Lecture 2

Motion along a straight line

(HR&W, Chapter 2, Sections 1–8)

Physics 105; Spring 2005

Class Organization

Week $n-1$		**Week n**		**Week $n+1$**	
Lecture	**Recitation**	**Lecture**	**Recitation**	**Lecture**	**Recitation**
❑ Read Chapter *n*		❑ Introduction to Chapter *n* and *QZ* ❑ Homework assignment for Chapter *n*	❑ Discuss Chapter *n* ❑ Sample problems Chapter *n*	❑ Deadline for the homework for Chapter *n* *(We. 9 am)*	

Motion along a straight line

- Motion
- Position and Displacement
- Average velocity and average speed
- Instantaneous velocity and speed
- Acceleration
- Constant acceleration: A special case
- Free fall acceleration

Motion along a straight line

- this is the simplest type of motion
- it lays the groundwork for more complex motion

Kinematic variables in one dimension

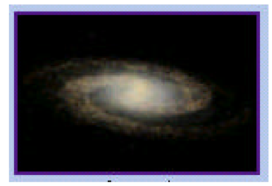
Position	**x(t)**	meters
Velocity	**v(t)**	meters/second
Acceleration	**a(t)**	meters/second ^{2}

All depend on time

All are vectors: have direction and magnitude.

Motion

- Everything moves!
- Classification and comparison of motion ⇨ **kinematics**
- Simplification
  - Motion along straight line
  - Forces cause motion
  - Moving object is a particle or moves like a particle



One Dimensional Position: x(t)

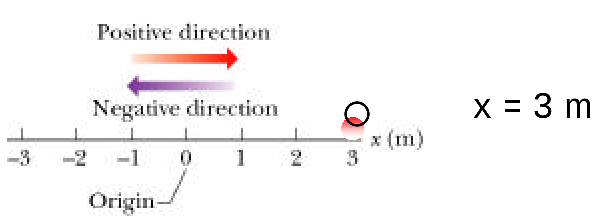
Position is a vector quantity.

Position has both a direction and magnitude.

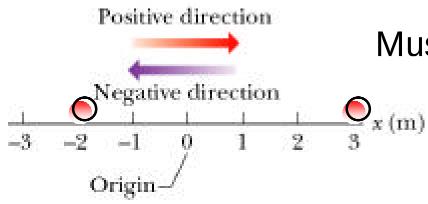
Position has units of [Length]: *meters*.

Must define:

- **x = 0** some position (Origin)
- positive direction for x



Displacement Along a Straight Line



Must define: t = 0

Displacement: $\Delta x = x_2 - x_1$

$x_1 = 3$ m

$x_2 = -2$ m

$\Delta x = -5$ m

Displacement is a change of position in time.

It is a vector quantity.

It has both a direction and magnitude.

It has units of [Length]: *meters*.

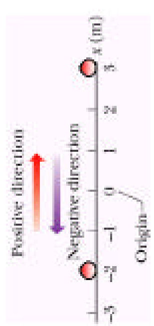
Displacement Along a Straight Line

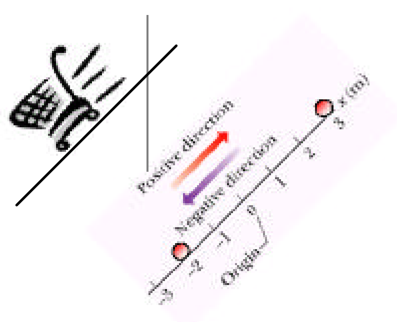
t=0; (start the clock) x = 0; (origin)

x(t=0) does not have to be 0

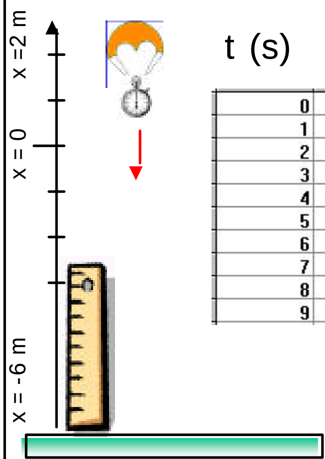
Straight line can be oriented

Horizontal, vertical, or at some angle

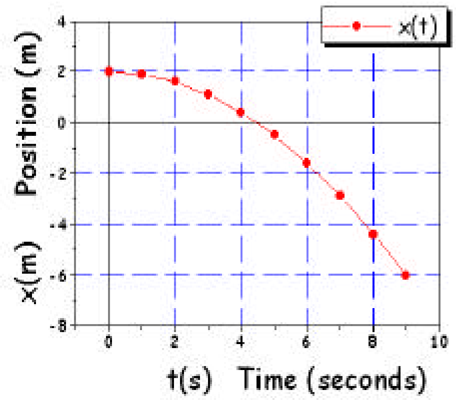




Displacement Along a Straight Line



t (s)	x (m)
0	2
1	1.9
2	1.6
3	1.1
4	0.4
5	-0.5
6	-1.6
7	-2.9
8	-4.4
9	-6



Motion with respect to the origin !

Displacement

Displacement in time:

$\Delta t = t_2 - t_1$

Displacement in Space:

$\Delta x = x(t_2) - x(t_1)$

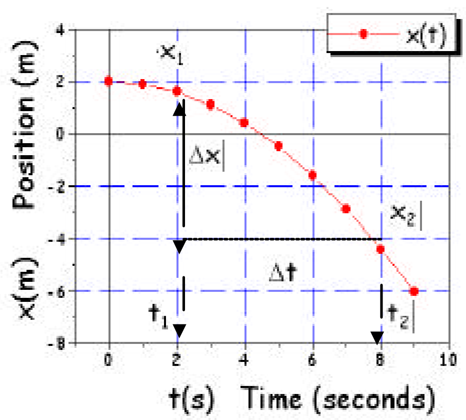
or

$\Delta x = x_2 - x_1$

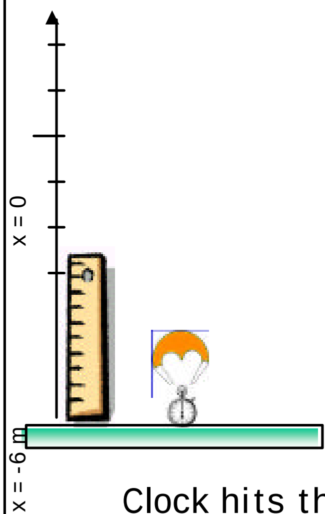
Avg:

$x(t_2) - x(t_1)$

is negative



Displacement Along a Straight Line



t (s)	x (m)
0	2
1	1.9
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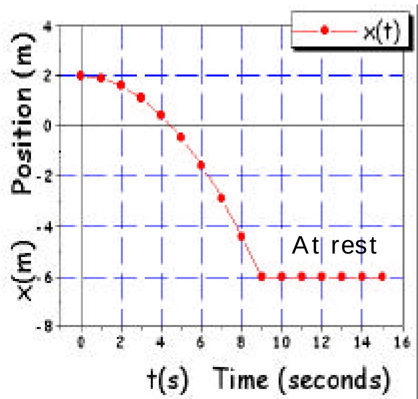
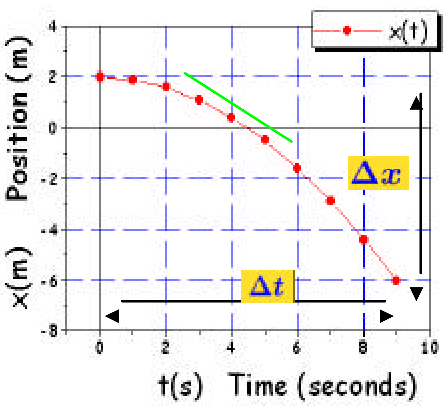
5	-0.5
6	-1.6
7	-2.9
8	-4.4
9	-6
10	-6
11	-6
12	-6
13	-6
14	-6
15	-6
16	-6



Clock hits the ground at t = 9 s

Velocity



Average velocity

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

=-8/9 m/s

≈ -0.9 m/s

Average speed

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$

=8/9 m/s

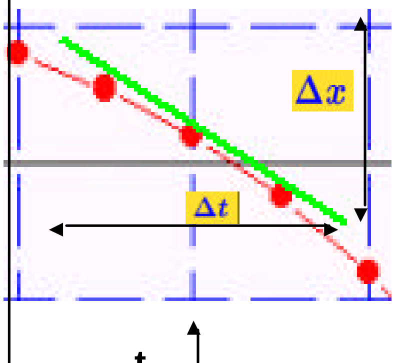
≈ 0.9 m/s

Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Velocity is the rate of change of position

Velocity
Consider smaller time intervals



Instantaneous velocity v(t) is the slope of the tangent line to x(t)

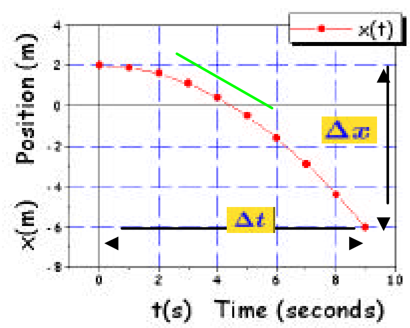
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

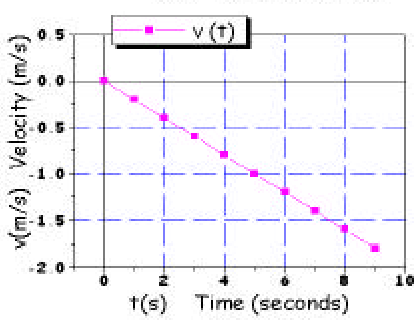
t

changes with time !

v(t) is a function of time !

Velocity





$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Example of negative **v**

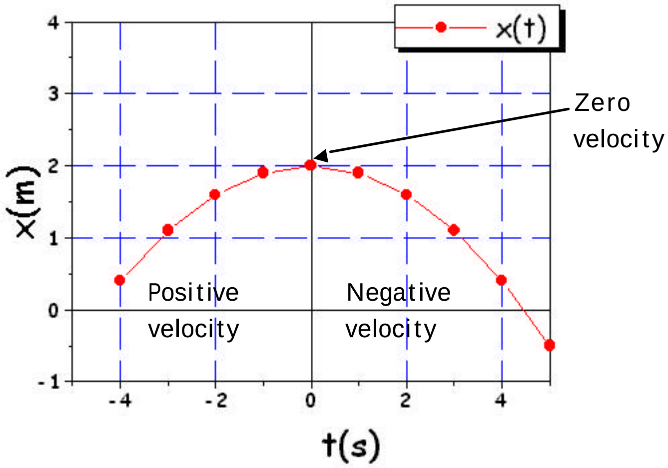
Velocity is positive in the same direction as x is positive

Example of a constant acceleration:

v(t) = at

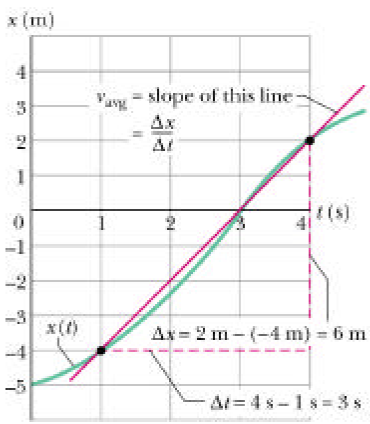
v(t) is a straight line

Sign of velocity



v(t) is a function of time !

Velocity



HR&W, p.16
"moving armadillo"

- Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$ = 6/3 m/s

- Average speed

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$ = 6/3 m/s

- Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Kinematic Variables

Position is a function of time: $x = x(t)$

Velocity is the rate of change of the position

Acceleration is the rate of change of the velocity

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$\text{Position} \xrightarrow{\frac{d}{dt}} \text{Velocity} \xrightarrow{\frac{d}{dt}} \text{Acceleration}$$

Acceleration

- Average acceleration

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

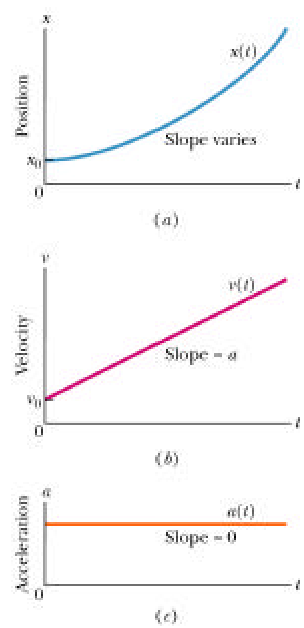
- Instantaneous acceleration

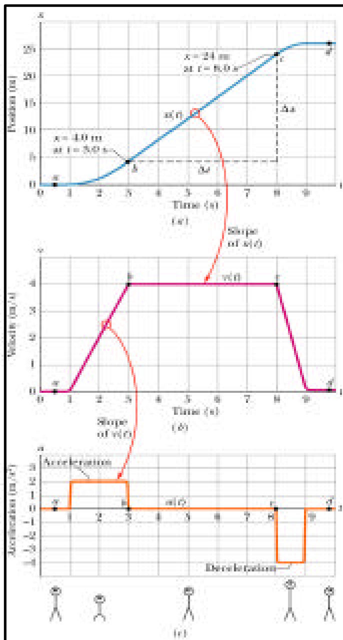
$$a = \frac{dv}{dt}$$

- Constant acceleration

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$





Sample Problem 11-2

p. 18

An elevator is initially stationary, then moves upward (which we take the positive direction of *x*), and then stops. Plot V as a function of time.

(a) x(t) curve for an upward moving elevator cab

(b) v(t) curve for the cab. Note $v = dx/dt$!

(c) a(t) curve for the cab. Note $a = dv/dt$!

http://webphysics.ph.msstate.edu/

Another Look at Constant Acceleration

- Integration of acceleration
- Indefinite integral
- Constant of integration

$$\begin{aligned} dv &= a\,dt \\ \int dv &= \int a\,dt \\ \int dv &= a\int dt \\ v &= at + C \\ v_0 &= (a)(0) + C = C \end{aligned}$$

$$\begin{aligned} dx &= v\,dt \\ \int dx &= \int v\,dt \\ \int dx &= \int (v_0 + at)\,dt \\ \int dx &= v_0\int dt + a\int t\,dt \\ x &= v_0 t + \frac{1}{2}at^2 + C' \\ x_0 &= (v_0)(0) + \frac{1}{2}(a)(0)^2 + C' = C' \end{aligned}$$

You will learn more about this in integral calculus!

What does zero mean ?

- $t = 0$ beginning of the process
- $x = 0$ is arbitrary; can set where you want it
- $x_0 = x(t=0)$; position at $t=0$; do not mix with the origin

- $v(t) = 0$ x does not change $x(t) - x_0 = 0$
- $v_0 = 0$ $v(t) = at$; $x(t) - x_0 = at^2/2$
- $a = 0$ $v(t) = v_0$; $x(t) - x_0 = v_0 t$

- $a \neq 0$ $v(t) = v_0 + at$; $x(t) - x_0 = v_0 t + at^2/2$

help: $t = (v - v_0)/a$ $x - x_0 = ½(v^2 - v_0^2)/a$

$a = (v - v_0)/t$ $x - x_0 = ½(v + v_0)t$

- Acceleration and velocity are positive in the same direction as displacement is positive

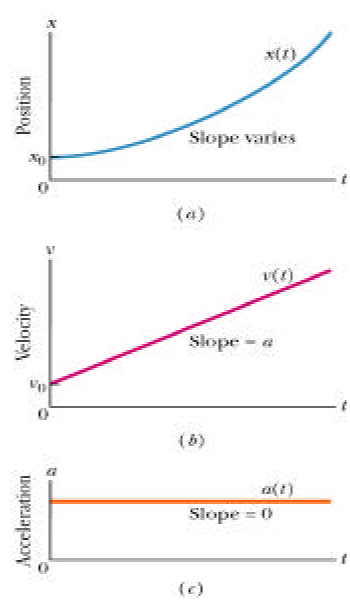
Constant Acceleration

(a > 0)

$v(t) = v_0 + at$;

$x(t) - x_0 = v_0 t + at^2/2$

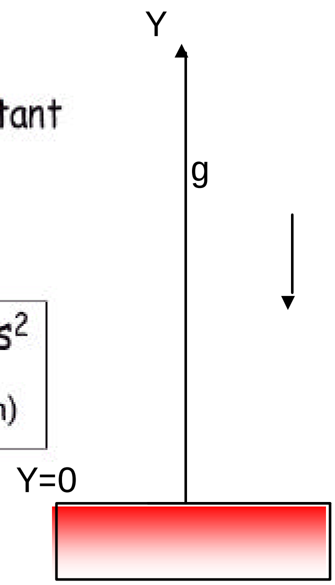
$x(t) - x_0 = (v(t)^2 - v_0^2)/2a$



Most important case of constant acceleration: Free fall

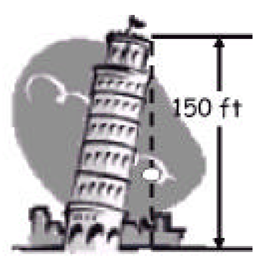
$a = -g$ where $g = 9.8\ m/s^2$

(defining "up" as the positive direction)



Gravitation is universal

Gravitational acceleration does not depend on the nature of the material or the mass of the object.

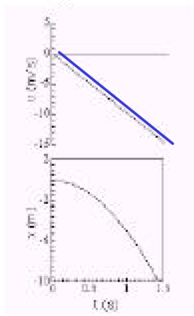


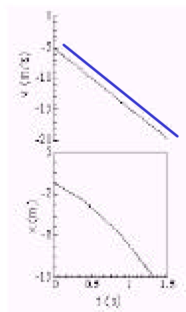
Three cases of free fall

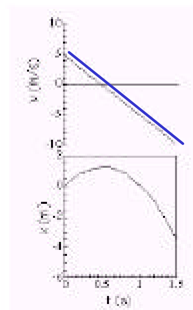
+

$$x(t) = x_0 + v_0 t + \tfrac{1}{2}(-g)t^2$$
$$v(t) = v_0 + (-g)t$$

$v_0 = 0$ $\qquad$ $v_0 < 0$ $\qquad$ $v_0 > 0$







Conclusions:
Motion along a straight line

- the simplest type of motion
- the groundwork for more complex motion

<u>Kinematical variables in one dimension</u>

Position:	**x(t)**	meters
Velocity:	**v(t)**	meters/second
Acceleration:	**a(t)**	meters/second ^{2}

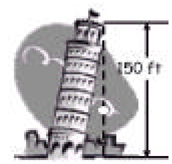
All depend on time

All are vectors: have direction and magnitude.

Lecture QZ2

1. What is the length of the vector **C** if **C**=**A**+**B**
   A=(5m)·**i** + (3m)·**j** and **B** = (–2m)·**i** + (1m)·**j** ?
   Hint: **i** and **j** are the unit vectors.

2. A stone is dropped from the height of 150 ft with no initial velocity. What is the rock's speed after the first 2 seconds. (Neglect the air resistance).
   Hint: The free fall acceleration g = 9.8 m/s^2



Homework

- Utexas Second HW