Lecture 4

- **Motions in Two and Three Dimensions**
- **Projectile Motion**
- **Circular Motion**

(HR&W, Chapters 3 and 4)

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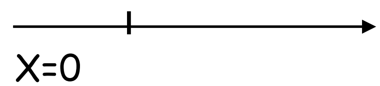
Physics 105, Summer 2005

Motion along the straight line + Vectors

One dimension (1D)

Position: $x(t)$ m
Velocity: $v(t)$ m/s
Acceleration: $a(t)$ m/s^2

All are vectors: have direction and magnitude.



Three dimension (2D)

Position: $\vec{r}(t)$ m

Velocity: $\vec{v}(t)$ m/s

Acceleration: $\vec{a}(t)$ m/s^2

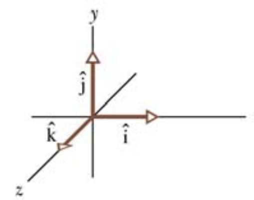


TABLE 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

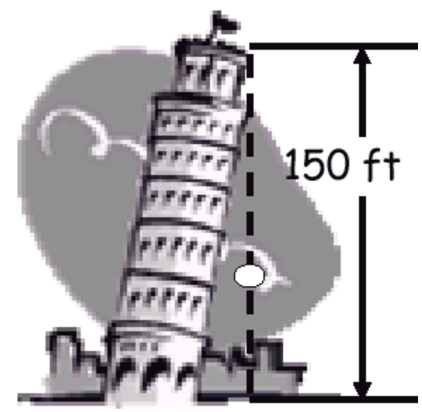
[a] Make sure that the acceleration is indeed constant before using the equations in this table.

Gravitation is universal

Most important case of constant acceleration: Free fall

$a = -g$ where $g = 9.8$ m/s^2
(defining "up" as the positive direction)

Gravitational acceleration does not depend on the nature of the material or the mass of the object.



Elephant and Feather

http://www.glenbrook.k12.il.us/gbssci/phys/mmedia/newtlaws/efar.html

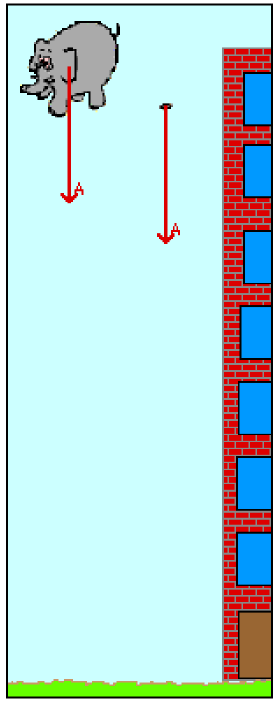
$a = -g$ where $g = 9.8\ m/s^2$
(defining "up" as the positive direction)

$a_y = -9.8\ m/s^2$

$v_y(t) = v_{y0} + a_y t;$

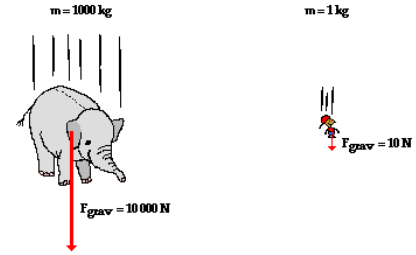
$y(t) - y_0 = v_{y0}\ t + a_y t^2/2$

If g is the same,
$H = y_f - y_0$ is the same,
and v_{y0} is the same, then the time of the fall should be the same for both Elethant and Feather !
What is wrong ???



Free Fall Motion

As learned in an earlier unit, free-fall is a special type of motion in which the only force acting upon an object is gravity. Objects which are said to be undergoing *free-fall*, are not encountering a significant force of air resistance; they are falling under the sole influence of gravity. Under such conditions, all objects will fall with the same rate of acceleration, regardless of their mass. But why? Consider the free-falling motion of a 1000-kg baby elephant and a 1-kg overgrown mouse.



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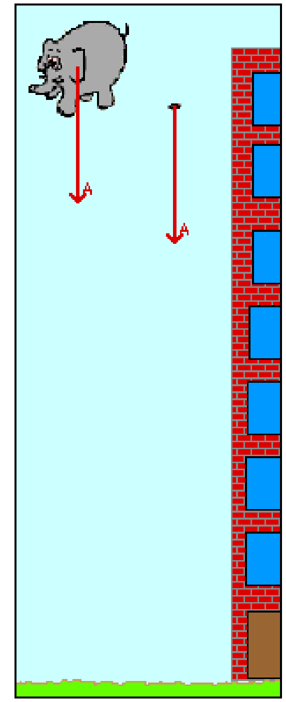


2D and 3D Motion

- Motion in 2D and 3D
  - Position, displacement, velocity, and acceleration
  - Projectile motion
  - Uniform circular motion
  - Relative motion
  - Reference frames

Motion in 3D: $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

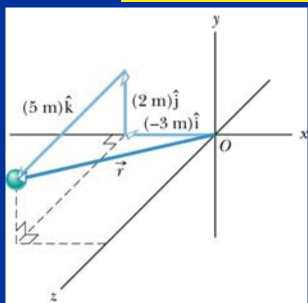
- $t = 0$ beginning of the process
- $\vec{r} = 0$ is arbitrary; can set where you want it
- $\vec{r}_0 = \vec{r}(t=0)$ position at t=0;

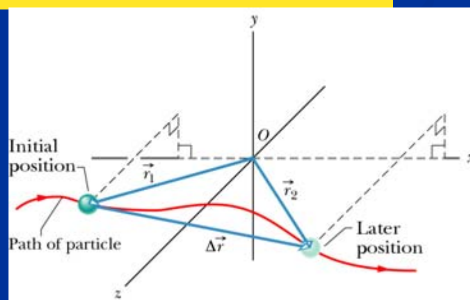
$$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z = x\cdot\hat{i} + y\cdot\hat{j} + z\cdot\hat{k}$$

$a_x \neq 0$	$v_x(t) = v_{x0} + a_x t;$	$x(t) - x_0 = v_{x0}\ t + a_x t^2/2$
$a_y \neq 0$	$v_y(t) = v_{y0} + a_y t;$	$y(t) - y_0 = v_{y0}\ t + a_y t^2/2$
$a_z \neq 0$	$v_z(t) = v_{z0} + a_z t;$	$z(t) - z_0 = v_{z0}\ t + a_z t^2/2$

Position and Displacement

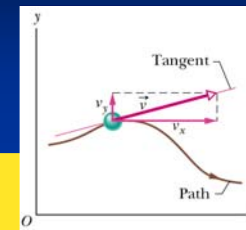
$$\begin{aligned}\vec r &= x\hat i + y\hat j + z\hat k\\ \Delta\vec r &= \vec r_2 - \vec r_1\\ \Delta\vec r &= (x_2\hat i + y_2\hat j + z_2\hat k) - (x_1\hat i + y_1\hat j + z_1\hat k)\\ &= (x_2 - x_1)\hat i + (y_2 - y_1)\hat j + (z_2 - z_1)\hat k\\ &= \Delta x\hat i + \Delta y\hat j + \Delta z\hat k\end{aligned}$$





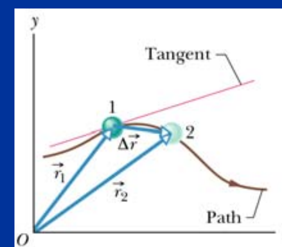
Average and Instantaneous Velocity

$$\begin{aligned}\vec v_{\rm avg} &= \frac{\Delta\vec r}{\Delta t}\\ &= \frac{\Delta x\hat i + \Delta y\hat j + \Delta z\hat k}{\Delta t}\\ &= \frac{\Delta x}{\Delta t}\hat i + \frac{\Delta y}{\Delta t}\hat j + \frac{\Delta z}{\Delta t}\hat k\end{aligned}$$



$$\begin{aligned}\vec v &= \frac{d\vec r}{dt}\\ &= \frac{d}{dt}(x\hat i + y\hat j + z\hat k)\\ &= \frac{dx}{dt}\hat i + \frac{dy}{dt}\hat j + \frac{dz}{dt}\hat k\\ &= v_x\hat i + v_y\hat j + v_z\hat k\end{aligned}$$

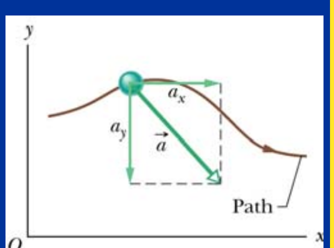
$$v_x = \frac{dx}{dt},\quad v_y = \frac{dy}{dt},\ \text{and}\quad v_z = \frac{dz}{dt}$$



Average and Instantaneous Acceleration

$$\vec a_{\rm avg} = \frac{\vec v_2 - \vec v_1}{\Delta t} = \frac{\Delta\vec v}{\Delta t}$$

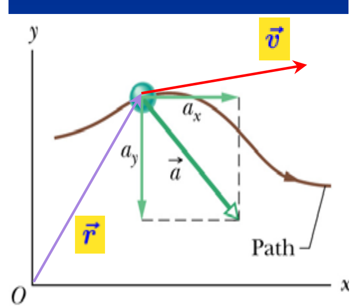
$$\begin{aligned}\vec a &= \frac{d\vec v}{dt}\\ &= \frac{d}{dt}(v_x\hat i + v_y\hat j + v_z\hat k)\\ &= \frac{dv_x}{dt}\hat i + \frac{dv_y}{dt}\hat j + \frac{dv_z}{dt}\hat k\\ &= a_x\hat i + a_y\hat j + a_z\hat k\end{aligned}$$



$$a_x = \frac{dv_x}{dt},\quad a_y = \frac{dv_y}{dt},\ \text{and}\quad a_z = \frac{dv_z}{dt}$$

Motion in 3D; Summary

Kinematic Variables in 2D or 3D



$\vec r(t) = x(t)\boldsymbol{i} + y(t)\ \boldsymbol{j} + z(t)\boldsymbol{k}$

$\vec v(t) = \frac{d\vec r}{dt} = \frac{dx}{dt}\boldsymbol{i} + \frac{dy}{dt}\boldsymbol{j} + \frac{dz}{dt}\boldsymbol{k} = v_x\boldsymbol{i} + v_y\boldsymbol{j} + v_z\boldsymbol{k}$

$\vec a(t) = \frac{d\vec v}{dt} = \frac{dv_x}{dt}\boldsymbol{i} + \frac{dv_y}{dt}\boldsymbol{j} + \frac{dv_z}{dt}\boldsymbol{k} = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k}$

$\vec r(t)$, $\vec v(t)$, and $\vec a(t)$ are not in same direction!

Projectile Motion

Horizontal motion + Vertical motion

"Free fall with horizontal motion"

$x \equiv$ horizontal
$y \equiv$ vertical (take positive direction as "up")
z is not relevant

$\vec{a}$ is only in the vertical direction: $\vec{a} = -g\, j$

$$a_y = -g \quad a_x = 0$$

Projectile Motion (continued)

$$a_x = 0 \qquad a_y = -g$$

In both directions the acceleration is constant

$$v_x = v_{0x} \equiv \text{constant} \qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Projectile Motion (continued)

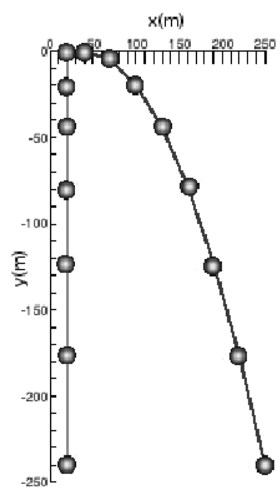
x and y motion happen independently so you can treat them separately

Connected by time:

$$x = x_0 + v_{0x}t$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Since y(t) is a parabola and x is linear in time: y(x) is a parabola too



Projectile Motion; General Case

Horizontal motion

$$x - x_0 = v_{0x}t = (v_0 \cos\theta_0)t$$

Trajectory and horizontal range

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

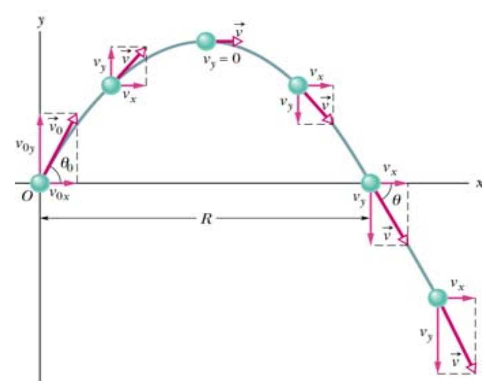
$$R = \frac{v_0^2}{g}\sin 2\theta_0$$

Vertical motion

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0\sin\theta_0)t - \frac{1}{2}gt^2$$

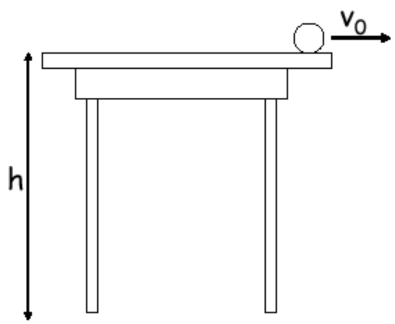
$$v_y = v_0\sin\theta_0 - gt$$

$$v_y^2 = (v_0\sin\theta_0)^2 - 2g(y - y_0)$$



Projectile Motion

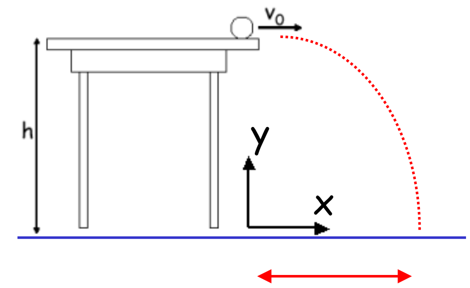
Sample Problem



A ball rolls off a table of height h. The ball has horizontal velocity v0 when it leaves the table.

How far away does it strike the ground?

How long does it take to reach the ground?



$$x - x_0 = v_{0x}t = (v_0 \cos\theta_0)t$$

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

Δx = ???

$v_{0x} = v_{0;}$ $x_0 = 0$

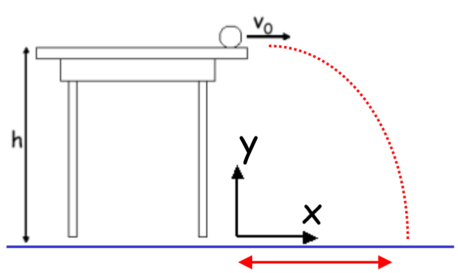
$v_{0y} = 0;$ $y_0 = h$

For **x** direction: $t = \Delta x / v_0$

For **y** direction: $y(t) = 0$

$y(t) - h = v_{0y}\, t - gt^2/2$

$\Delta x = v_0 \cdot (2h/g)^{\frac{1}{2}}$



For **x** direction: $t = \Delta x / v_0$

For **y** direction: $y(t) = 0$

$y(t) - h = v_{0y}\, t - gt^2/2$

$\Delta x = v_0 \cdot (2h/g)^{\frac{1}{2}}$

Δx = ???

$v_{0x} = v_{0;}$ $x_0 = 0$

$v_{0y} = 0;$ $y_0 = h$

For

v_0= 2.2 m/s

h = 1.0 m

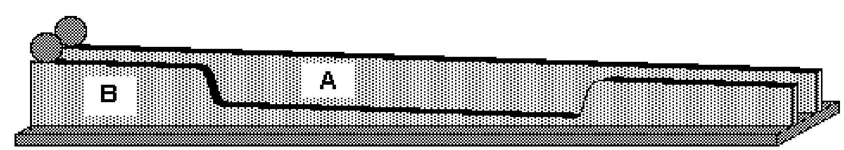
Δx= 1.0 m

QZ#3 How long is the fall?

QZ#4 (continued)

Velocity, Position, and Acceleration

HIGH ROAD LOW ROAD



Balls A and B start with equal velocities.

Which path (A or B) is quicker and why?

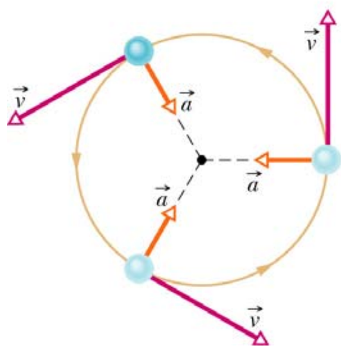
Uniform Circular Motion

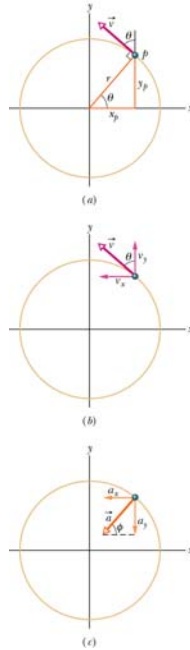
Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$





Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

What is her centripetal acceleration?

Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$

Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

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Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period: $T = \frac{2\pi r}{v}$

v = 100 m /12 s = 8.33 s; R = 100/π = 31.8 m

a = $(8.33)^2$/31.8 m/s ^{2} = 2.2 m/s ^{2}