Lecture 9

Potential Energy and Conservation of Energy

(HR&W, Chapter 8)

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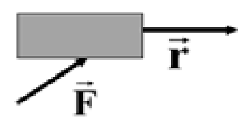
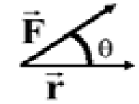
Physics 105 Summer 2006

What does $W = \vec{F} \bullet \vec{r}$ mean?





$$W = \vec{F} \bullet \vec{r} = F_x r_x + F_y r_y = Fr\cos\theta$$

$W > 0$ if $\theta < 90^\circ$ ⟶ force is adding energy to object

$W < 0$ if $\theta > 90^\circ$ ⟶ force is reducing energy of object

$W = 0$ if $\boxed{r = 0}$ or $\boxed{F = 0}$ or $\boxed{\vec{F} \perp \vec{r}}$

Work Examples

Push on a wall

W = 0 since wall does not move ($\vec{r} = 0$)

Power

Work doesn't depend on the time interval

Work to climb a flight of stairs~3000 J
10 s
1 min
1 hour

Power is work done per unit time

Average Power $P_{avg} = \dfrac{W}{\Delta t}$

Instantaneous Power P = dW/dt = F dx/dt = Fv

Units $\dfrac{\text{Work}}{\text{time}}$ $\dfrac{1\ \text{J}}{1\ \text{s}}$ = 1 Watt 1 hp = 746 W

P = ½*60kg*(5m/s)^2

Power

Average Power

$$P_{avg} = \frac{W}{\Delta t}$$

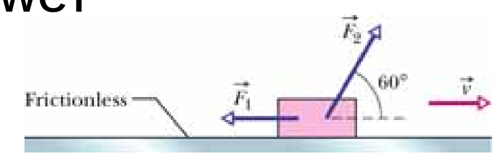
Units: Watts

Instantaneous Power

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = \frac{dE}{dt}$$



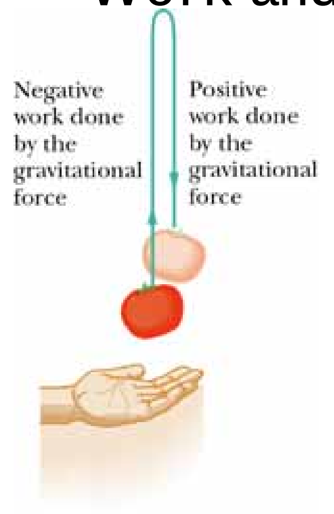
Sample Problem 7-10: Two constant forces F_1 and F_2 acting on a box as the box slides rightward across a frictionless floor. Force F_1 is horizontal, with magnitude *2.0 N*, force F_2 is angled upward by *60º* to the floor and has a magnitude of *4.0 N*. The speed *v* of the box at a certain instant is *3.0 m/s*.

a) What is the power due to each force acting on the box? Is the net power changing at that instant?

b) If the magnitude F_2 is, instead, *6.0 N*, what is now the net power, and is it changing?

- **Potential Energy** and Conservation of Energy
- Conservative Forces
- Gravitational and Elastic Potential Energy
- Conservation of (Mechanical) Energy
- Potential Energy Curve
- External and Internal Forces

Work and Potential Energy



Potential Energy

$$\Delta U = -W$$

General Form:

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)\,dx$$

Gravitational Potential Energy

$$U = mgy$$

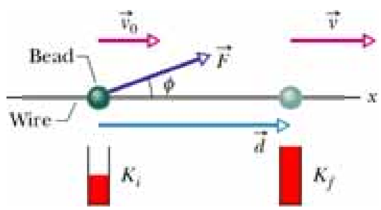
Elastic Potential Energy

$$U = \frac{1}{2}kx^2$$

Energy and Work

Kinetic energy

$$K = \frac{1}{2}mv^2 \qquad \left[\mathrm{J} = \mathrm{kg}\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]$$



Units of Work and Energy: Joule

Work done by a constant force

$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

Work–kinetic energy theorem

$$\Delta K = K_f - K_i = W$$

Conservation of Mechanical Energy

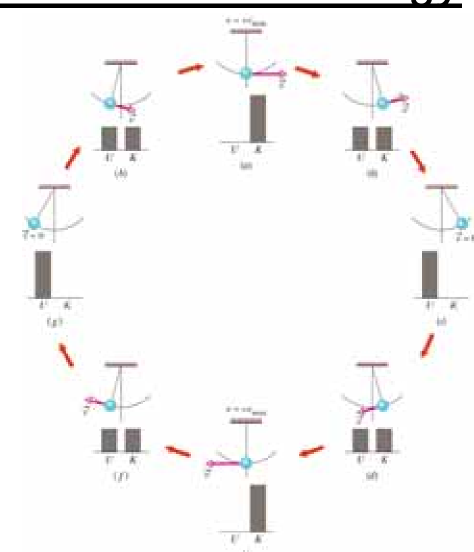
Mechanical Energy

$$E_{\mathrm{mec}} = K + U$$

Conservation of Mechanical Energy

$$K_2 + U_2 = K_1 + U_1$$

In an isolated system where only conservative forces cause energy changes, the kinetic and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

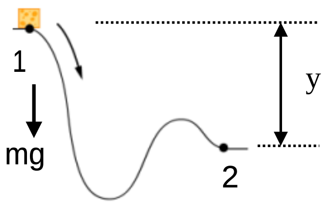


Kinetic Energy: $K = \frac{1}{2}mv^2$

Potential Energy:

$\Delta U = -W$

- Gravitation: $U = mgy$
- Elastic (due to spring force): $U = \frac{1}{2}kx^2$



U → K

$E_{mec} = K + U$

Conservation of Mechanical Energy

$K_2 + U_2 = K_1 + U_1$

Kinetic Energy: $K = \frac{1}{2}mv^2$

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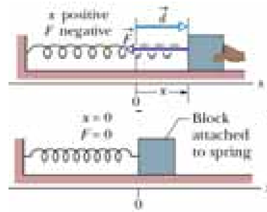


$U = \frac{1}{2}kx^2$ K = 0

U = 0 $K = \frac{1}{2}mv^2$

U ← → K

$E_{mec} = K + U$

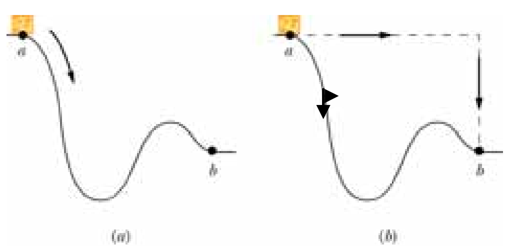
Conservation of Mechanical Energy

$K_2 + U_2 = K_1 + U_1$

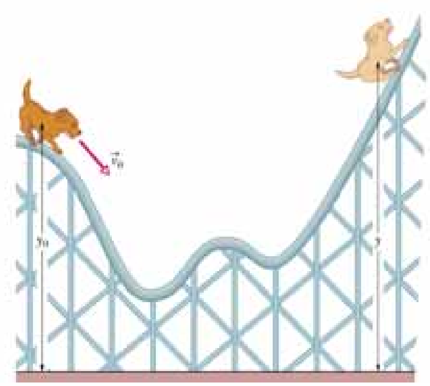
Path Independence of Conservative Forces

Sample Problem 8-1: A 2.0 kg block slides along a frictionless track from *a* to point *b*. The block travels through a total distance of 2.0 m, and a net vertical distance of 0.8 m. How much work is done on the block by the gravitational force?

$U = mgy$



Sample Problem



A circus beagle of mass m = 6.0 kg runs onto the left end of a curved ramp with speed v_0 = 7.8 m/s at height y_0 = 8.5 m above the floor. It then slides to the right and comes to a momentary stop when it reaches a height y = 11.1 m from the floor. The ramp is not frictionless. What is the increase ΔE_{th} in the thermal energy of the beagle and the ramp because of the sliding?

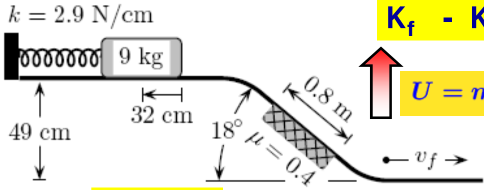
Answer: 30J

Examples for Energy Conservation

Kinetic Energy changes
\+ Gravitational Potential Energy
\+ Elastic Potential Energy

Total Mechanical Energy = *Const.*

$U = \frac{1}{2}kx^2$ → $K = \frac{1}{2}mv^2$



$K_f - K_i = W = mgy - |W_{friction}|$

$U = mgy$

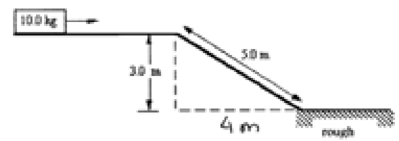
$K = \frac{1}{2}mv^2$

$W_{friction}$

$E_f - E_i = K_f - (K_i + mgy) = -|W_{friction}| = f_k \cdot d \cdot \cos 180° =$
$= - f_k \cdot d = -mg\,\mu \cdot d \cdot \cos 18°$

Problems:

A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.



$K_i = \frac{mv_i^2}{2}$;

$U_i = mg\Delta y \quad | \Delta y = 3m$

--- Ø Pot. Energy

(a) [2 points] What is the speed of the crate when it arrives at the lower surface?

$E_i = K_i + U_i = \frac{mv_i^2}{2} + mg\Delta y$

$E_f = K_f + \varnothing = \frac{mv_f^2}{2}$

$v_f = \sqrt{\frac{2}{m}} \cdot \sqrt{\frac{mv_i^2}{2} + mg\Delta y} = \sqrt{v_i^2 + 2g\Delta y} =$
$= \sqrt{(4\,m/s)^2 + 2 \cdot 9.8\,\frac{m}{s^2} \cdot 3m} = 8.6\,\frac{m}{s}$

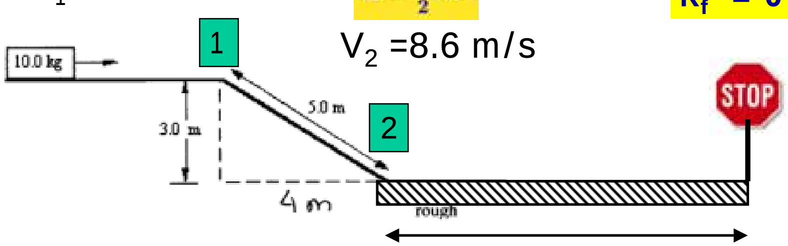
A 10.0-kg crate slides along a horizontal frictionless surface at a constant speed of 4.0 m/s. The crate then slides down a frictionless incline and across a second rough horizontal surface as shown in the figure.

$V_1 = 4.0$ m/s

$K = \frac{1}{2}mv^2$

$K_f = 0$

$V_2 = 8.6$ m/s



What minimum coefficient of kinetic friction μ_k is required to bring the crate to a stop over a distance of 10 m along the lower surface ?

$K_f - K_i = W_{friction} = -mg\mu_k d$

$0 - \frac{1}{2}mv^2 = -mg\mu_k d$

$\frac{1}{2}mv^2 = mg\mu_k d$; $\mu_k = v^2/2gd =$

$(8.6\,m/s)^2/(2 \cdot 10\,m/s^2 \cdot 10\,m) =$ **0.37**

Example of the 3rd Common Exam

Problem 1: What is the work done by a force **F** = (2 N)**i** + (-4 N)**j** that causes a displacement **d** = (-3 m)**i** + (2 m)**j**?

A) 2 J
B) 14 J
C) -14 J
D) -2 J
E) 16 J

$W = \vec{F} \cdot \vec{d} = 2N \cdot (-3m) + (-4N) \cdot 2m =$
$= -6 - 8 = -14\ J$

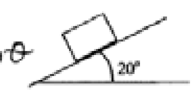
Problem 2: A man pushes a 2-kg block 5 m along a frictionless incline at an angle of 20° with the horizontal at constant speed. What is the work done by his force?

A) 0 J
B) 98 J
C) 34 J
D) 92 J
E) 100 J

$\Delta K = \varnothing$

$W = \Delta U = mg \cdot \Delta y = mg \cdot s \cdot \sin\theta$

$W = 2kg \cdot 9.8\,\frac{m}{s^2} \cdot 5m \cdot \sin 20° = 33.5\,J \approx 34\,J$



Problem 3: Starting from rest, it takes 8.00 s to lower with constant acceleration an 80.0-kg couch from a 16.0-m high rooftop of a building all the way to the ground with a single vertical rope tied to its body. What is the work done by the tension in the rope?

A) 1.57 kJ
B) -1.28 kJ
C) -12.5 kJ
D) 12.5 kJ
E) -11.9 kJ

$\vec{T}$ $\downarrow \vec{a}$ $m\vec{g}$ $\downarrow \vec{d}$

$ma = mg - T$; $T = mg - ma$

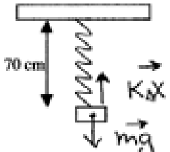
$W = T \cdot d \cdot \cos 180° = -T \cdot d =$

$= -(mg - ma) \cdot d$; *) $\frac{at^2}{2} = d \Rightarrow a = \frac{2d}{t^2}$

$W = -(mg - m \cdot 2d/t^2) \cdot d = -11904\,J$

Problem 4: A 10-kg mass is attached to one end of a 50-cm-long unstretched spring. When the other end of the spring is attached to the ceiling the mass reaches a stable stationary position as shown in the adjacent diagram. What is the spring constant of the spring?

A) 490 N/m
B) 245 N/m
C) 980 N/m
D) 140 N/m
E) 196 N/m



$\Delta x = 70cm - 50cm = 20cm$

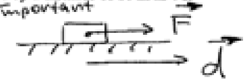
$k\Delta x = mg$

$k = \frac{mg}{\Delta x} = \frac{10kg \cdot 9.8\, m/s^2}{0.2\, m} = \underline{490\, N/m}$

Problem 5: A dog must apply its full power of 100 W in order to move a 5-kg sled by a distance of 10 m in 4 s. What average force does the dog exert on the sled?

not important

A) 49 N
B) 250 N
C) 8 N
D) 40 N
E) 200 N



$P \cdot t = W$ (work)

$F \cdot d = W$ (work)

$F = \frac{P \cdot t}{d} = \frac{100W \cdot 4s}{10m} = 40N$

Problem 6. A bicyclist is traveling on a horizontal track at a speed of 20.0 m/s as he approaches the bottom of a hill. He decides to coast up the hill and stops upon reaching the top. Determine the vertical height of the hill.

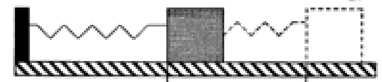
A) 28.5 m
B) 3.70 m
C) 11.2 m
D) 40.8 m
E) 20.4 m

$\frac{mv^2}{2} = mg\Delta y$; $\Delta y = \frac{v^2}{2g} = \frac{20^2 (\frac{m}{s})^2}{2 \cdot 9.8 \frac{m}{s^2}} = \underline{20.4\, m}$

Problem 7. A mass m = 2.5 kg is sliding left along a frictionless table with initial speed *v*. It strikes a coiled spring that has a force constant k = 500 N/m and compresses it a distance 5.0 cm before coming to a momentary rest. The initial speed *v* of the block was

A) 0.71 m/s
B) 1.0 m/s
C) 1.4 m/s
D) 0.50 m/s
E) 1.7 m/s

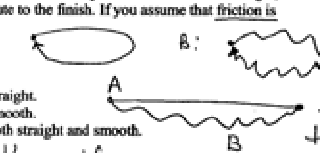
$\frac{1}{2}k\Delta x^2$ $\quad$ $\frac{mv^2}{2}$



$\frac{1}{2}k\Delta x^2 = \frac{mv^2}{2}$; $V = \Delta x \cdot \sqrt{\frac{k}{m}} = 0.05 \cdot \sqrt{500/2.5} = 0.71\, m/s$ ($x_1 = -5$ cm; $x_1 = 0$)

Problem 8. Two skiers start from rest at the same place and finish at the same place. Skier A takes a straight, smooth route to finish whereas skier B takes a curvy, bumpy route to the finish. If you assume that friction is negligible, which of the following statements is true?

A) Skier A has the same speed as skier B at the finish.
B) Skier B has greater speed at the finish.
C) Skier A has greater speed at the finish because the route is straight.
D) Skier B has greater speed at the finish because the route is smooth.
E) Skier A has greater speed at the finish because the route is both straight and smooth.



$\Delta K = \Delta U$
if $\Delta U = \varnothing$,
then $\Delta K = \varnothing$
then $V_A = V_B$

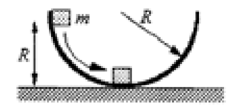
if $\Delta U \neq 0$, then $\Delta K_A = \Delta K_B$; if $m_A = m_B$, then $V_A = V_B$;

Problem 9. A block of mass *m* is released from rest at a height *R* above a horizontal surface. The acceleration due to gravity is *g*. The block slides along the inside of a frictionless circular hoop of radius *R*. Which one of the following expressions gives the speed of the mass at the bottom of the hoop?

A) zero m/s
B) $v = mgR$
C) $v = mg/2R$
D) $v^2 = g^2/R$
E) $v^2 = 2gR$

$mgR = \frac{mv^2}{2}$

$\underline{v^2 = 2gR}$



Problem 10. A 60-kg skier starts from rest from the top of a 50-m high slope. If the work done by friction is –6.0 x 10^3 J, what is the speed of the skier on reaching the bottom of the slope?

A) 17 m/s
B) 24 m/s
C) 28 m/s
D) 31 m/s
E) 42 m/s

$K_i = 0$; $U_i = mg\Delta y$

$K_f = \frac{mv_f^2}{2}$; $U_f = \varnothing$

$\frac{mv_f^2}{2} = mg\Delta y + W_{fr}$

$\frac{mv_f^2}{2} = (60 \cdot 9.8 \cdot 50)J - 6 \cdot 10^3 J = 23.4 \times 10^3 J$

$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \cdot 23400}{60}} = \underline{28\, m/s}$

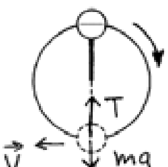
is not important !!!

Problem 11. A 2.0-kg ball is attached to a light rod that is 1.2 m long. The other end of the rod is loosely pinned at a frictionless pivot. The rod is raised until it is inverted, with the ball above the pivot. The rod is released and the ball moves in a vertical circle. The tension in the rod as the ball moves through the bottom of the circle is closest to:

A) 40 N
B) 100 N
C) 20 N
D) 60 N
E) 80 N

$T - mg = \frac{mv^2}{R}$

$T = mg + \frac{mv^2}{R}$

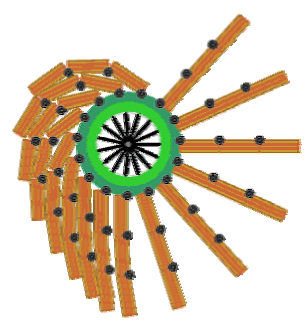


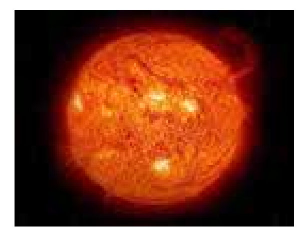
$5 \cdot mg = 5 \cdot 2kg \cdot 9.8 \frac{m}{s} \approx 98N$

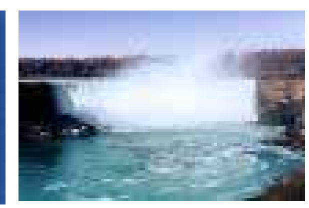
$\frac{mv^2}{2} = mg \cdot 2R \Rightarrow$

$\Rightarrow \frac{mv^2}{R} = 4 \cdot mg$; $T = mg + 4mg = 5 \cdot mg \approx \underline{100N}$

Perpetual Motion and "Free Energy"



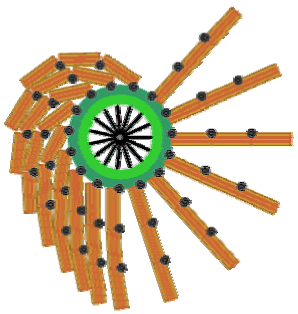


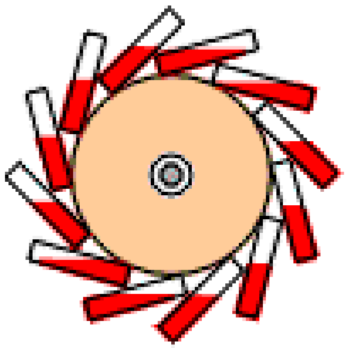


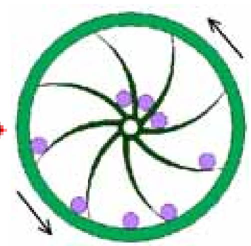
Perpetuum Mobile

"Machine, which works itself forever"

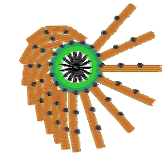
English: Perpetual Motion







Example 1

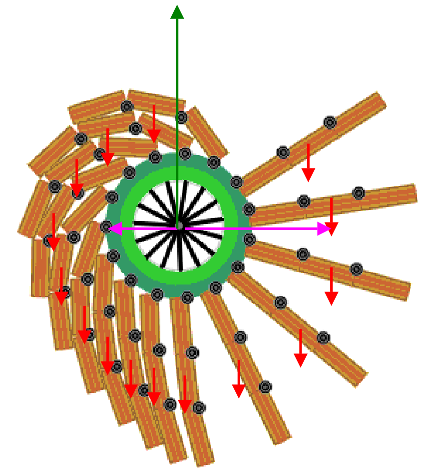


Balance of Forces:

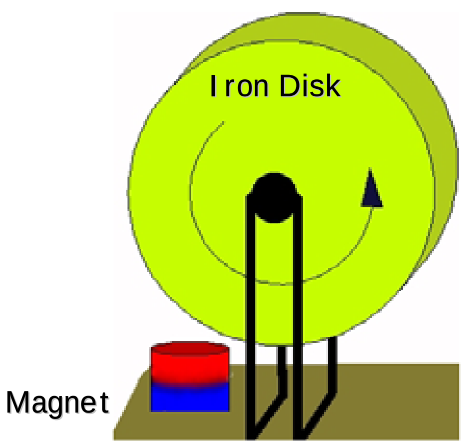
$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

Balance of Torques:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

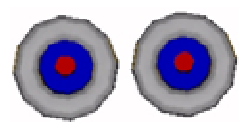


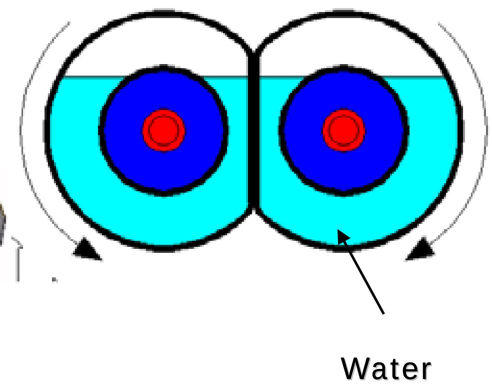
More Examples:



All parts of the cylinder that fall in the greater gravity (magnetic) level must be pushet out, as well. All the work, that a part of cylinder gets when it's moving toward the greater gravity (magnetic) level is needed when it is pushed back out of it..

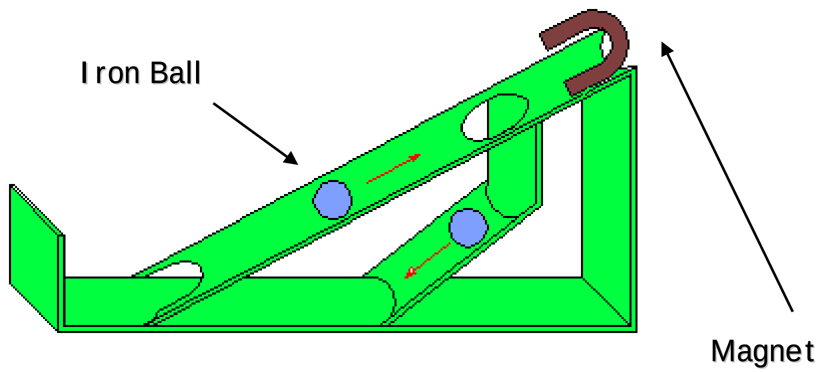
More Examples:





Put an innertube on a wheel. Fill it two thirds with wather. Put an axle through it so it can spin. Now make another one like it. Now hold the axels and push the wheel up against each other so that they can squeez each others wather to the outside. The results are that one side of each wheel is lighter than its other side. That is why the wheel spins.

More Examples:

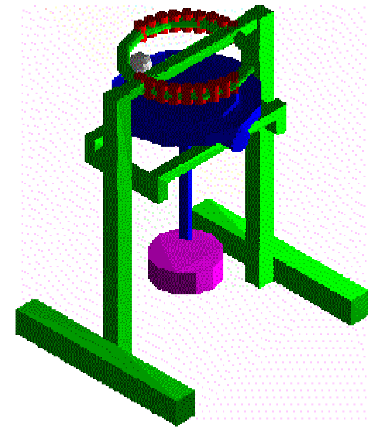


More Examples:

Iron Ball

Magnets

Pendulum

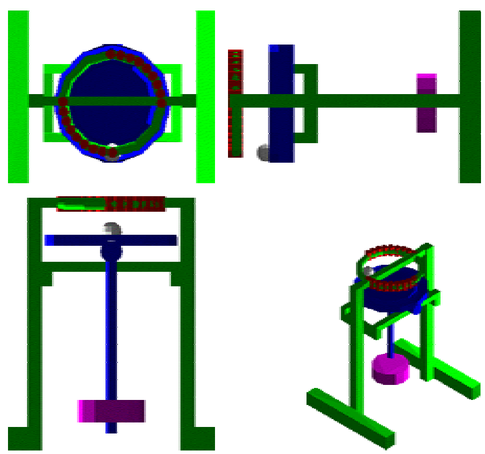


More Examples:

Iron Ball

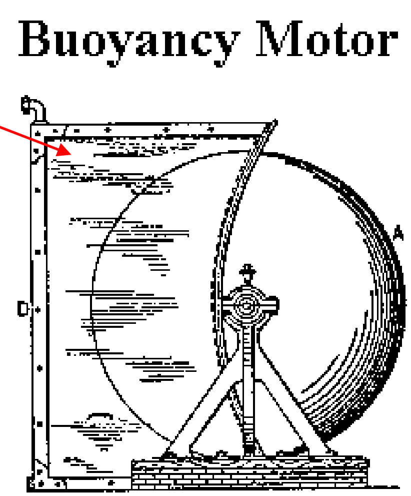
Magnets

Pendulum



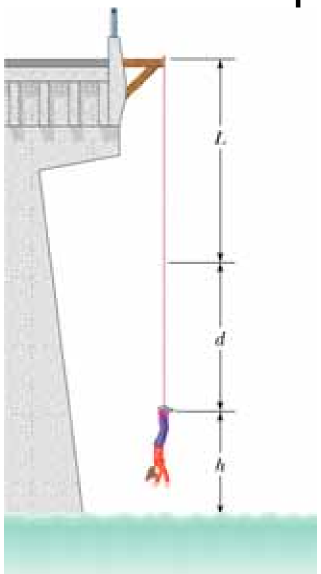
More Examples:

Water



buoyant force of Archimedes' principle: "A body immersed in liquid experiences and upward buoyant force equal to the weight of the displaced liquid."

Sample Problem



A 61 kg bungee-cord jumper is on a 45 m bridge above a river. The elastic bungee cord has a relaxed length of L = 25 m. Assume that the cord obeys Hooke's law, with a spring constant of 160 N/m. If the jumper stops before reaching the water, what is the height h of her feet above the water at her lowest point?

L + d + h = 45m

$E = K + U_g + U_e = \text{Const}; \quad (\Delta K = 0)$

$\Delta U_g = -mgy = -61\text{ kg} \cdot 9.8\text{m/s}^2 \cdot * (L+d)$

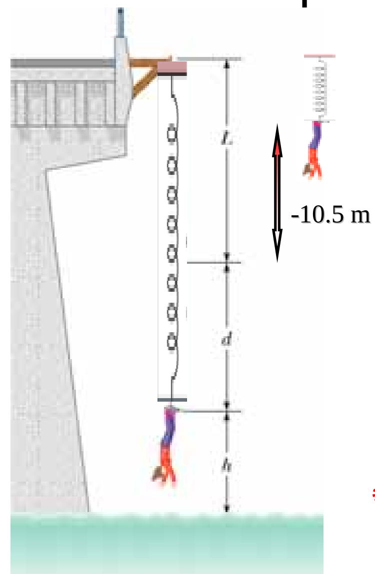
$\Delta U_e = kd^2/2 = 160\text{ N/m} \cdot d^2/2$

$80d^2 - 600d - 15000 = 0; (d^2 - 7.5d - 187.5 = 0)$

d = 18 m and d = -10.5 m

h = 45m – 25m – 18m = <u>2m</u>

Sample Problem



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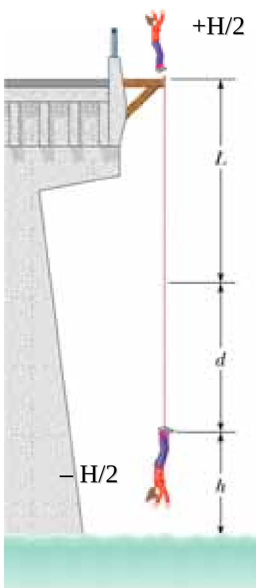

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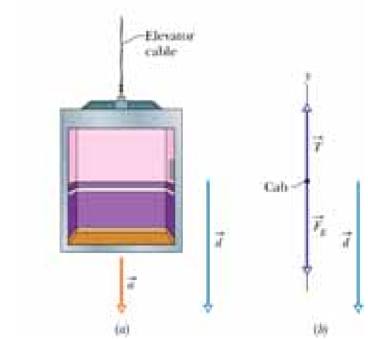
$80d^2 - 600d - 15000 = 0; (d^2 - 7.5d - 202.5 = 0)$

d = 18.5 m and d = – 11 m

h = 45m – 25m – 18.5m = <u>1.5m but H=2m …</u>

Sample Problem

An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration <u>a</u> = <u>g</u>/5.



During the 12 m fall, what is the work W_T done by the upward pull <u>T</u> of the elevator cab?

T – mg = – ma = – mg/5

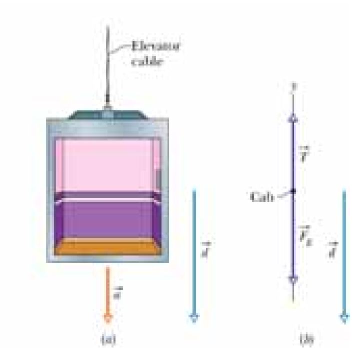
T=0.8·mg;

$W = -0.8mgd = -0.8 \cdot 500\text{kg} \cdot 9.8\text{m/s}^2 \cdot 12\text{m} =$

= – 47,000 J

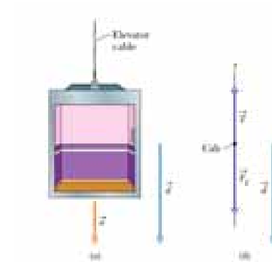
Sample Problem

An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration <u>a</u> = <u>g</u>/5.



During the fall through a distance d = 12 m, what is the work W_g done on the cab by the gravitational force F_g?

W_{mg}= +mgd = 500kg · 9.8m/s2 · 12m = 59000J

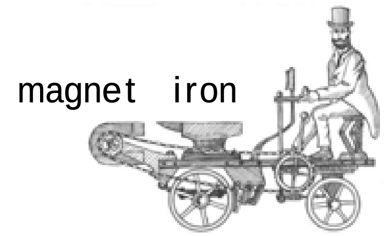


QZ#9:

<u>1.</u> An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration a = g/5, d=12 m

What is the elevator's kinetic energy at the end of the fall? Hint: W_{mg} = 59000 J ; W_T = – 47000 J

<u>2</u> We are not using this type of vehicle because



a) perpetual motion is forbidden by the Newton's Laws

b) police does not allow it

c) sitting next to a strong magnet is not good for the driver's health

d) there are no such strong magnets so far

e) this vehicle is not going to start moving by itself, so it is not very practical for plane roads. Can be only used to go down the hill.