal Force is a combination of:

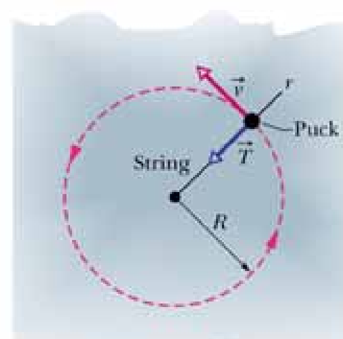
- Gravitational Force: $\vec{mg}$
  down to the ground
- Tension Force: $\vec{T}$
  along the string
- Normal Force: $\vec{N}$
  perpendicular to the support
- Static Friction Force
  maximum value $F_{fr}^{max} = \mu_{st} N$



ma = N – mg

ma = mv^2/R

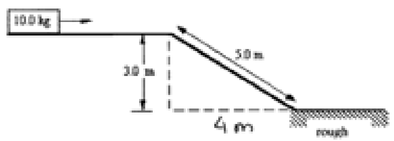
Centripetal Force and Tension Force:



$ma_c = mv^2/R = \Sigma$(all forces along the direction towards the center)

Problems:

A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.



$K_i = \frac{m v_i^2}{2}$;

$U_i = mg\Delta y$ | $\Delta y = 3m$

--- ∅ Pot. Energy

(a) [2 points] What is the speed of the crate when it arrives at the lower surface?

$$E_i = K_i + U_i = \frac{m v_i^2}{2} + mg\Delta y$$

$$E_f = K_f + \varnothing = \frac{m v_f^2}{2}$$

$$v_f = \sqrt{\frac{2}{m}} \cdot \sqrt{\frac{m v_i^2}{2} + mg\Delta y} = \sqrt{v_i^2 + 2g\Delta y} =$$

$$= \sqrt{(4\,m/s)^2 + 2 \cdot 9.8\,\frac{m}{s^2} \cdot 3\,m} = 8.6\,\frac{m}{s}$$

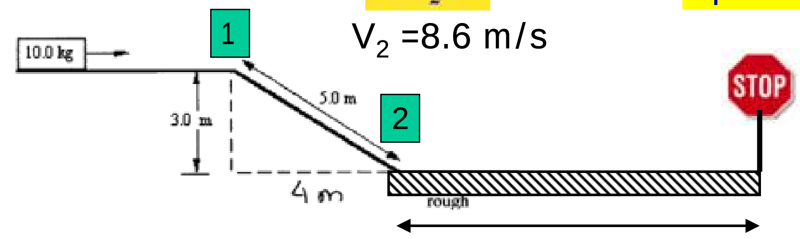
A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.

V_1 = 4.0 m/s

$K = \frac{1}{2}mv^2$

$K_f = 0$

V_2 = 8.6 m/s



What minimum coefficient of kinetic friction μ_k is required to bring the crate to a stop over a distance of 10 m along the lower surface ?

$K_f - K_i = W_{friction} = -mg\mu_k d$

$0 - \frac{1}{2}mv^2 = -mg\mu_k d$

$\frac{1}{2}mv^2 = mg\mu_k d$; $\mu_k = v^2/2gd =$

$(8.6 m/s)^2/(2 \cdot 10 m/s^2 \cdot 10 m) =$ **0.37**

Example of the 3rd Common Exam

Problem 1: What is the work done by a force **F** = (2 N)**i** + (-4 N)**j** that causes a displacement **d** = (-3 m)**i** + (2 m)**j**?

A) 2 J
B) 14 J
C) -14 J
D) -2 J
E) 16 J

$$W = \vec{F} \cdot \vec{d} = 2N \cdot (-3m) + (-4N) \cdot 2m =$$
$$= -6 - 8 = \underline{-14\ J}$$

Problem 2: A man pushes a 2-kg block 5 m along a frictionless incline at an angle of 20° with the horizontal at constant speed. What is the work done by his force?

A) 0 J
B) 98 J
C) 34 J
D) 92 J
E) 100 J

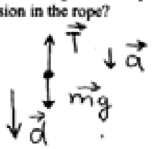
$\Delta K = \varnothing$

$$W = \Delta U = mg \cdot \Delta y = mg \cdot s \cdot \sin\theta$$

$$W = 2kg \cdot 9.8\,\frac{m}{s^2} \cdot 5m \cdot \sin 20^\circ = 33.5\,J \approx \underline{34\ J}$$

Problem 3: Starting from rest, it takes 8.00 s to lower with constant acceleration an 80.0-kg couch from a 16.0-m high rooftop of a building all the way to the ground with a single vertical rope tied to its body. What is the work done by the tension in the rope?

A) 1.57 kJ
B) −1.28 kJ
C) −12.5 kJ
D) 12.5 kJ
E) −11.9 kJ



$ma = mg - T$; $T = mg - ma$

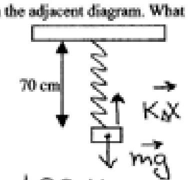
$W = T \cdot d \cdot \cos 180^\circ = -T \cdot d =$

$= -(mg - ma) \cdot d$; *) $\frac{at^2}{2} = d \Rightarrow a = \frac{2d}{t^2}$

$W = -(mg - m \cdot 2d/t^2) \cdot d = -11\,904\,J$

Problem 4: A 10-kg mass is attached to one end of a 50-cm-long unstretched spring. When the other end of the spring is attached to the ceiling the mass reaches a stable stationary position as shown in the adjacent diagram. What is the spring constant of the spring?

A) 490 N/m
B) 245 N/m
C) 980 N/m
D) 140 N/m
E) 196 N/m



$$\Delta x = 70cm - 50cm = 20cm$$

$$K\Delta x = mg$$

$$K = \frac{mg}{\Delta x} = \frac{10kg \cdot 9.8\,m/s^2}{0.2\,m} = \underline{490\ N/m}$$

Problem 5: A dog must apply its full power of 100 W in order to move a 5-kg sled by a distance of 10 m in 4 s. What average force does the dog exert on the sled?

not important

A) 49 N
B) 250 N
C) 8 N
D) 40 N
E) 200 N

$$P \cdot t = W \text{ (work)}$$

$$F \cdot d = W \text{ (work)}$$

$$F = \frac{P \cdot t}{d} = \frac{100W \cdot 4s}{10m} = 40N$$

Problem 6. A bicyclist is traveling on a horizontal track at a speed of 20.0 m/s as he approaches the bottom of a hill. He decides to coast up the hill and stops upon reaching the top. Determine the vertical height of the hill.

A) 28.5 m
B) 3.70 m
C) 11.2 m
D) 40.8 m
E) 20.4 m

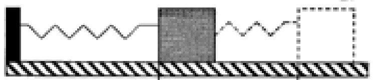
$$\frac{mv^2}{2} = mg\Delta y \;;\; \Delta y = \frac{v^2}{2g} = \frac{20^2 (\frac{m}{s})^2}{2 \cdot 9.8\,\frac{m}{s^2}} =$$

$$= \underline{20.4\ m}$$

Problem 7. A mass m = 2.5 kg is sliding left along a frictionless table with initial speed *v*. It strikes a coiled spring that has a force constant k = 500 N/m and compresses it a distance 5.0 cm before coming to a momentary rest. The initial speed *v* of the block was

A) 0.71 m/s
B) 1.0 m/s
C) 1.4 m/s
D) 0.50 m/s
E) 1.7 m/s

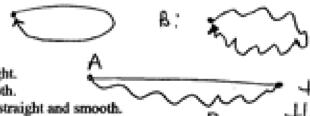
$\frac{1}{2}k\Delta x^2 \qquad \frac{mv^2}{2}$



$\frac{1}{2}k\Delta x^2 = \frac{mv^2}{2}$; $V = \Delta x \cdot \sqrt{\frac{k}{m}} = 0.05 \cdot \sqrt{500/2.5} = 0.71$ m/s

Problem 8. Two skiers start from rest at the same place and finish at the same place. Skier A takes a straight, smooth route to finish whereas skier B takes a curvy, bumpy route to the finish. If you assume that friction is negligible, which of the following statements is true?

A) Skier A has the same speed as skier B at the finish.
B) Skier B has greater speed at the finish.
C) Skier A has greater speed at the finish because the route is straight.
D) Skier B has greater speed at the finish because the route is smooth.
E) Skier A has greater speed at the finish because the route is both straight and smooth.



$\Delta K = \Delta U$
if $\Delta U = \emptyset$,
then $\Delta K = \emptyset$
then $V_A = V_B = \emptyset$

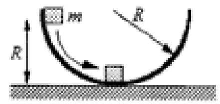
if $\Delta U \neq 0$, then $\Delta K_A = \Delta K_B$; if $m_A = m_B$, then $V_A = V_B$;

Problem 9. A block of mass *m* is released from rest at a height *R* above a horizontal surface. The acceleration due to gravity is *g*. The block slides along the inside of a frictionless circular hoop of radius *R*. Which one of the following expressions gives the speed of the mass at the bottom of the hoop?

A) zero m/s
B) $v = mgR$
C) $v = mg/2R$
D) $v^2 = g^2/R$
E) $v^2 = 2gR$

$mgR = \frac{mv^2}{2}$

$\underbrace{v^2 = 2gR}$



Problem 10. A 60-kg skier starts from rest from the top of a 50-m high slope. If the work done by friction is -6.0×10^3 J, what is the speed of the skier on reaching the bottom of the slope?

A) 17 m/s
B) 24 m/s
C) 28 m/s
D) 31 m/s
E) 42 m/s

$K_i = 0$; $U_i = mg\Delta y$

$K_f = \frac{mv_f^2}{2}$; $U_f = \emptyset$

$\frac{mv_f^2}{2} = mg\Delta y + W_{fr}$

$\frac{mv_f^2}{2} = (60 \cdot 9.8 \cdot 50)\,J - 6 \cdot 10^3\,J = 23.4 \times 10^3\,J$

$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \cdot 23400}{60}} = 28$ m/s

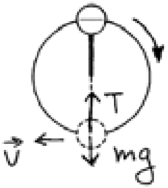
is not important !!!

Problem 11. A 2.0-kg ball is attached to a light rod that is 1.2 m long. The other end of the rod is loosely pinned at a frictionless pivot. The rod is raised until it is inverted, with the ball above the pivot. The rod is released and the ball moves in a vertical circle. The tension in the rod as the ball moves through the bottom of the circle is closest to:

A) 40 N
B) 100 N
C) 20 N
D) 60 N
E) 80 N

$T - mg = \frac{mv^2}{R}$

$T = mg + \frac{mv^2}{R}$



$5 \cdot mg = $
$= 5 \cdot 2\,kg \cdot 9.8\,\frac{m}{s^2} \approx$
$= 98$ N

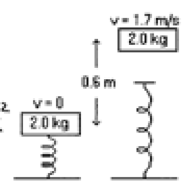
$\frac{mv^2}{2} = mg \cdot 2R \Rightarrow$

$\Rightarrow \frac{mv^2}{R} = 4 \cdot mg$; $T = mg + 4mg = 5 \cdot mg \approx \underbrace{100N}$

Problem 12. A block of mass 2.0 kg is placed on a vertical spring, which is kept compressed 0.050 m by a clamp (The clamp is not shown in the diagram). The spring and the block are not attached. When the clamp is removed, the spring propels the block vertically upward. When the block has risen 0.60 m above its initial position its velocity is 1.7 m/s. How much potential energy was originally stored in the spring?

A) 5.5 J
B) 8.2 J
C) 11 J
D) 15 J
E) 26 J

$E_i = \frac{1}{2}k\Delta x^2$



$E_f = K + U =$
$= \frac{mv^2}{2} + mg\Delta y$

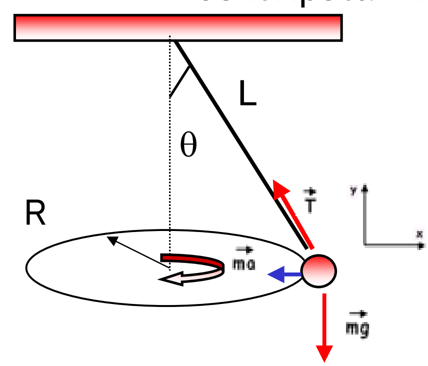
$E_i = \frac{1}{2}k\Delta x^2$

$E_f = \frac{1}{2}mv^2 + mg\Delta y$

$E_i = E_f = \frac{1}{2}mv^2 + mg\Delta y = 2\,kg\left(\frac{(1.7\,\frac{m}{s})^2}{2} + 9.8\,\frac{m}{s^2} \cdot 0.6\,m\right) =$

$= 14.7\,J \approx \underbrace{15\,J}$

Centripetal Motion:



m = 5 kg
L = 5 m
R = 2 m
Find v,T, and a

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/(1-(R/L)^2)^{1/2} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = -mg + T * \cos\theta$

WORKOUT PROBLEM 2:

(a) **[3 points]** A 2000 kg race car is rounding a level curve at a speed of 50m/s. If the coefficient of static friction between the road and the tires is 0.5, what is the minimum radius of the curve for which the car can round the curve without skidding?

$V = 50\ m/s$

$\mu_{st} = 0.5$

$R_{min} = ?$

$$\frac{mV^2}{R} = f_{st} \quad ; \quad f_{st} = \mu \cdot N = \mu \cdot mg$$

$$\frac{\cancel{m}V^2}{R} = \mu \cancel{m} g \quad ; \quad \frac{V^2}{R} = \mu \cdot g \ ;$$

$$R = \frac{V^2}{\mu \cdot g} = \frac{50 \cdot 50}{0.5 \cdot 9.8} = \boxed{510\ m}$$

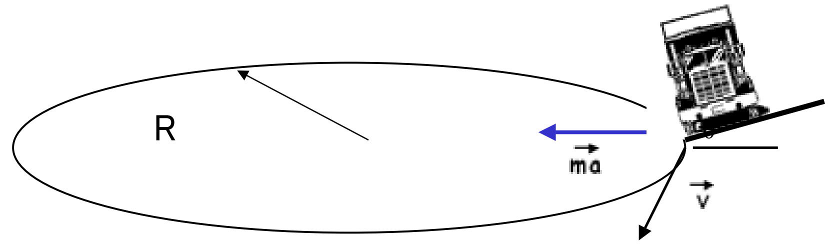
What if the road is banked?

v_{max} = 50 m;

μ_{st} **= 0.5**

Angle 10°

Find R_{min}



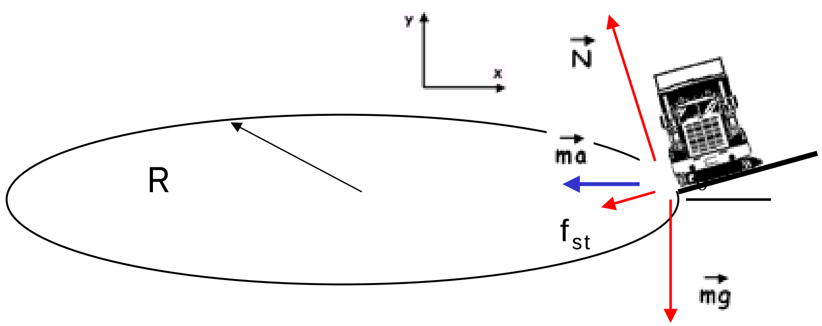
What if the road is banked?

v_{max} = 50 m;

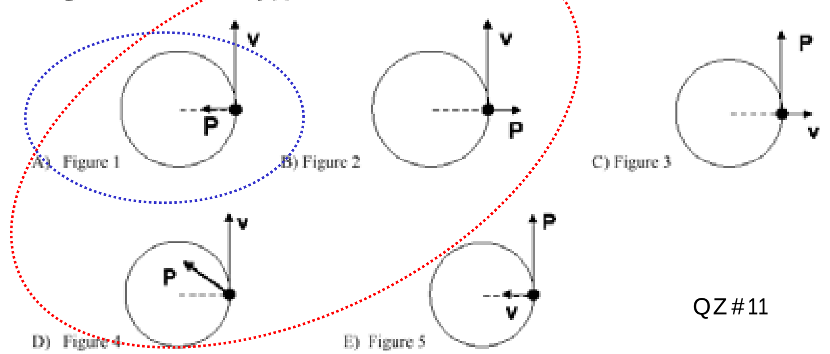
μ_{st} **= 0.5**

Angle 10°

Find R_{min}



Problem 12: A ball a mass 0.5kg is tied to a string. The ball is swung in a circle (in the absence of gravity) in a circle of radius 2m. For the diagrams below, which correctly shows the relative directions of the centripetal force (P) acting on the ball and the velocity (v) of the ball.



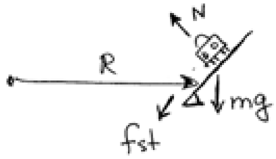
QZ#11

Problem 13: Referring to the problem above with the ball moving in circular motion, if the time for the ball to complete one revolution is 0.5 seconds, the magnitude of the centripetal acceleration is

A) 9.8 m/s^2
B) 4 m/s^2
C) 25 m/s^2
D) 157 m/s^2
E) 316 m/s^2

(b) **[1 point]** If the curve is banked rather than flat, does the minimum radius at which the car can turn without skidding at 50m/s increase or decrease compared to the case of a flat curve? **IN ORDER TO RECEIVE CREDIT, YOU MUST JUSTIFY YOUR ANSWER using a diagram and a *brief* explanation.**

b)



for this case R will decrease since $N \cdot \sin\theta$ will contribute to the centrepetal force

$$ma^c = f_{st} \cdot \cos\theta + N \sin\theta$$

$$ma^c = N \cdot \mu \cdot \cos\theta + N \cdot \sin\theta \qquad N \approx mg / \cos\theta$$

$$ma^c = N \cdot (\mu \cos\theta + \sin\theta) = mg \cdot \frac{\mu \cos\theta + \sin\theta}{\cos\theta} =$$

$$= mg(\mu + \tan\theta)$$

$$\frac{\cancel{m} V^2}{R'} = \cancel{m} g (\mu + \tan\theta)$$

$$R' = \frac{V^2}{g(\mu + \tan\theta)} < \frac{V^2}{g \cdot \mu} = R \quad \Rightarrow \{R' < R\}$$