Lecture 8

Energy

Work and Kinetic Energy

Kinetic and Potential Energy

(HR&W, Chapters 7 and 8)

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Physics 105 Summer 2006

ENERGY

Energy is a property of the state of an object: hard to define precisely

Energy is a scalar quantity. It does not have a direction associated with it

Energy is conserved. It can be transferred from one object to another or change in form, but not created or destroyed.

Units: joule = $kg \cdot m^2/s^2$

Kinetic Energy

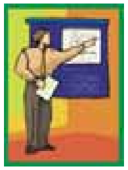
Kinetic Energy ≡ Energy of motion

$K = \frac{1}{2}mv^2$ for object moving with velocity v

$$K = \frac{1}{2}mv^2 \qquad \left[\mathrm{J} = \mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$$

Kinetic Energy: Orders of Magnitude

$K = \frac{1}{2}mv^2$ for object moving with velocity v

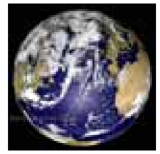
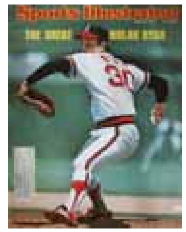
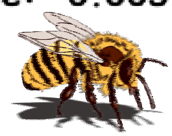
Earth orbiting sun: 2×10^{29} J

Car at 60 mph: 100,000 J

Nolan Ryan pitch: 300 J

Professor walking: 40 J

Angry bee: 0.005 J

Why $K = \frac{1}{2}mv^2$?

Why $K = \frac{1}{2}mv^2$?

Special case: Constant Acceleration

Remember result eliminating *t*: $v^2 - v_0^2 = 2a(x - x_0)$

Multiply by ½ m: $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = ma(x - x_0)$

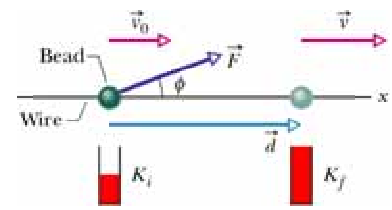
$= ma\Delta x$

But *F=ma!* $\Delta(\frac{1}{2}mv^2) = F\Delta x$

Energy and Work

Kinetic energy

$$K = \frac{1}{2}mv^2 \qquad \left[\mathrm{J} = \mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$$



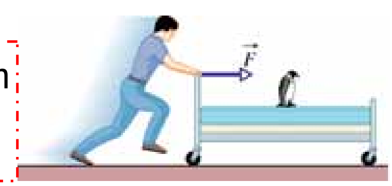
Units of Work and Energy: Joule

Work done by a <u>constant</u> force

$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

Work–kinetic energy theorem

$$\Delta K = K_f - K_i = W$$



Work

Work ≡ Energy transferred by a force

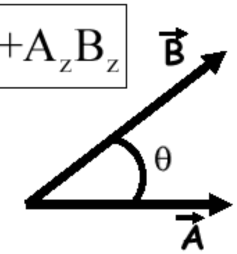
Work done on an object is the energy transferred to/from it

W > 0 → energy added
W < 0 → energy taken away

$\mathbf{W} = \vec{\mathbf{F}} \bullet \vec{\mathbf{r}}$ ≡ Work done on an object by a constant force $\vec{F}$ while moving through a displacement $\vec{r}$

Dot Product: Physical Meaning

$$\vec{A} \bullet \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

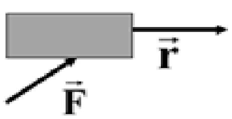


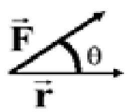
$$\theta = 0 \rightarrow \vec{A} \bullet \vec{B} = AB$$

$$\theta = 90^o \rightarrow \vec{A} \bullet \vec{B} = 0$$

Dot product measures how much vectors are along each other

What does $W = \vec{F} \bullet \vec{r}$ mean?





$$W = \vec{F} \bullet \vec{r} = F_x r_x + F_y r_y = Fr\cos\theta$$

W > 0 if θ < 90° ⟶ force is adding energy to object

W < 0 if θ > 90° ⟶ force is reducing energy of object

W = 0 if $r = 0$ or $F = 0$ or $\vec{F} \perp \vec{r}$

Work Examples

Push on a wall

W = 0 since wall does not move ($\vec{r} = 0$)

Work due to Gravity

A weightlifter does work when lifting a weight

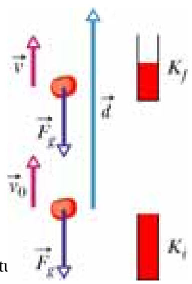
W=mgh
(h is the vertical drop)

Work Done by a Gravitational Force

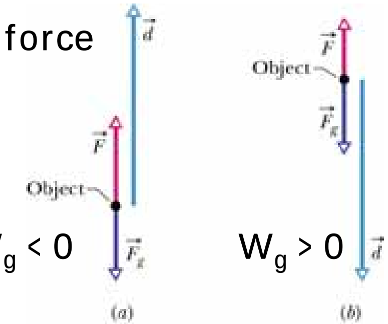
Work done by gravitational force

$$W_g = mgd\cos\theta$$

Tomato thrown upward



$W_g < 0$ $W_g > 0$

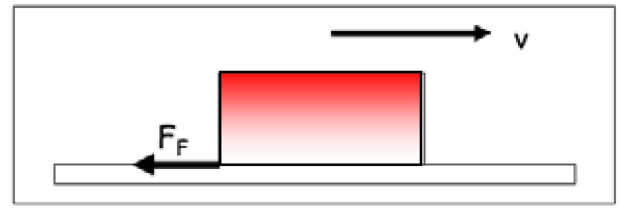


Lifting/lowering an object

Change in kinetic energy:

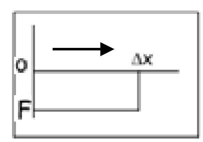
$$\Delta K = K_f - K_i = W_a + W_g$$

Work due to Friction



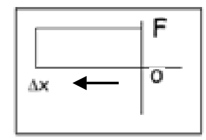
The frictional force always opposes the motion

Moving to the right



W=-|F| Δx
Δx>0

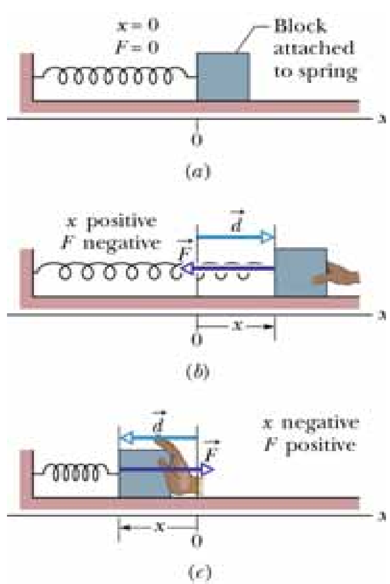
Moving to the left



W=+|F| Δx
Δx<0

W negative in both cases

Restoring Force

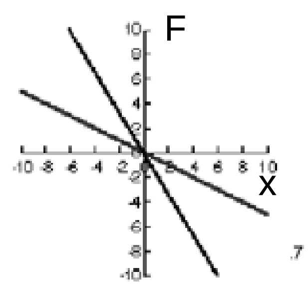


Equilibrium - no force

Stretched - force towards equilibrium point

F=-k x

Hooke's Law



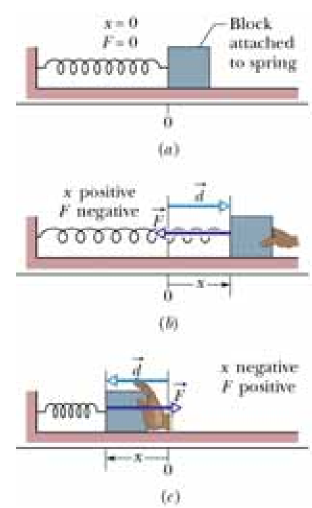
Work Done by a Spring Force

Hooke's law: $\vec{F} = -k\vec{d}$

Work done by a spring force:

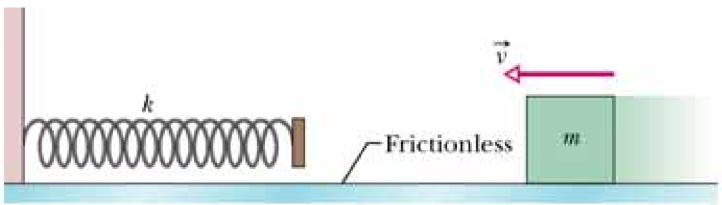
$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$\begin{aligned}
W_s &= \sum F_j\,\Delta x \\
W_s &= \int_{x_i}^{x_f} F\,dx \\
&= \int_{x_i}^{x_f} -kx\,dx = -k\int_{x_i}^{x_f} x\,dx \\
&= -\frac{1}{2}k\left[x^2\right]_{x_i}^{x_f} \\
&= -\frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2
\end{aligned}$$

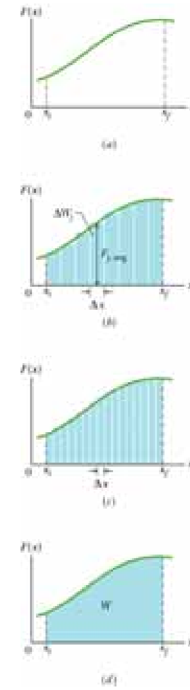


Sample Problem 7-8

A block of mass $m = 0.40\ kg$ slides across a horizontal frictionless counter with a speed of $v = 0.50\ m/s$. It runs into and compresses a spring of spring constant $k = 750\ N/m$. When the block is momentarily stopped by the spring, by what distance $\boldsymbol{d}$ is the spring compressed?



Work Done by a General Variable Force



Work: variable force

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

- Calculus
- Divide area under curve
- Add increments of W (numerically)
- Analytical form?
- Integration!!!

Power

Work doesn't depend on the time interval

Work to climb a flight of stairs~3000 J — 10 s / 1 min / 1 hour

Power is work done per unit time

Average Power $P_{\text{avg}} = \dfrac{W}{\Delta t}$

Instantaneous Power P = dW/dt = F dx/dt = Fv

Units $\dfrac{\text{Work}}{\text{time}}$ $\dfrac{1\ J}{1\ s}$ = 1 Watt 1 hp = 746 W

$P = ½*60kg*(5m/s)^2$

Power

Average Power

$$P_{\text{avg}} = \frac{W}{\Delta t}$$

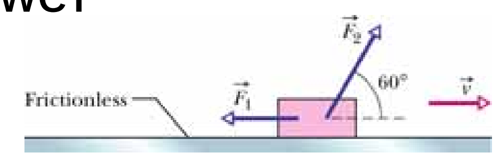
Units: Watts

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = \frac{dE}{dt}$$



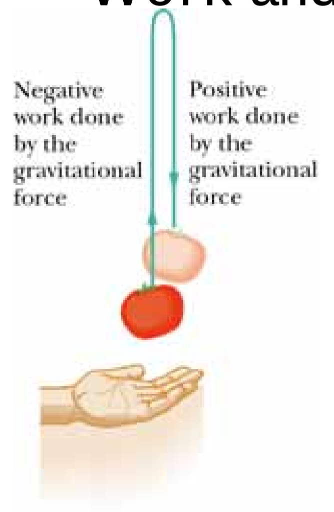
Sample Problem 7-10: Two constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude *2.0 N*, force F_2 is angled upward by *60°* to the floor and has a magnitude of *4.0 N*. The speed *v* of the box at a certain instant is *3.0 m/s*.

a) What is the power due to each force acting on the box? Is the net power changing at that instant?

b) If the magnitude F_2 is, instead, *6.0 N*, what is now the net power, and is it changing?

- **Potential Energy** and Conservation of Energy
- Conservative Forces
- Gravitational and Elastic Potential Energy
- Conservation of (Mechanical) Energy
- Potential Energy Curve
- External and Internal Forces

Work and Potential Energy



Potential Energy

$$\Delta U = -W$$

General Form:

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

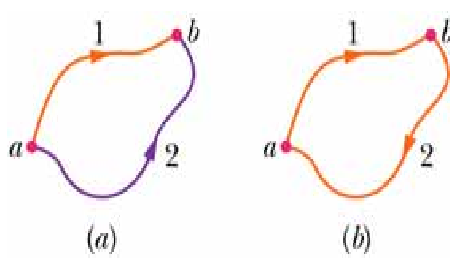
Gravitational Potential Energy

$$U = mgy$$

Elastic Potential Energy

$$U = \frac{1}{2}kx^2$$

Path Independence of Conservative Forces

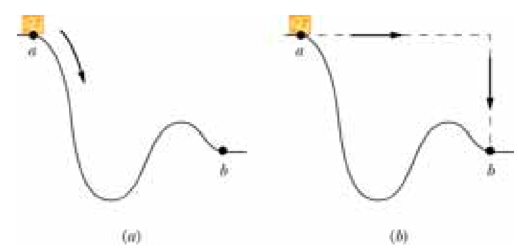


- The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.
- The net work done by a conservative force on a particle moving around every closed path is zero.

Path Independence of Conservative Forces

Sample Problem 8-1: A 2.0 kg block slides along a frictionless track from *a* to point *b*. The block travels through a total distance of 2.0 m, and a net vertical distance of 0.8 m. How much work is done on the block by the gravitational force?

$$U = mgy$$



Conservation of Mechanical Energy

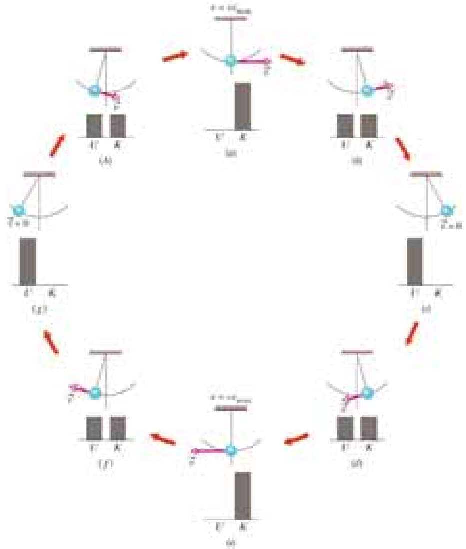
Mechanical Energy

$$E_{mec} = K + U$$

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1$$

In an isolated system where only conservative forces cause energy changes, the kinetic and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.



Kinetic Energy: $K = \frac{1}{2}mv^2$

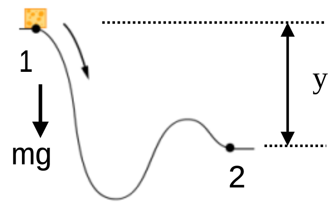
Potential Energy:

$$\Delta U = -W$$

• Gravitation: $U = mgy$

• Elastic (due to spring force): $U = \frac{1}{2}kx^2$

$U = mgy$



$K = \frac{1}{2}mv^2$

U → K

$$E_{mec} = K + U$$

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1$$

Kinetic Energy: $K = \frac{1}{2}mv^2$

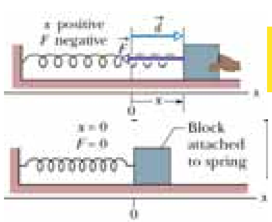
Potential Energy:

$$\Delta U = -W$$

• Gravitation: $U = mgy$

• Elastic (due to spring force): $U = \frac{1}{2}kx^2$

1



2

$U = \frac{1}{2}kx^2$ K = 0

U = 0 $K = \frac{1}{2}mv^2$

U ← → K

$$E_{mec} = K + U$$

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1$$

QZ#8
Name, ID#, Section

SF 4 days

NYC

NYC – SF by train. On a regular schedule
It takes 4 days for 1 way trip
One train per day; starts at 1 pm in NYC
and arrives in 4 days at 1 pm to SF.
Quickly unload – load and go back.

(a) Calculate the work of the engine when the train of the mass 10,000 kg accelerates to v = 72 km/h from zero at the departure from NYC.

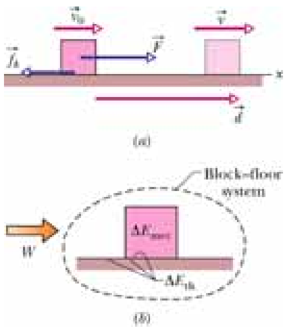
(b) Calculate the work done by the breaks (friction force) when the train slows down from v = 72 km/h to v = 0 arriving to SF.

(c) How many other trains will our train meet during one way trip?

Conservation of Energy

Thermal Energy/Friction

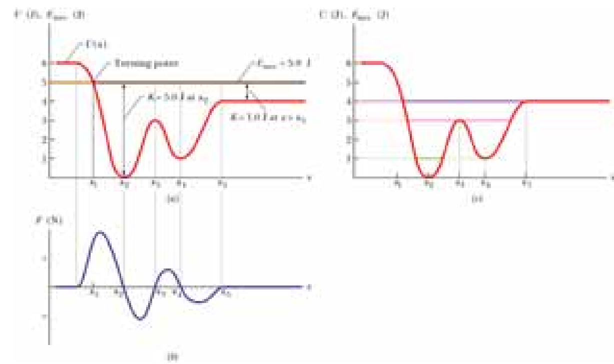
$$\Delta E_{th} = f_k d$$



- The total energy of a system can change only by amounts of energy that are transferred to or from the system.
- The total energy E of an isolated system cannot change.

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

Potential Energy Curve



A plot of U(x), the potential energy function of a system containing a particle confined to move along the x axis. There is no friction, so mechanical energy is conserved.

1D Motion

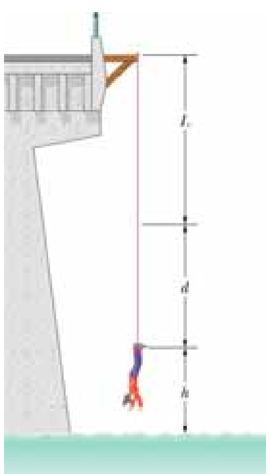
$$F(x) = -\frac{dU(x)}{dx}$$

Turning Points

Equilibrium Points

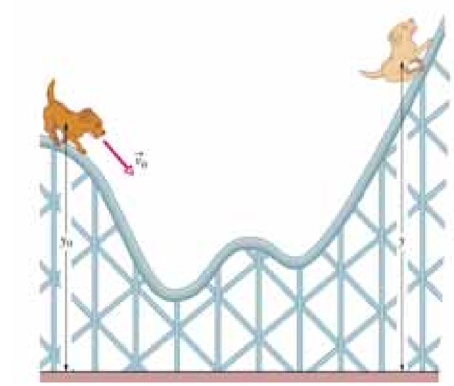
- Neutral Equilibrium
- Unstable Equilibrium
- Stable Equilibrium

Sample Problem 8-4



A 61.0 kg bungee-cord jumper is on a 45.0 m bridge above a river. The elastic bungee cord has a relaxed length of L = 25.0 m. Assume that the cord obeys Hooke's law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height h of her feet above the water at her lowest point?

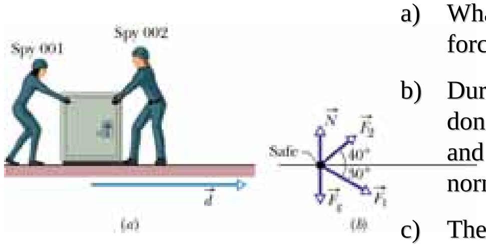
Sample Problem 8-8



A circus beagle of mass m = 6.0 kg runs onto the left end of a curved ramp with speed v_0 = 7.8 m/s at height y_0 = 8.5 m above the floor. It then slides to the right and comes to a momentary stop when it reaches a height y = 11.1 m from the floor. The ramp is not frictionless. What is the increase ΔE_{th} in the thermal energy of the beagle and the ramp because of the sliding?

Sample Problem 7-2

Two industrial spies sliding an initially stationary *225 kg* floor safe a displacement *d* of magnitude *8.50 m*, straight toward their truck. The push $\boldsymbol{F_1}$ of spy 001 is *12.0 N*, directed at an angle of *30º* downward from the horizontal; the pull $\boldsymbol{F_2}$ of spy 002 is *10.0 N*, directed at *40º* above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.



a) What is the net work done on the safe by the forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ during the displacement *d* ?

b) During the displacement, what is the work W_g done on the safe by the gravitational force $\boldsymbol{F_g}$ and what is the work done on the safe by the normal force ***N*** from the floor?

c) The safe is initially stationary. What is its speed v_f at the end of the *8.50 m* displacement?