Lecture 7

Problem-Solving Tactics:
Friction and Centripetal Motion

(HR&W, Chapters **5-6**)

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Physics 105; Summer 2006

Newton's Laws

I. If no net force acts on a body, then the body's velocity cannot change.

II. The net force on a body is equal to the product of the body's mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction ($\mathbf{F}_{12} = -\mathbf{F}_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration:

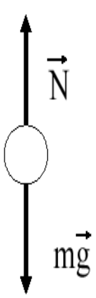
$$\vec{F}_{tot} = 0 \Leftrightarrow \vec{a} = 0 \text{ (constant velocity)}$$

$$\vec{F}_{tot} = m\vec{a} \text{ for any object}$$

$$F_{tot,x} = ma_x \qquad F_{tot,y} = ma_y \qquad F_{tot,z} = ma_z$$

How do we jump ?

A standing person



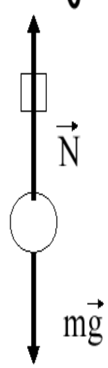
No acceleration

$\Rightarrow$

Net force is zero

$F_{NET}=|N|-mg=0$

A jumping person



Acceleration

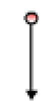
$\Rightarrow$

$F_{NET} = ma$

$|N|-mg=ma > 0$

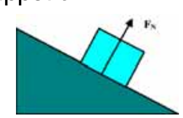
Forces:

- Gravitational Force: $\vec{\mathbf{F}}_g = \vec{\mathbf{mg}}$ down to the ground



- Tension Force: $\vec{\mathbf{T}}$ along the string

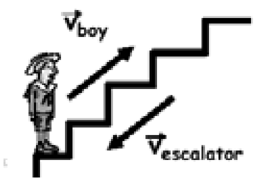
- Spring Force: $\mathbf{F}_s = -k\mathbf{x}$
- Normal Force: $\vec{\mathbf{N}}$ perpendicular to the support



- Friction Force
  - Static; maximum value $\mathbf{f}_s = \mu_{st}\mathbf{N}$
    opposite to the component of other forces parallel to the support
  - Kinetic; value $\mathbf{f}_k = \mu_{kin}\mathbf{N}$
    opposite to the **velocity**, parallel to the support

$$\mu_{st} > \mu_{kin}$$

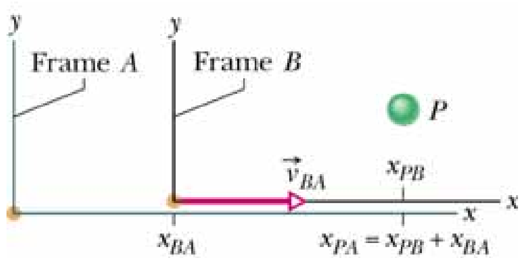
Relative Motion/Reference Frames

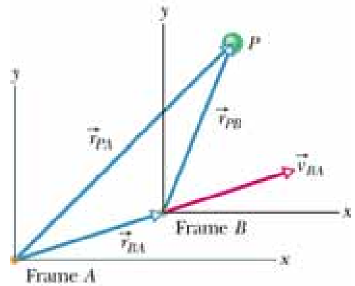


$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA} \quad \text{and} \quad \vec{v}_{BA} = \text{const.}$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$

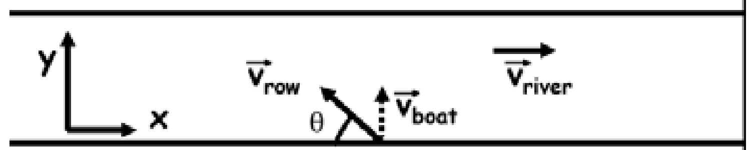




Relative Motion/Reference Frames

Relative Velocity: Rowing a Boat

You can row a boat at v_{row} = 3 m/s, and you want to go straight across a river which flows with v_{river} = 2 m/s. At what angle should you row?



$\vec{v}_{boat} = \vec{v}_{row} + \vec{v}_{river}$

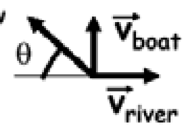
you want $\vec{v}_{boat}$ in y-direction to go straight across

Relative Motion/Reference Frames

Rowing a Boat (continued)

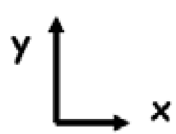
$\vec{v}_{boat} = \vec{v}_{row} + \vec{v}_{river}$

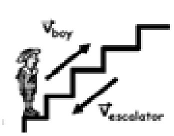
you want $\vec{v}_{boat}$ in y direction



need $v_{row,x} = - v_{river,x}$

$v_{row} \cos\theta = v_{river}$

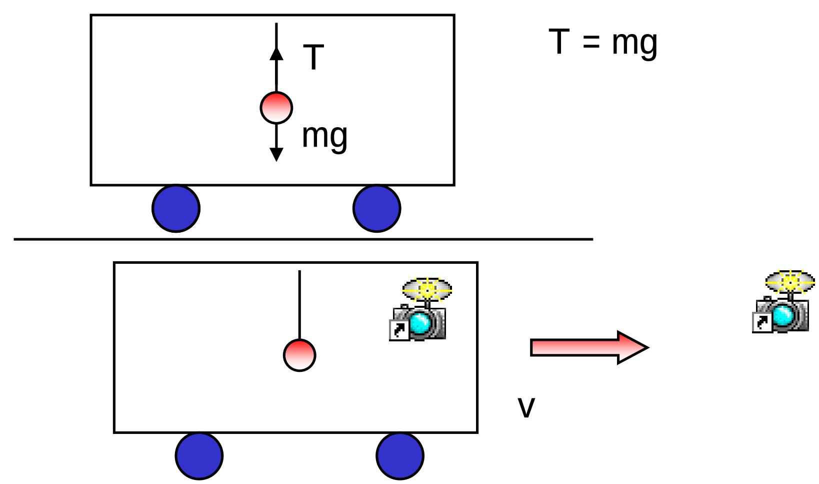


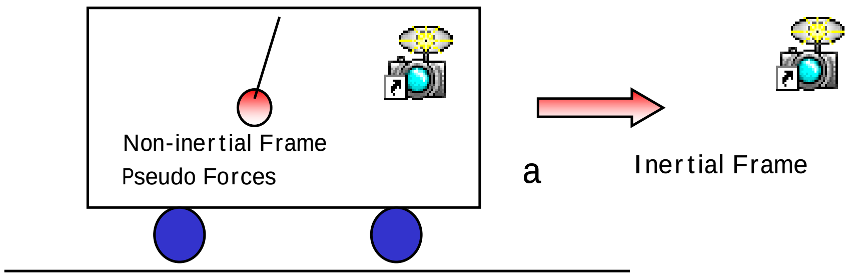


48.2 degrees; v_{boat} = 2.2 m/s

Inertial Frames:

T = mg





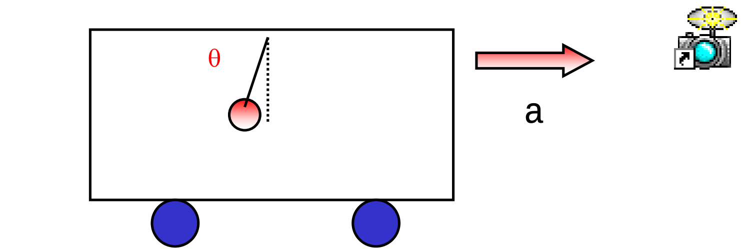
Inertial Frame;

There are no Pseudo Forces:

$T \cos\theta = mg$

$T \sin\theta = ma$

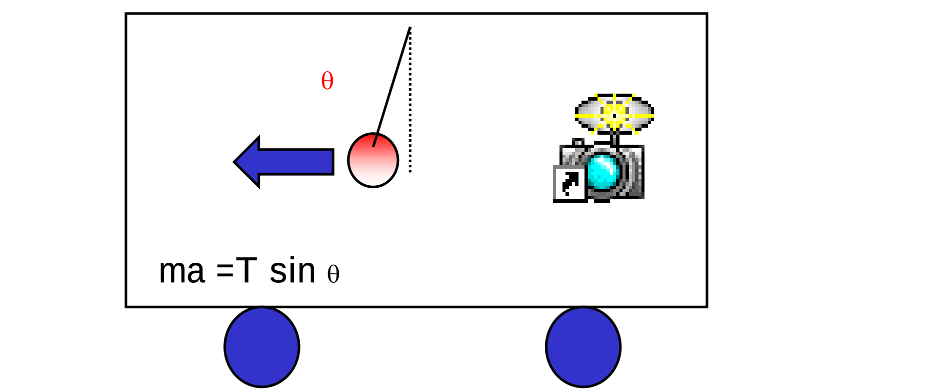
$a = g \cdot \tan\theta$



Non-Inertial Frame;

There is a Pseudo Force: ma

Newton's Laws do not work !!!



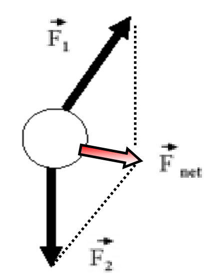
Combination of Forces:
Net Force
Dealing with Multiple Forces

If multiple forces are acting on the same object, the net force determines the acceleration.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_{net} = m\vec{a}$$

Use a free body diagram to keep track of the forces on *one object*.



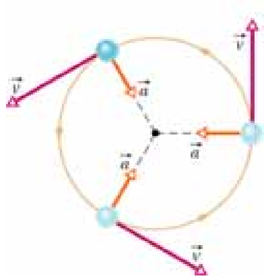
Uniform Circular Motion

Centripetal acceleration

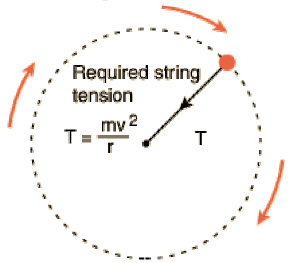
$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$



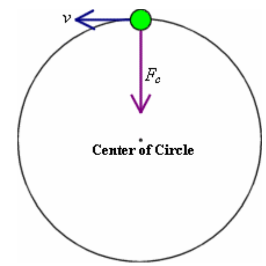
Top view:



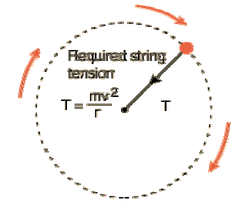
Centripetal force : F = ma

$$F = m\frac{v^2}{r}$$

Net Force and Centripetal Force

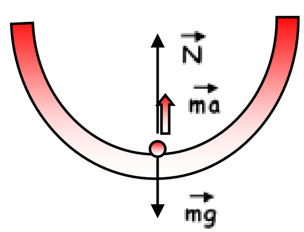


$$F = m\frac{v^2}{r}$$



Centripetal Force is a combination of:

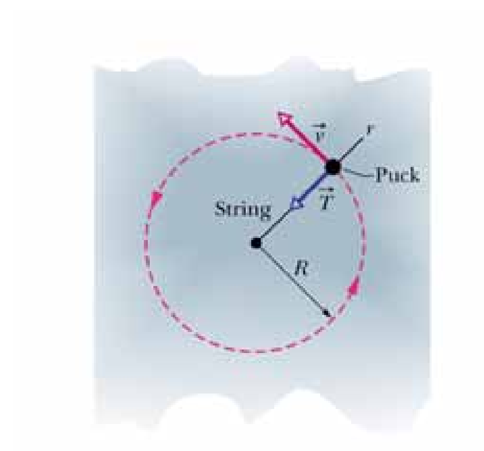
- Gravitational Force: $\vec{mg}$
  down to the ground
- Tension Force: $\vec{T}$
  along the string
- Normal Force: $\vec{N}$
  perpendicular to the support
- Static Friction Force
  maximum value $F_{fr}^{max} = \mu_{st}N$



ma = N – mg

$ma = mv^2/R$

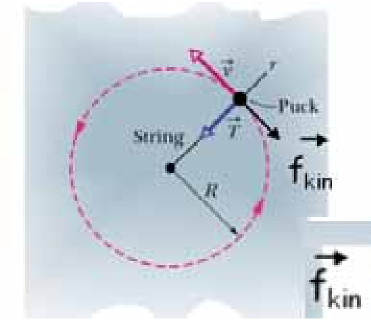
Centripetal Force and Tension Force:



ma = T

$ma = mv^2/R = T$

Centripetal Force and Kinetic Friction Force:



$ma_c = T$

$ma_c = mv^2/R = T$

For uniform circular motion $\vec{f}_{kin}$ is perpendicular to $\vec{ma}_c$

Kinetic friction does not affect Centripetal acceleration directly

Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

What is her centripetal acceleration?

Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$

Uniform Circular Motion

Sample Problem

A runner takes 12 seconds round a 180° curve at one end of an oval track. The distance covered on the curve is 100 meters.

What is her centripetal acceleration?

Centripetal acceleration

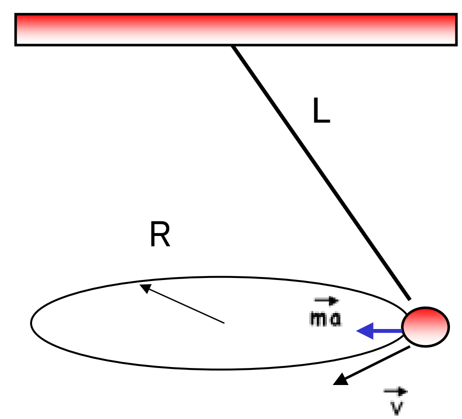
$$a = \frac{v^2}{r}$$

Period: $T = \frac{2\pi r}{v}$

v = 100 m /12 s = 8.33 s; R = 100/π = 31.8 m

a = $(8.33)^2/31.8$ m/s ^{2} = 2.2 m/s ^{2}

Problem #1



m = 5 kg

L = 5 m

R = 2 m

Find v, T, and a

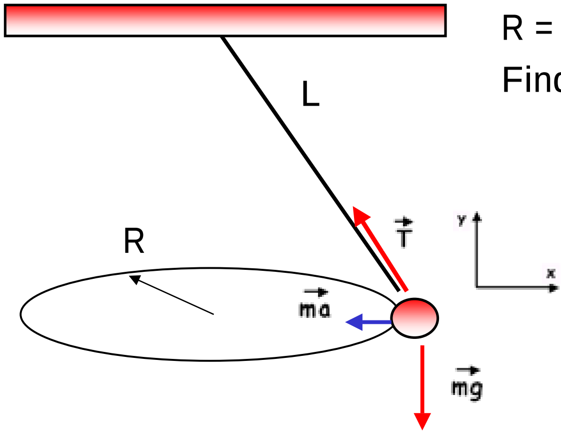
Problem solving tactics:

m = 5 kg

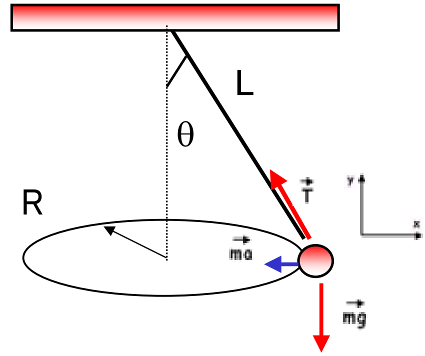
L = 5 m

R = 2 m

Find v,T, and a



Problem solving tactics:



m = 5 kg

L = 5 m

R = 2 m

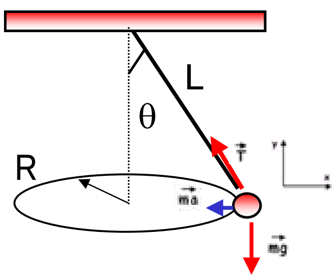
Find v,T, and a

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/\ (1-(R/L)^2)^{½} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = - mg + T * \cos\theta$

Problem solving tactics:



m = 5 kg

L = 5 m

R = 2 m

Find v,T, and a

Centripetal Force originates from the tension force!

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/\ (1-(R/L)^2)^{½} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = - mg + T * \cos\theta$

$ma = mg*\sin\theta\ /\ \cos\theta = mg*\tan\theta$

$T = mg/\ \cos\theta \quad a = g*\tan\theta$

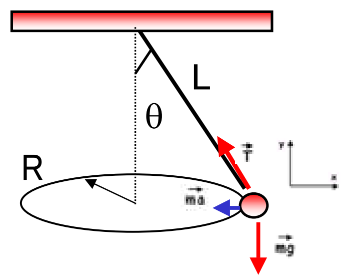
Circular motion:

$ma = mv^2/R$

$a = v^2/R$

$v = (aR)^{½}$

Problem is solved:



m = 5 kg

L = 5 m

R = 2 m

Find v,T, and a

$\sin\theta = R/L = 0.4$; $\tan\theta = (R/L)/\ (1-(R/L)^2)^{½} = 0.44$

<u>X:</u> $ma = T * \sin\theta$

<u>Y:</u> $ma = 0 = - mg + T * \cos\theta$

$ma = mg*\sin\theta\ /\ \cos\theta = mg*\tan\theta$

$T = mg/\ \cos\theta \quad a = g*\tan\theta$

$T = 5\ kg * 9.8\ m/s^2\ /(1-(2m/5m)^2)^{½} = 53\ N$

$a = 4.3\ m/s^2$; $ma = 5\ kg * 4.3\ m/s^{\wedge}2 = 21\ N$;

Circular motion:

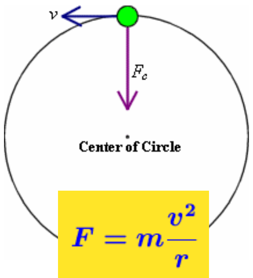
$ma = mv^2/R$

$a = v^2/R$

$v = (aR)^{½}$

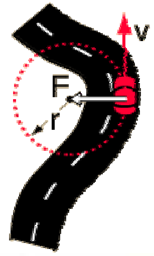
$v = (4.3*2)^{½}\ m/s = 2.9\ m/s$

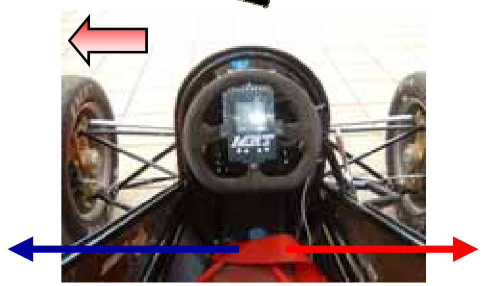
Net Force and Centripetal Force



$$F_{centripetal} = m\frac{v^2}{r}$$

$\frac{v^2}{r}$ is the centripetal acceleration



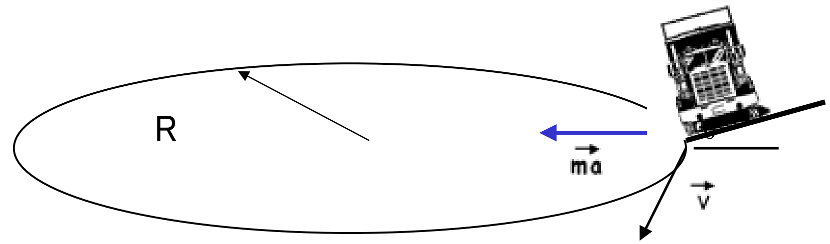


Problem #2

R = 20 m; μ_{st} = **0.5**

Angle 10°

Find v_{max}



Problem #2

R = 20 m; μ_{st} = **0.5**

Angle 10°

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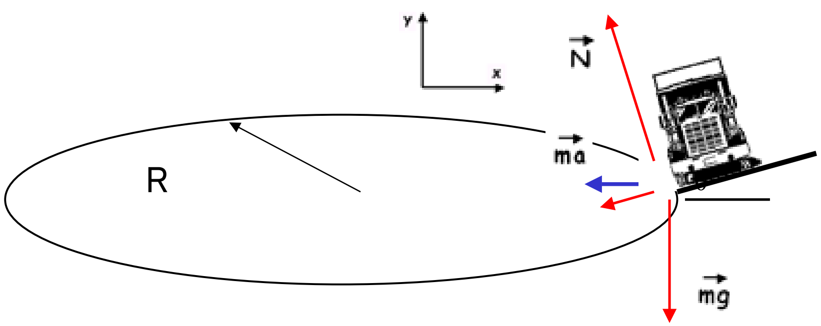


Problem #2

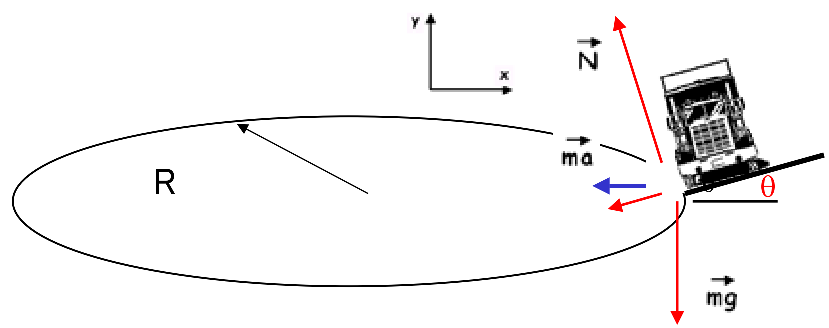
R = 20 m; μ_{st} = **0.5**

Angle 10°

Find v_{max}

X: $ma = N \sin\theta + \underline{\mu_{st}\ N \cos\theta}$

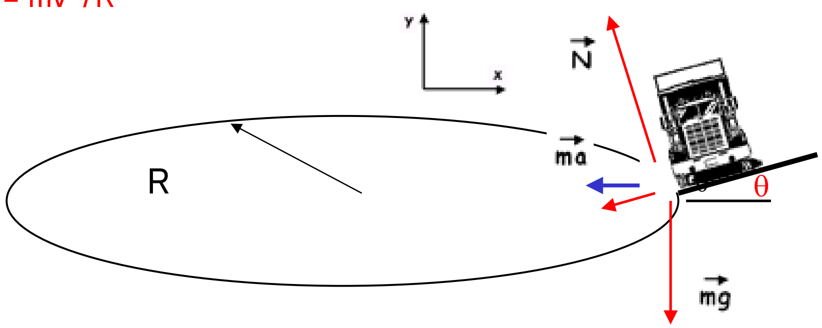
Y: $0 = N \cos\theta - mg - \underline{\mu_{st}\ N \sin\theta}$



Problem #2

$R = 20$ m; $\mu_{st} = \mathbf{0.5}$
Angle 10°
Find v_{max}

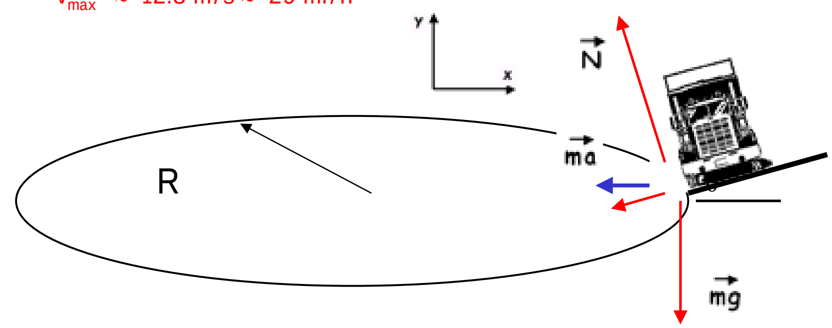
X: $ma = N * (\sin\theta + \mu_{st} \cos\theta)$
Y: $N = mg / (\cos\theta - \mu_{st} \sin\theta)$
$ma = mv^2/R$



Problem #2

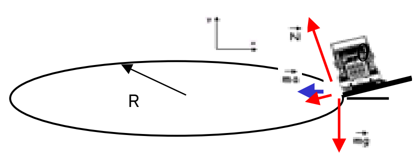
$R = 20$ m; $\mu_{st} = \mathbf{0.5}$
Angle 10°
Find v_{max}

$ma = N * (\sin\theta + \mu_{st} \cos\theta)$
$N = mg / (\cos\theta - \mu_{st} \sin\theta)$
$ma = m v_{max}^2 / R$
$m v_{max}^2 / R = mg * (\sin\theta + \mu_{st} \cos\theta) / (\cos\theta - \mu_{st} \sin\theta)$
$v_{max}^2 = g*R * (\sin\theta + \mu_{st} \cos\theta) / (\cos\theta - \mu_{st} \sin\theta)$
Mass m disappeared !!!
$v_{max} \approx 12.8$ m/s $\approx$ 29 ml/h



QZ # 7 Analyze the previous problem

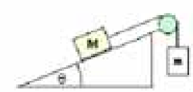
1. $R = 20$ m; $\mu_{st} = \mathbf{0.5}$; Angle 10°
What is going to happen to
the <u>static friction force</u> for
the case when the velocity of the
track is doubled: $v = v_{max} * 2$
What is going to happen to the track?
(describe)



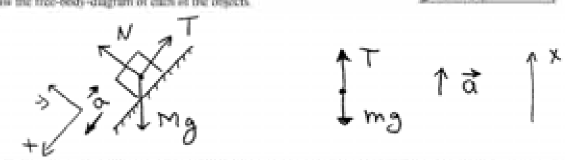
2. $R = 20$ m; $\mu_{st} = \mathbf{0.5}$, $v = 5$ m/s; $m = 3000$ kg,
and $\theta = 0$
What is the value and direction of the static friction force ?

WORKOUT PROBLEM I: [1 point for each part]

For the diagram shown, assume that the pulley is massless and frictionless, the incline is frictionless, the string is massless, M = 4.0 kg and θ = 43°. Starting from rest, the mass M moves downhill with a speed that is increasing at a rate of 2.0 m/s².



a) Draw the free-body-diagram of each of the objects



b) Write components of Newton's 2nd Law of Motion for each of the masses using symbols only (M, m, Θ, g etc.)

$\boxed{M}$
x: $Ma = Mg\sin\theta - T$ *
y: $\emptyset = N - Mg\cos\theta$

$\boxed{m}$
$ma = T - mg$

$m(a+g) = T$
$m = \frac{T}{a+g}$
$m = \frac{M(g\sin\theta - a)}{a+g}$
$m = \frac{4(9.8\sin 43 - 2)}{2+9.8}$
$m = 1.6$ kg

c) Solve the equations in part b using the data supplied and find the tension in the string T and the mass m

* $T = M(g\sin\theta - a)$
$T = 4(9.8 \cdot \sin 43° - 2) =$
$= 18.7$ N

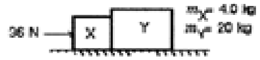
d) What value of m would have given an uphill motion of M with constant speed ?

$a = 0$
$T = m'g$
$T = M \cdot (g\sin\theta)$
$m' = \frac{T}{g} = M \cdot \sin\theta = 4 \cdot \sin 42° \text{ [kg]} = 2.7\text{kg}$

Final Exam Phys 105 - Fall 2003 Version A

Name______________________ ID______________________ Section________________

9. Two blocks (X and Y) are in contact on a horizontal frictionless surface. A 36-N constant force is applied to X as shown. The magnitude of the force exerted by Y on X is:



A) 1.5 N
B) 6.0 N
C) 29 N
D) 30 N
E) 36 N

$a = \frac{F}{m_x + M_y} = \frac{36N}{20kg+4kg} = \frac{36}{24} = 1.5\ \frac{m}{s^2}$

F_y ← X → 36N

IInd Newton's law for the Block X:

$a_x = \frac{36N - F_y}{m_x} = 1.5\ m/s^2$; $F_y = 36N - 4kg \cdot 1.5\ \frac{m}{s^2} = 30N$

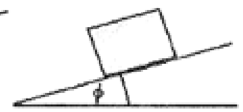
11. A block is at rest on a horizontal plank of wood. The plank is slowly lifted at one end while the other end stays on the floor. If the coefficient of static friction between the block and the plank is 0.5, what is the steepest angle the plank can have before the block begins to slide without being pushed?

A) 27°
B) 37°
C) 45°
D) 53°
E) 59°

$\mu_{st}^{max} = \tan\theta$

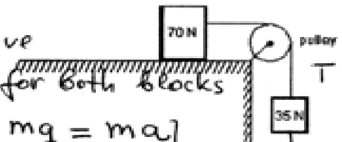
$\theta = \tan^{-1} 0.5 = $

$= \underline{26.5°}$



13. Two blocks with the weights of 70-N and 35-N are connected by a string as shown. If the pulley is massless and the surface is frictionless, the magnitude of the acceleration of the 70-N block is:

A) 1.6 m/s^2
B) 3.3 m/s^2
C) 4.9 m/s^2
D) 6.7 m/s^2
E) 9.8 m/s^2



Accelation should have the same magnitude for both blocks

for 35N Block: $-T + mg = ma$

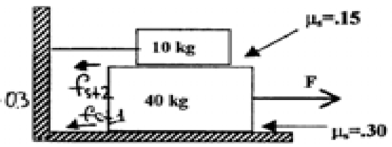
for 70N Block: $T = Ma$ } ⇒

⇒ $Ma + ma = mg$; $m = \frac{35N}{9.8\ m/s^2}$

$a = \frac{m}{M+m} \cdot g = \underline{3.3\ \frac{m}{s^2}}$; $M = 70\ N / 9.8\ m/s^2$

14. A horizontal force F is gradually increased until the 40 kg block begins moving to the right. The 10 kg block cannot move because of the cord attaching it to the wall at left. For what force F does the lower block just start to move?

A) 14.7 N.
B) 117.6 N
C) 132.3 N.
D) 147 N.
E) 161.7 N.



$F \geq f_{st1} + f_{st2}$

$f_{st1} = (10kg + 40kg) \cdot g \cdot 0.3$

$f_{st2} = 10kg \cdot g \cdot 0.15$

$F \geq 50kg \cdot 9.8\ \frac{m}{s^2} \cdot 0.3 + 10kg\ 9.8\ m/s^2 \cdot 0.15 = 161.7\ N$