Physics 105 NJIT EQUATION SHEET Fall 2003

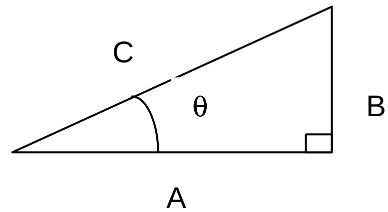
Name: ________________________

CONSTANTS AND TRIGONOMETRY

$g = 9.80\ m/s^2$

$\sin\theta = \frac{B}{C}$ $\cos\theta = \frac{A}{C}$ $\tan\theta = \frac{B}{A}$

$C = \sqrt{A^2 + B^2}$



MOTION ALONG A STRAIGHT LINE

$v = v_0 + at$ $x - x_0 = v_0 t + \frac{1}{2}at^2$

$x - x_0 = \frac{1}{2}(v_0 + v)t$ $v^2 = v_0^2 + 2a(x - x_0)$

TWO-DIMENSIONAL MOTION:

$\mathbf{r} = (v_{ox}t + \frac{1}{2}a_x t^2)\,\mathbf{i} + (v_{oy}t + \frac{1}{2}a_y t^2)\mathbf{j};$ $\mathbf{v} = (v_{ox} + a_x t)\,\mathbf{i} + (v_{oy} + a_y t)\,\mathbf{j};$ $\mathbf{a} = \frac{v_x - v_{ox}}{t}\mathbf{i} + \frac{v_y - v_{oy}}{t}\mathbf{j}$

PROJECTILE MOTION

$v_{0x} = v_0\cos\theta_0$ $v_{0y} = v_0\sin\theta_0$ $\Delta x = v_{0x}t$ $v_y = v_{0y} - gt$

$\Delta y = v_{0y}t - \frac{1}{2}gt^2$ $v_y^2 = v_{0y}^2 - 2g(\Delta y)$

$\Delta y = \frac{v_y^2 - v_{oy}^2}{-2g}$ $\Delta y = \left(\frac{v_{0y} + v_y}{2}\right) \times t$

FORCE AND MOTION

$\mathbf{F_{net}} = m\mathbf{a}$ $F_g = mg$ $f_{s,max} = \mu_s N$ $f_k = \mu_k N$

Uniform circular motion: centripetal acceleration: $a = v^2/r$
centripetal force: $F = mv^2/r$

WORK AND ENERGY:

$W = Fd(\cos\theta)$ $W = F_x d_x + F_y d_y + F_z d_z$ $W_{net} = \Delta K$ $W_{spring} = -\frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}k(x_i^2 - x_f^2)$

$K = \frac{1}{2}mv^2$ $\Delta U_{grav} = mg(y_f - y_i)$

Spring: $F_s = -kx$ $\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$

Power: $P_{avg} = \frac{W}{\Delta t}$

CONSERVATION OF ENERGY

$W = \Delta E_{mec} + \Delta E_{th}$ $\Delta E_{mec} = \Delta K + \Delta U$

Or

Work due to *nonconservative* forces: $W_{nc} = \Delta E_{mec}$

$W_{nc} = \Delta K + \Delta U_g + \Delta U_s$ or $K_i + U_{gi} + U_{si} + W_{nc} = K_f + U_{gf} + U_{sf}$

If the system is isolated (no friction or applied forces do work on system):

$0 = \Delta K + \Delta U_g + \Delta U_s$ or $K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$

CENTER OF MASS:

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \qquad y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i$$

LINEAR MOMENTUM:

$\vec{p} = m\vec{v}$ Impulse: $\vec{F}_{avg}\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_o$

For system of particles: $\Delta\vec{p}_{sys} = \sum(m\vec{v})_f - \sum(m\vec{v})_o$

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Closed book exam - Calculators are allowed.

Only the official formula sheet downloaded from the course web page can be used. You are allowed to write notes on the back of the formula sheet.

Use the scantron forms (pencil only!) for the multiple-choice problems. Circle the answers on the examination sheet as well, and return it together with the scantron form. Use the back of these pages, or attach your own pages with solutions for the problems, which require calculations.

The multiple-choice problems are 1 point each.. Passing of the exam requires at least 50% of the maximum number of points.

Clearly print your last name and indicate your section number on both the scantron form and the examination sheet. Failure to do any of these will result in a penalty of 2 points.

Problem 1. 1 mi is equivalent to 1609 m so 55 mph is:

A) 16 m/s
B) 25 m/s (circled)
C) 66 m/s
D) 88 m/s
E) 1500 m/s

$v = 55\,mph = 55 \cdot 1609\ m/h = \frac{55 \cdot 1609}{3600}\frac{m}{s} = 25\ m/s$

Problem 2. A cubic box with an edge of exactly 1 cm has a volume of:

A) 10^{-9} m^3
B) 10^{-6} m^3 (circled)
C) 10^{-3} m^3
D) 10^3 m^3
E) 10^6 m^3

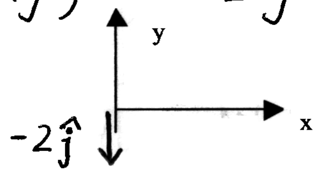
$1\ cm = 0.01\ m$

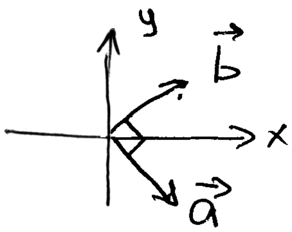
$(1\ cm)^3 = (0.01\ m)^3 = 10^{-6}$

Problem 3. Consider a two-dimensional system of coordinates with the *x*-axis pointing right and the *y*-axis pointing up as shown in the figure below. In vector component notation, $\vec{a} = \hat{i} - \hat{j}$ and $\vec{b} = \hat{i} + \hat{j}$. The difference of these vectors, $\vec{a} - \vec{b}$, points

A) up
B) down (circled)
C) left
D) right
E) is zero (no direction)

$\vec{a} - \vec{b} = \hat{i} - \hat{j} - (\hat{i} + \hat{j}) = -2\hat{j}$





Problem 4. For the previous problem (*Problem 3*), the scalar product of the two vectors, $\vec{a} \bullet \vec{b}$, is

A) 4
B) 3
C) 2
D) −2
E) 0 (circled)

$\vec{a} \cdot \vec{b} = (\hat{i} - \hat{j}) \cdot (\hat{i} + \hat{j}) = \hat{i}^2 - \hat{j}^2 = 1 - 1$

$\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = \emptyset$

Problem 5. For vectors $\vec{a}$ and $\vec{b}$ from problem 3, the magnitude of a vector $7\vec{a} + \vec{b}$ is

A) 0
B) 2
C) 8
D) 10 (circled)
E) 12

$7\vec{a} + \vec{b} = 7 \cdot (\hat{i} - \hat{j}) + \hat{i} + \hat{j} =$

$= 7\hat{i} - 7\hat{j} + \hat{i} + \hat{j} = 8\hat{i} - 6\hat{j}$

$|7\vec{a} + \vec{b}| = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

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Problem 6. How far will an object under earth's gravity drop in 2 s, starting from rest and neglecting air friction?

A) 0.49 m
B) 4.9 m
C) 9.8 m
D) 19.6 m
E) 29.4 m

$y = \frac{g \cdot t^2}{2} = \frac{9.8 \cdot 2^2}{2} m = 19.6 m$

Problem 7. A 40 kg ball is dropped from a height of 5.1 m above a table. What is the velocity of the ball just before impact?

A) 5 m/s
B) 10 m/s
C) 15 m/s
D) 20 m/s
E) 25 m/s

$\frac{mv^2}{2} = mgh \; ; \; v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 5.1} \; m/s = 10 \frac{m}{s}$

Problem 8. An object with an initial velocity of 12 m/s west experiences a constant acceleration of 4 m/s^2 west for 3 seconds. During this time the object travels a distance of:

A) 12 m
B) 24 m
C) 36 m
D) 54 m
E) 144 m

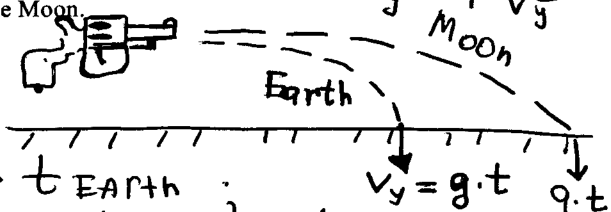
$\vec{V_0} = 12 \; m/s \quad \vec{a} = 4 \; m/s^2$

$X = V_0 \cdot t + \frac{at^2}{2} = \left(12 \cdot 3 + \frac{4 \cdot 3^2}{2}\right) m = 54 m$

Problem 9. Identical guns fire identical bullets horizontally at the same speed from the same height above level planes, one on the Earth and one on the Moon (g=1.67 m/s^2 on the moon.. Which of the following three statements is/are true?

I. The horizontal distance traveled by the bullet is greater for the Moon.
II. The flight time is less for the bullet on the Earth.
III. The velocity of the bullets at impact are the same.

A) III only
B) I and II only
C) I and III only
D) II and III only
E) I, II, III



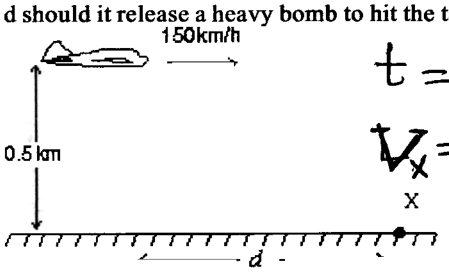
$V_y = \sqrt{2gh}$ $\quad V_y^M \neq V_y^E$

$h = gt^2/2$

$t = \sqrt{\frac{2h}{g}} \Rightarrow t_{MOON} > t_{EARTH}$;

$L = V \cdot t$; $L_{moon} > L_{EARTH}$

Problem 10. The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance d should it release a heavy bomb to hit the target X? Take $g = 10$ m/s^2.



$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 500 m}{10 m/s^2}} = 10 s$

$V_x = 150 \; km/h = 41.7 \; m/s$

$d = V_x \cdot t = (41.7 \cdot 10) m = 417 m$

A) 150 m ; B) 295 m; C) 417 m ; D) 2550 m ; E) 15,000 m

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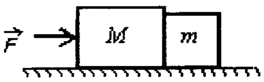
Problem 11. A force of 6 newtons acts on a 3-gram particle. Find the acceleration (watch for dimensions!):

A) 2 m/s^2
B) 0.5 m/s^2
C) 2000 m/s^2
D) 18 m/s^2
E) 9 m/s^2

$a = F/m = \frac{6\ N}{0.003\ kg} = 2000\ \frac{m}{s^2}$

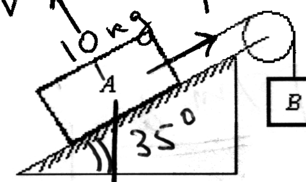
Problem 12. Two blocks with masses *m* and *M* are pushed along a horizontal frictionless surface by a horizontal applied force F as shown. The acceleration of the system is is:



A) $(m + M)/F$
B) F/M
C) $F/(M - m)$
D) $F/(M + m)$
E) F/m

$a = \frac{Force}{mass}$ $\frac{F}{M+m}$

Problem 13. Block A, with a mass of 10 kg, rests on a frictionless, 35° inclined plane. An attached string is parallel to the incline and passes over a massless, frictionless pulley at the top. The mass m_B, attached to the dangling end, for which A remains at rest is:



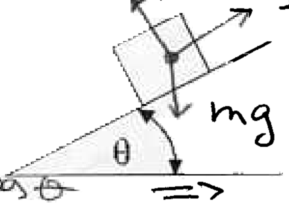
A: $T = m_A g \cdot \sin\theta$
B: $T = m_B g$

A) 2.74 kg
B) 3.74 kg
C) 5.74 kg
D) 9.74 kg
E) 10.74 kg

$m_B = m_A \cdot \sin\theta$

$m_B = 10\,kg \cdot \sin 35° = 5.7\ kg$

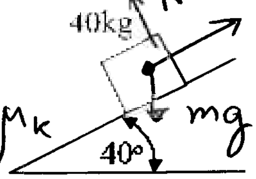
Problem 14. The coefficient of static friction between a 4-kg block and an inclined plane is 0.3. The block is at rest. As the inclined plane is slowly raised, the block will begin to slide when the



A) tangent of angle θ = 0. 25
B) cosine of angle θ = 0.3
C) cosine of angle θ = 0.25
D) tangent of angle θ = 0.3
E) sine of angle θ = 0.25

$f_{st} = \mu \cdot N$
$N = mg \cdot \cos\theta$
$mg \cdot \sin\theta = \mu \cdot mg \cos\theta$ ⇒ $\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

Problem 15. What is the frictional force between a 40 kg block and the ramp? The coefficient of static friction is 0.8, and the coefficient of kinetic friction is 0.4.



A) 100 (96) N
B) 120 (85) N
C) 200 (74) N
D) 240 (63) N
E) 400 (54) N

$f = \mu \cdot N = \mu \cdot mg \cdot \cos\theta$

we have to decide μ_{st} or μ_k

at rest $\tan\theta = \tan 40° = 0.84$

$\mu_s = 0.8 < 0.84$ ⇒

$f_{kin} = 0.4 \cdot 40 \cdot 9.8 \cdot \cos 40° = 120N$

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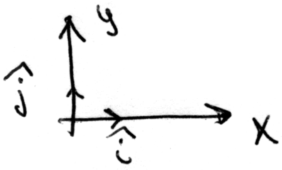
Problem 16. A particle moves 5 m in the $+x$ direction while being acted upon by a constant force $\vec{F} = 4\hat{i} + 2\hat{j}$ (Newtons). The work done on the particle by this force is:



A) 20 J
B) 10 J
C) –20 J
D) 30 J
E) impossible to calculate without knowing other forces

$$\vec{d} = (5m)\cdot\hat{i}$$
$$W = \vec{F}\cdot\vec{d} = 5\hat{i}\cdot 4\hat{i} + \underbrace{\emptyset\hat{j}\cdot 2\hat{j}}_{\emptyset} = 20\,J$$

Problem 17. An object with mass of 4 kg is traveling in a circular path of radius 1 meter at a constant speed of 10 m/s. The acceleration of the object is

A. 25 m/s^2
B. 50 m/s^2
C. 100 m/s^2
D. 250 m/s^2
E. 0.1 m/s^2

$$a = \frac{V^2}{R} = \frac{10\cdot 10}{1\,m}(m/s)^2 = 100\,m/s^2$$

Problem 18. The iron ball shown is being swung in a vertical circle at the end of a 0.7-m string. How slowly, in m/s, can the ball go through its top position without having the string go slack [HINT: at this point, the tension in the string is zero]?

A) 1.3 m/s
B) 2.6 m/s
C) 3.9 m/s
D) 6.9 m/s
E) 9.8 m/s

$$m\cdot a^c = mg + T,$$
since $T = \emptyset$, then
$$ma^c = mg$$
$$a^c = g;\quad \frac{V^2}{R} = g$$
$$V = \sqrt{g\cdot R} = \sqrt{9.8\cdot 0.7} = 2.6\,\frac{m}{s}$$

Problem 19. A 5.0-kg cart is moving horizontally at 6.0 m/s. In order to increase its speed to 10.0 m/s, the net work done on the cart must be:

A) 40 J
B) 90 J
C) 160 J
D) 400 J
E) 550 J

$$W_N = \Delta K;\quad \Delta K = \frac{mV_f^2}{2} - \frac{mV_i^2}{2} =$$
$$= \frac{5kg}{2}\left(10^2_{\frac{m}{s}} - 6^2_{\frac{m}{s}}\right)$$
$$160\,J$$

$$W_{Net} = 160\,J$$

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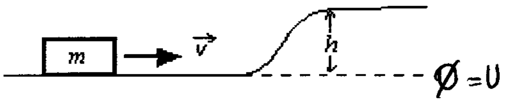
Problem 20. For a block of mass m to slide without friction up the rise of height h shown, it must have a minimum initial speed of:



$\Delta K \Rightarrow \Delta U$

$\frac{mv^2}{2} \Rightarrow mgh$

$v = \sqrt{2gh}$

A) $1/2\sqrt{gh}$

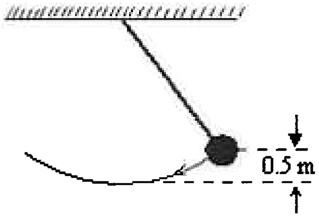
B) $\sqrt{gh/2}$

C) $\sqrt{2gh}$

D) $2\sqrt{2gh}$

E) $2\sqrt{gh}$

Problem 21. The long pendulum shown is drawn aside until the ball has risen 0.5 m. It is then released from rest. The speed of the ball at its lowest position is:



$\Delta K + \Delta U = 0$

$mgh = \frac{mv^2}{2}$

$V = \sqrt{2gh} =$

$= \sqrt{2 \cdot 9.8 \cdot 0.5}$ m/s

$= 3$ m/s

A)	3.1 m/s
B)	4.2 m/s
C)	5.8 m/s
D)	20 m/s
E)	cannot be determined unless the mass is known

Problem 22. The spring constant of a spring is 4.0 newtons per millimeter. The work done in compressing this spring from x = 10 millimeters to x = 30 millimeters (with x=0 corresponding to uncompressed spring) is

A. 0.16 J

B. 0. 32 J

C. 0. 64 J

D. 1.28 J

E. 2.54 J

?

$W = \frac{kx_f^2}{2} - \frac{kx_i^2}{2} = \frac{k}{2}(x_f^2 - x_i^2)$

$= \frac{4 \cdot 10^3 N/m}{2} \cdot (0.03^2_{m^2} - 0.01^2_{m^2}) = 1.6\ J$

Problem 23. A 0.5-kg block slides along a horizontal frictionless surface at 2 m/s. It is brought to a momentarily rest by compressing a very long spring of spring constant 800 N/m. The maximum spring compression is:

A)	0.1 cm
B)	3 cm
C)	5 cm
D)	80 cm
E)	80 m

$K_i = \frac{0.5 kg\ (2\ m/s)^2}{2} = 1\ J$

$\Delta U_s = \Delta K_{f-i}; \quad \frac{k(\Delta x)^2}{2} = 1\ J$

$\Delta x = \sqrt{\frac{2 \cdot 1 J}{800\ N/m}} = 0.05 m$

$0.05 m = 5 cm$

unit conversion

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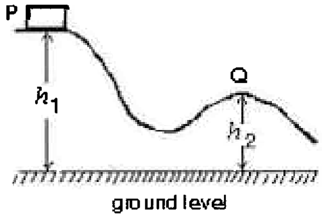
Problem 24 A block is released fro m rest at point P and slides along the frictionless track shown. At point Q, its speed is:



A) $2g\sqrt{h_1 - h_2}$
B) $2g(h_1 - h_2)$
C) $(h_1 - h_2)/2g$
D) $\sqrt{2g(h_1 - h_2)}$
E) $(h_1 - h_2)^2/2g$

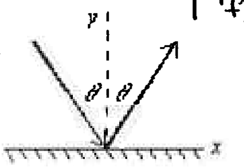
$\Delta U = -mg(h_1 - h_2)$
$\Delta K = \frac{mv^2}{2}$
$\Delta E = \emptyset \Rightarrow$
$\Delta U + \Delta K = \emptyset \Rightarrow$
$\frac{mv^2}{2} = mg(h_1 - h_2)$; $V = \sqrt{2g(h_1 - h_2)}$ [m/s]

Problem 25. A 4-kg block moves from rest down a frictionless inclined plane, which is 4 m long and has one end elevated by 2 m with respect to the other. The final speed of the block is most nearly

A. 6.26 m/s
B. 5.25 m/s
C. 4.24 m/s
D. 3.23 m/s
E. 2.22 m/s

$V = \sqrt{2g \cdot \Delta y}$; $\Delta y = 2m$
$V = \sqrt{2 \cdot 9.8 \cdot 2} \approx \sqrt{40} \approx 6.3 m/s$

Problem 26. A ball hits a wall and rebounds with the same speed, as diagrammed below. The changes in the components of the momentum of the ball are:



$\vec{P}_{i,y} \downarrow$ $\uparrow \vec{P}_{f,y}$ y, x

no forces in X direction
$\Delta P_x = \emptyset$
$\Delta P_y = P_{fy} - P_{iy} > \emptyset$

A) $\Delta p_x > 0, \Delta p_y > 0$
B) $\Delta p_x < 0, \Delta p_y > 0$
C) $\Delta p_x = 0, \Delta p_y > 0$
D) $\Delta p_x = 0, \Delta p_y < 0$
E) $\Delta p_x > 0, \Delta p_y < 0$

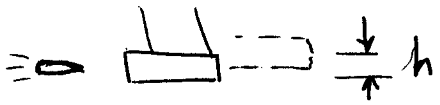
Problem 27. A 60-g projectile with a horizontal speed 60 m/s collides with and is embedded in an 40-g block of wood that is initially at rest on a horizontal frictionless surface. What is the speed of the block after impact?

A. 16 m/s
B. 25 m/s
C. 36 m/s
D. 49 m/s
E. 64 m/s

$P_i = 0.06 kg \cdot 60 m/s = 3.6 m/s \cdot kg$
$P_f = (m+M) \cdot V_f$; $V_f = \frac{3.6}{(0.06 + 0.04)} = 36 m/s$

Problem 28. A 3-gram bullet is fired horizontally into a 10-kg block of wood suspended by a rope from the ceiling. The block swings in an arc, rising 3 mm above its lowest position. The velocity of the bullet was:

A) unknown since the heat generated in the collision was not given
B) 8.0×10^2 m/s
C) 24.0 m/s
D) 8.0 m/s
E) 2.4×10^4 m/s



$m \cdot V_b = (m+M) \cdot V_f$; $V_f = \sqrt{2gh}$
$V_b = \frac{m+M}{m} \cdot \sqrt{2gh} =$
$= \frac{(10 + 0.003) kg}{0.003 kg} \sqrt{2 \cdot 9.8 \, m/s^2 \cdot 0.003 m} =$
$= 808 m/s$

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Problem 29. Two objects are on a head-on collision course. The first object with mass m=1 kg is moving from left to right with a speed of 10 m/s. The second object with mass M=5 kg is moving from right to left with a speed of 2 m/s. After the (partially inelastic) collision the first object is moving left, with a speed of 2.5 m/s. What is the velocity of object 2 after impact?

A) 0.25 m/s, left
B) 0.25 m/s, right
C) 0.5 m/s, left
D) 0.5 m/s, right
E) 0 m/s

$P_i = m_1 V_1 - M_2 V_2 = 10 - 10 = \emptyset$

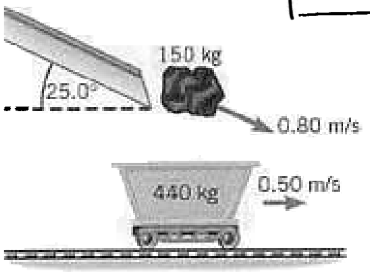
$P_f = -2.5 \cdot 1 + M_2 \cdot V_f = \emptyset$

$P_i = P_f$

$V_f = \frac{2.5}{5} = +0.5\, m/s$

Problem 30. A mine car, whose mass is 440 kg, rolls at a speed of 0.50 m/s on a horizontal track, as the drawing shows. A 150-kg chunk of coal has a speed of 0.80 m/s when it leaves the chute. Determine the velocity of the car/coal system after the coal has come to rest in the car.

A. 0.23 m/s
B. 0.34 m/s
C. 0.45 m/s
D. 0.56 m/s
E. 0.67 m/s



$P_{ch}^x = m \cdot V \cdot \cos\theta$

$P_{ch}^y = m\, V \sin\theta$

$= 150\,kg \cdot 0.8\,\frac{m}{s} \cdot \cos 25^\circ$

$= -150\,kg \cdot 0.8 \cdot \cos 25^\circ$

$\vec{P}_{ch} = P_{ch}^x \hat{i} + P_{ch}^y \hat{j}$

$P_{car}^x = M\, V = 440\,kg \cdot 0.5\,\frac{m}{s}$

$P_{car}^x = 220\,kg\,\frac{m}{s}$

$P_{x,i} = P_{ch}^x + P_{car}^x = 50 \cdot 0.8 \cdot \cos 25^\circ + 220\,kg \cdot \frac{m}{s} =$

$= (108 + 220)\,kg\,\frac{m}{s} = 328\,kg\,\frac{m}{s}$

$P_{x,f} = V_f \cdot (m + M) = V_f\; 590\,kg$

$V_f = \frac{328}{590}\, m/s = 0.56\, m/s$