Lecture 13

Momentum Conservation
And Collisions

(HR&W, Chapters 9)

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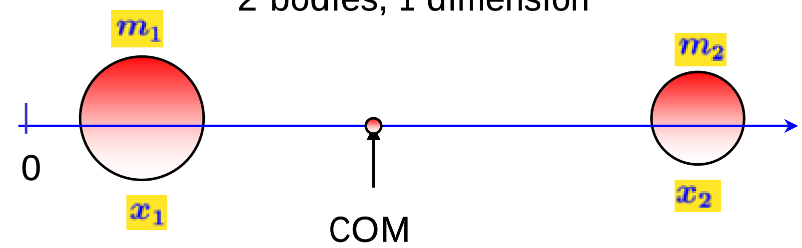
Physics 105   Summer 2006

Center of Mass for a system of particles

2 bodies, 1 dimension



$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Linear Momentum

New fundamental quantity (like force,energy,..)

Particle: $\vec{p} = m\vec{v}$

System of Particles: $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots$

Extended objects: $\vec{P} = M\vec{v}_{com}$

Relation to Force: $\vec{F}_{tot} = m\vec{a}$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

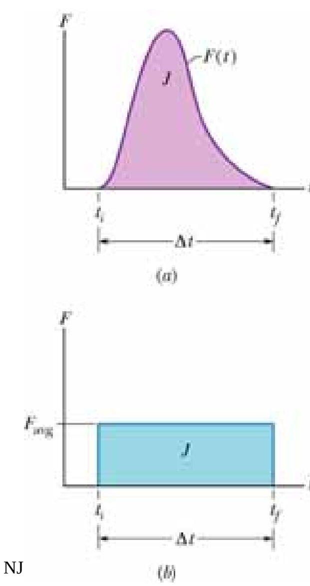
$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

Impulse and Linear Momentum

Definition of Impulse

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J}$$

Impulse–Linear Momentum Theorem



TODAY:

- Collisions
- Impulse and Linear Momentum
  - Single Collision
- Momentum and Kinetic Energy
- Inelastic Collisions in One Dimension
  - One-Dimensional Collision
  - Completely Inelastic Collision
- Elastic Collisions in One Dimension

Conservation of Linear Momentum

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

If $F_{tot} = 0$, then momentum is constant

For an isolated system (no external forces):

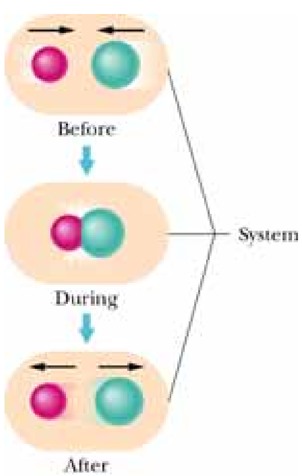
$$\vec{P} = \text{const.} \quad \Rightarrow \quad \vec{P}_i = \vec{P}_f$$

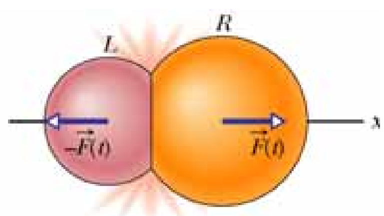
Even if there are internal forces inside the system

If no net external force acts on a system of particles, the total linear momentum ***P*** of the system cannot change

If the component of the net external force on a closed system is zero along an axis, then the component of the linear momentum along that axis cannot change

Collision of two particle-like bodies



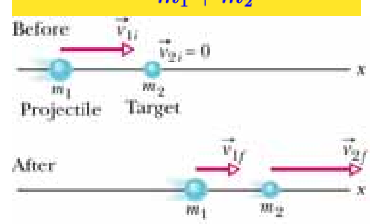


Elastic Collisions in 1D

In an elastic collision, the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change

Stationary Target

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$
$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$



Moving Target

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$
$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Elastic Collisions in 1D

$m_1 = m_2$

$$v_{1,f} = v_{2,i}$$
$$v_{2,f} = v_{1,i}$$

Objects exchange velocities

$m_1 >> m_2$

$$v_{1,f} = v_{1,i}$$
$$v_{2,f} = 2\,v_{1,i} - v_{2,i}$$

Heavy object unchanged; light object has large change in velocity

Elastic Collisions in 1D

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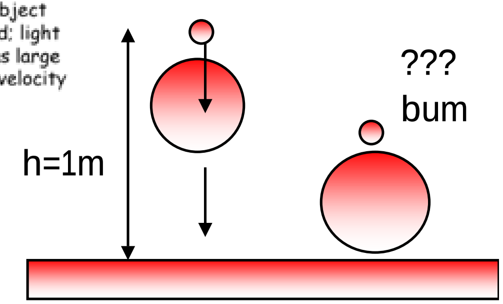
Heavy object unchanged; light object has large change in velocity

What is the rebound heights of the Basketball ?
What is the rebound heights of the tennis ball ?



Elastic and Inelastic Collisions

Two types of collisions: Elastic and Inelastic

<u>Inelastic</u> (most collisions): Momentum is conserved but energy is lost

Sound, heat, distortion of objects, sticking together

Only one equation:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

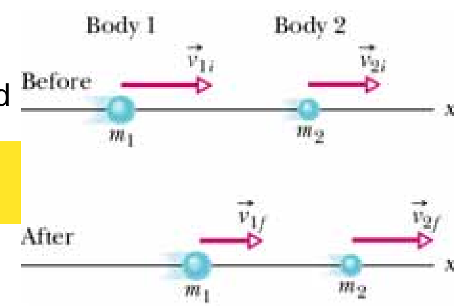
<u>Inelastic</u> Collisions in 1D

Conservation of Linear Momentum

Kinetic Energy is not conserved

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$



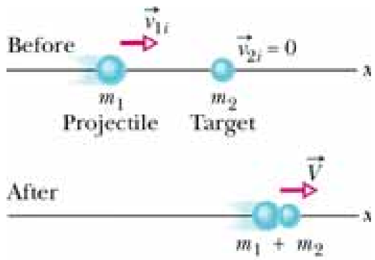
Cannot solve based only on the information about the state before the collision

Completely Inelastic Collision
Collisions in 1D

Conservation of Linear Momentum works !

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

$$m_1 v_{1i} = (m_1 + m_2)V$$
$$V = \frac{m_1}{m_1 + m_2} v_{1i}$$



Example: Two equal objects, one initially at rest

$mv_i = 2mv_f \longrightarrow v_f = v_i/2$

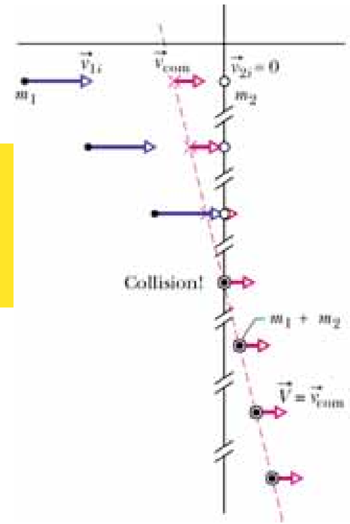
Final Kinetic Energy = $\frac{1}{2}(2m)(v_i/2)^2$
= $\frac{1}{4}m(v_i)^2$

Half the original Kinetic Energy

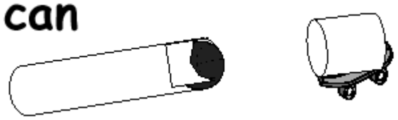
Inelastic Collisions in 1D

Velocity of Center of Mass

$$\vec{P} = M\vec{v}_{com} = (m_1 + m_2)\vec{v}_{com}$$
$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}$$

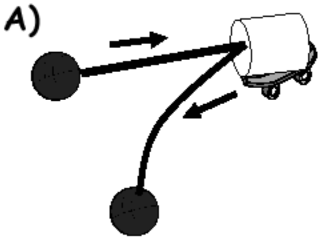


Shoot the trash can

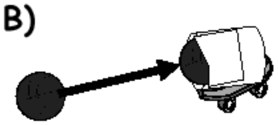


Two possible outcomes

A)



B)



Ball bounces out

Ball sticks

Which way should I shoot the trashcan to make it go faster ???

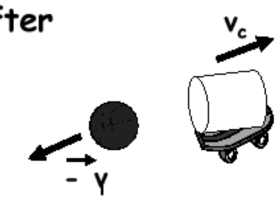
Trash can continued

Before

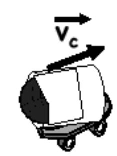
$\vec{P}_i = m_b\vec{v}_i + m_c(0)$
$= m_b\vec{v}_i$

After

A)



B)



$P_f = m_b(-\vec{v}_i) + m_c\vec{v}_c$
$= m_c\vec{v}_c - m_b\vec{v}_i$

$P_f = m_b\vec{v}_c + m_c\vec{v}_c$
$= (m_b + m_c)\vec{v}_c$

Trash can (finish)

A) Bouncing out

$$\vec{P}_i = \vec{P}_f$$

$$m_b\vec{v}_i = m_c\vec{v}_c - m_b\vec{v}_i$$

B) Sticking

$$\vec{P}_i = \vec{P}_f$$

$$m_b\vec{v}_i = (m_c+m_b)\vec{v}_c$$

Solving for v_c

$$\vec{v}_c = \vec{v}_i \frac{2m_b}{m_c}$$

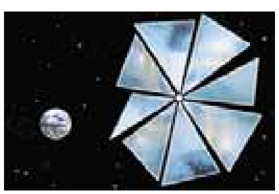
$$\vec{v}_c = \vec{v}_i \frac{m_b}{m_c+m_b}$$

Which is greater?

Always: $\frac{2m_b}{m_c} > \frac{m_b}{m_c+m_b}$

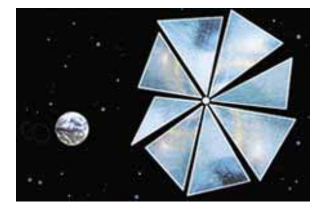
Who wins ?

How can we reach another star ?



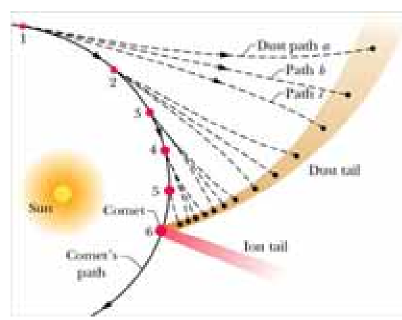
Combination of
- Regular rocket
- Ion-drive engine
- And Solar sail

Light pressure



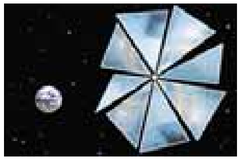
$$\Delta p = \frac{\Delta U}{c} \quad \text{(total absorption)},$$

$$\Delta p = \frac{2\Delta U}{c} \quad \text{(total reflection back along path)}$$



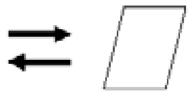
$$p_r = \frac{I}{c} \quad \text{(total absorption)}$$

$$p_r = \frac{2I}{c} \quad \text{(total reflection back along path)}$$



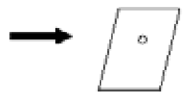
Light Sail

White Sail

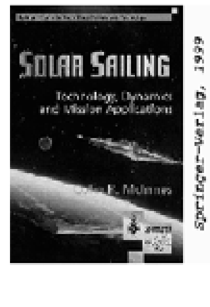


Photons bounce off

Black Sail



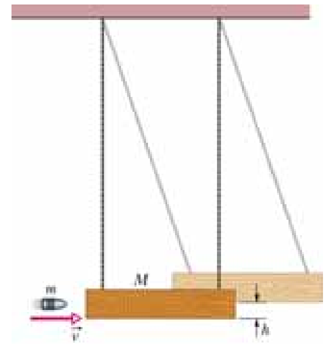
Photons adsorbed



What color/material is the best for the Light Sail?

A) Black; B) Mirror-type; C) Blue; D) any

Sample Problem

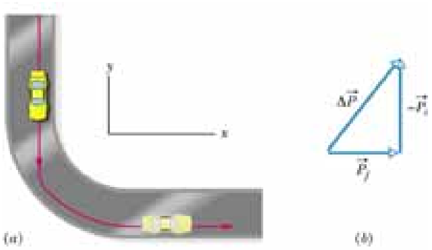


Ballistic Pendulum. A bullet is fired into the block (M = 5 kg, m = 10 g). The block/bullet is then swinging upward, their center of mass rising h = 6 cm. What is the speed of the bullet just prior to the collision?

a) Linear Momentum is conserved at the collision

b) After collision the Energy is conserved

QZ#13
Linear Momentum



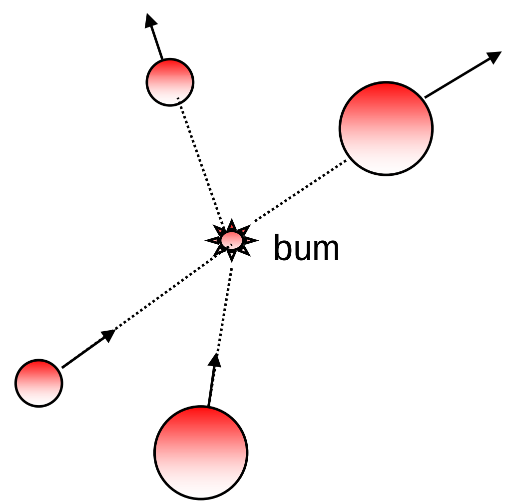
$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt}$$

The figure shows a **2.0 kg** toy car before and after taking a turn on a track. Its speed is **0.30 km/s** before the turn and **0.40 km/s** after the turn. The turn takes **0.33** seconds.

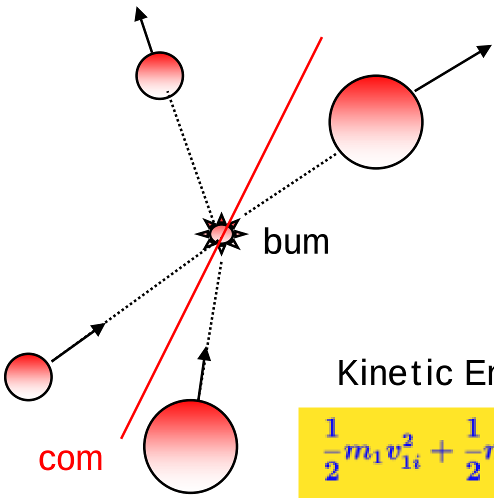
(a) What is the change ΔP in the linear momentum of the car due to the turn?

(b) What is the **average force** of friction between the car and the road during the turn ?

Elastic Collisions in 2D



Elastic Collisions in 2D



$$\vec{v}_{com} = \frac{\vec{P}}{m_1 + m_2} = \frac{\vec{p}_{1i} + \vec{p}_{2i}}{m_1 + m_2}$$

Conservation of Linear Momentum works !

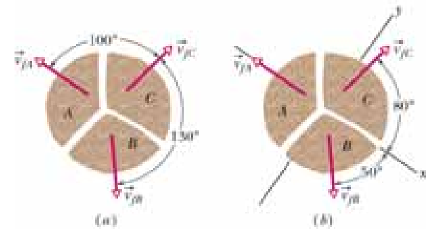
$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

Kinetic Energy is conserved

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

Conservation of Linear Momentum

$$\vec{P} = \text{const.} \quad \Rightarrow \quad \vec{P}_i = \vec{P}_f$$



A firecracker placed inside a coconut of mass ***M***, initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. An overhead view is shown. Piece *C*, with mass **0.30 *M***, has final speed v_{fC} **= 5.0 m/s**.

(a) What is the speed of piece *B*, with mass **0.2 *M*** ?

(b) (b) What is the speed of piece *A*?