REVIEW 1

Review for the First Common QZ

(HR&W, Chapters 1-4)

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Physics 105; Summer 2006

Common QZ includes:

- Unit Conversion
- Vectors (addition, subtraction, multiplication, angle between vectors)
- Motion along the Straight line with constant acceleration
- Projectile Motion

Unit Conversions

Multiply quantities and units:

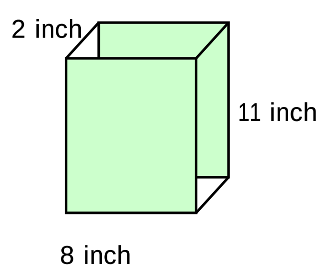
$$60\frac{\cancel{mi}}{\cancel{hr}}\bullet 5280\frac{\cancel{ft}}{\cancel{mi}}\bullet 12\frac{\cancel{in}}{\cancel{ft}}\bullet 0.0254\frac{m}{\cancel{in}}\bullet \frac{1}{3600}\frac{\cancel{hr}}{s}$$

$$26.8\frac{m}{s}$$

EXAMPLE:

What is the volume of the book in cm^3.

(hint: 1 inch = 2.54 cm)



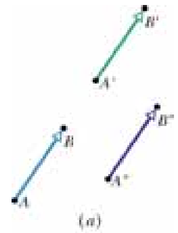
Solution:

V = (2x2.54)(8x2.54)(11x2.54)cm^3=

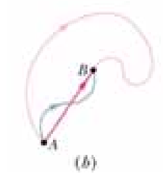
=$2.9x10^3$ cm^3

Vectors:

(variables with magnitude and direction)

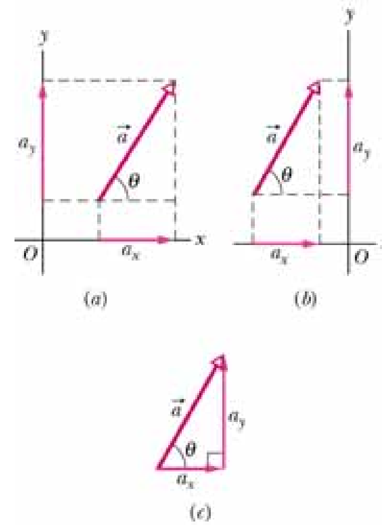


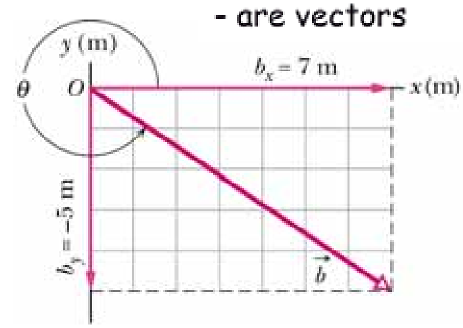
Displacement:



Components of Vectors:

- aligned along axis
- add to give vector
- are vectors





$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

Length (Magnitude)

$\hat{\imath}$

Unit Vectors

Components of a vector are still vectors

$$\vec{D} = \vec{D}_x + \vec{D}_y$$

Vectors have units (i.e. m/s)

Unit vectors

$\hat{\imath} \rightarrow x$

$\hat{\jmath} \rightarrow y$

$\hat{k} \rightarrow z$

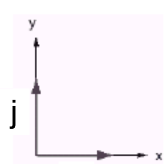
Unit Magnitude
Dimensionless

Used to specify direction

$$\vec{D} = D_x\hat{\imath} + D_y\hat{\jmath}$$

Magnitude + sign

Unit Vector



Vector Addition

Consider Two Vectors

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{A} + \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + \left(B_x\hat{\imath} + B_y\hat{\jmath}\right)$$

$$= \left(A_x + B_x\right)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath}$$

Just add components.

Example

$$\vec{A} = 12m\cdot\hat{i} + 5m\cdot\hat{j}$$

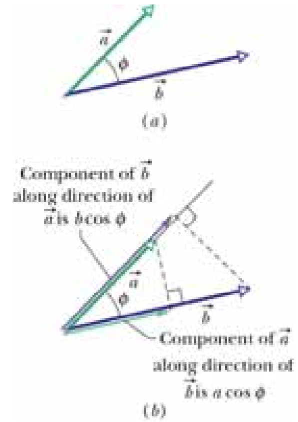
$$\vec{B} = 2m\cdot\hat{i} - 5m\cdot\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$= (12m\cdot\hat{i} + 5m\cdot\hat{j}) + (2m\cdot\hat{i} - 5m\cdot\hat{j})$$

$$= 14m\cdot\hat{i}$$

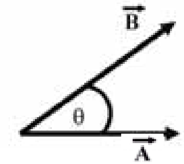
Vector Multiplication



Scalar product

$$\vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

θ is the angle between the vectors if you put their tails together



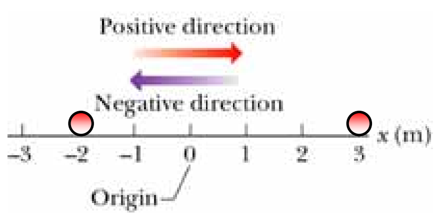
$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

since cos(θ) =cos(-θ)

$$\vec{a}\cdot\vec{b} = ab\cos\phi$$

Scalar product

Motion Along a Straight Line



Displacement:

Displacement is a change of position in time.

It is a vector quantity.

It has both a direction and magnitude.

It has units of [Length]: *meters*.

$$\Delta x = x_2 - x_1$$

Kinematic Variables

Position is a function of time: x = x(t)

Velocity is the rate of change of the position

Acceleration is the rate of change of the velocity

$$v(t) = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$a(t) = \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Position $\xrightarrow{\frac{d}{dt}}$ Velocity $\xrightarrow{\frac{d}{dt}}$ Acceleration

TABLE 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2} a t^2$	v_0

[a] Make sure that the acceleration is indeed constant before using the equations in this table.

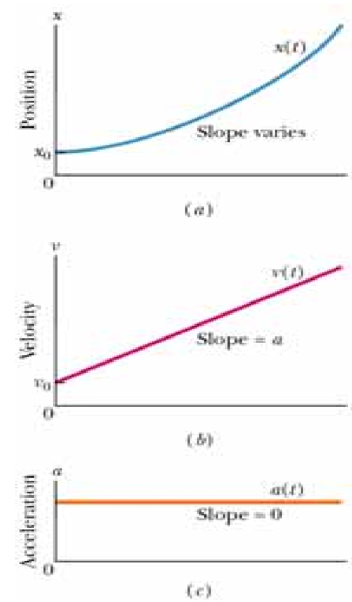
Constant Acceleration (a > 0)

$v(t) = v_0 + at;$

$x(t) - x_0 = v_0 t + at^2/2$

$x(t) - x_0 = (v(t)^2 - v_0^2)/2a$

$x - x_0 = ½ (v + v_0)t$



What does zero mean ?

- t = 0 beginning of the process
- x = 0 (origin) is arbitrary; can set where you want it
- $x_0 = x(t=0)$; position at t=0; *do not mix with the origin !*

- v (t) = 0 x does not change $x(t) - x_0 = 0$
- $v_0 = 0$ $v(t) = at;$ $x(t) - x_0 = at^2/2$
- a = 0 $v(t) = v_0;$ $x(t) - x_0 = v_0 t$

- $a \neq 0$ $v(t) = v_0 + at;$ $x(t) - x_0 = v_0 t + at^2/2$

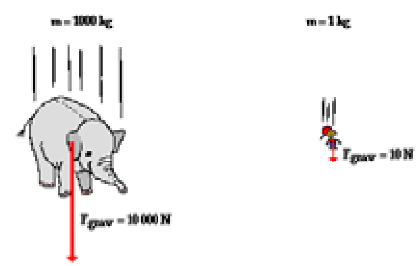
help: $t = (v - v_0)/a$ $x - x_0 = ½(v^2 - v_0^2)/a$

$a = (v - v_0)/t$ $x - x_0 = ½ (v + v_0)t$

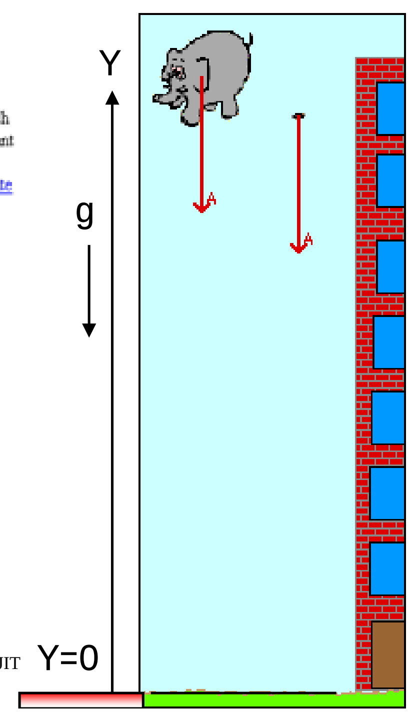
- Acceleration and velocity are positive in the same direction as displacement is positive

Free Fall Motion

As learned in an earlier unit, free-fall is a special type of motion in which the only force acting upon an object is gravity. Objects which are said to be undergoing *free-fall*, are not encountering a significant force of air resistance; they are falling under the sole influence of gravity. Under such conditions, all objects will fall with the same rate of acceleration, regardless of their mass. But why? Consider the free-falling motion of a 1000-kg baby elephant and a 1-kg overgrown mouse.



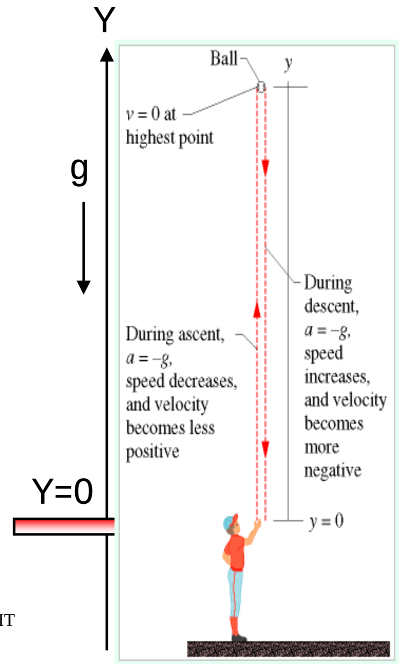
a = -g where g = 9.8 m/s²
(defining "up" as the positive direction)



More general case of the Free Fall Motion

$a = -g$ where $g = 9.8\ m/s^2$

(defining "up" as the positive direction)



Example:

Car starts at rest and accelerates for 10 seconds with $a=+5\ m/s^2$. Then the driver pushes the breaks and comes to a complete stop with accelerates of $a=-3\ m/s^2$. What is the total traveled distance?

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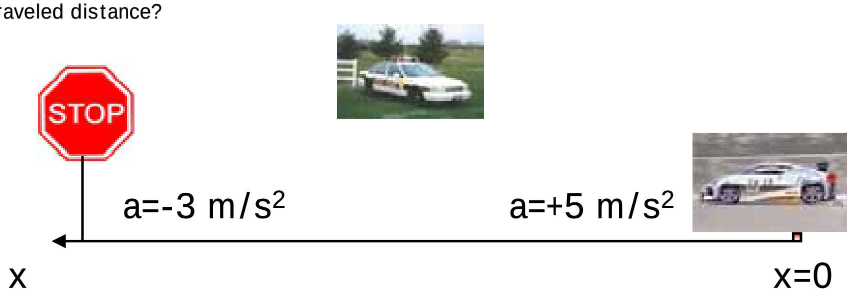


$x_1= v_0\ t + at^2/2 = 5\ m/s^2 \cdot 100\ s^2\ /2 = 250\ m$;

$v = at = 5\ m/s^2 \cdot 10\ s = 50\ m/s$ (ticket ?)

$x_2= (v(t)^2 - v_0^2)/2a = (0-50 \cdot 50\ m^2/s^2)/(-2 \cdot 3\ m/s^2)= 417\ m$

$x_1 + x_2= 250m + 417m = 667\ m$

Projectile Motion

Horizontal motion + Vertical motion

"Free fall with horizontal motion"

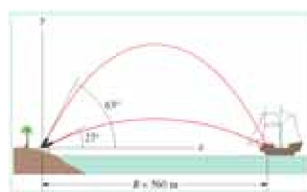
x ≡ horizontal

y ≡ vertical (take positive direction as "up")

z is not relevant

$\vec{a}$ is only in the vertical direction: $\vec{a} = -g\ \hat{j}$

$a_y = -g \qquad a_x = 0$



Projectile Motion

Horizontal motion

$a_x = 0$

Vertical motion

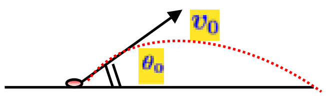
$a_y = -g$

In both directions the acceleration is constant

$v_x = v_{0x} \equiv \text{constant}$

$x = x_0 + v_{0x}t$

$$x - x_0 = v_{0x}t = (v_0 \cos\theta_0)t$$



$v_y = v_{0y} - gt$

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

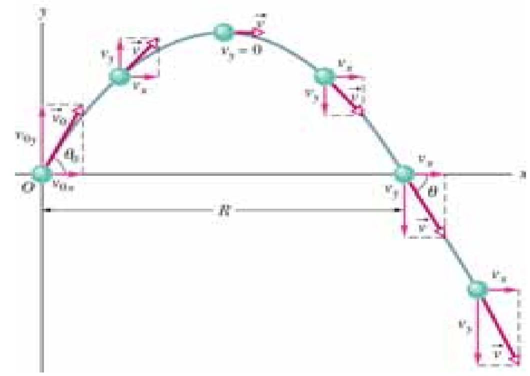
$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

Projectile Motion; General Case

Trajectory and horizontal range

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

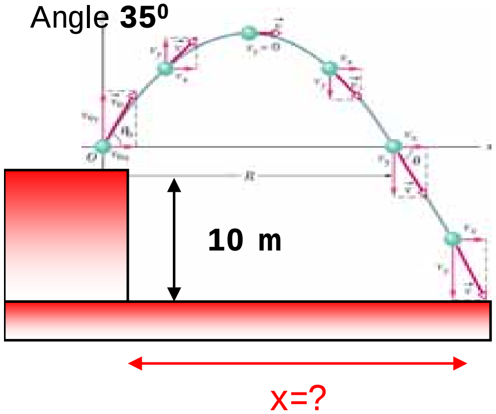
$$R = \frac{v_0^2}{g}\sin 2\theta_0$$



Example: Projectile Motion

V_0=20 m/s

Angle **35^0**

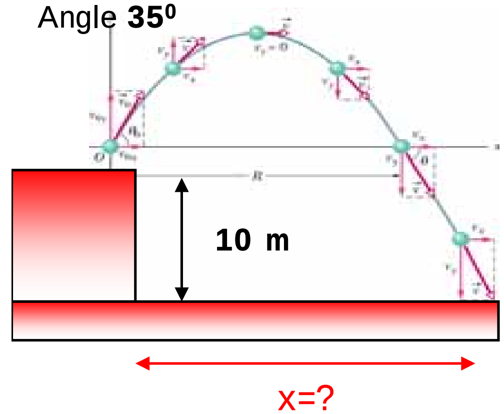


1. Find the magnitude of the final velocity and find the velocity components when it touches the ground
2. What is the horizontal distance **X** ?

Example: Projectile Motion

V_0=20 m/s

Angle **35^0**



1. v_x = Constant
2. $v_x = v_0 \cos 35^0$ = 16.4 m/s
3. $v_y = v_0 \sin 35^0 - gt$
4. Find time of flight using

$0 = h + v_0 \sin 35^0\ t - gt^2/2$

Plug the numbers in:

$0 = 10m + 11.5m/s*t - 9.8m/s^2\ t^2/2$

$4.9\ t^2 - 11.5m/s*t - 10m = 0$

Solve it for t and find two roots:

t_1= -0.7 s (what does it mean ?)

t_2= 3.0 s

v_y= 11.5m/s–9.8m/ s^2 *3s = -18m/s

$v = (v_x^2 + v_y^2)^{1/2}$ = 24 m/s

x = 16.4 m/s * 3s = 49 m

Projectile Motion

EXAMPLE for the Horizontal range:

A football is thrown toward a receiver with an initial speed of 20 m/s at an angle of 25° above the horizontal. At what horizontal distance the receiver should be to catch the football at the level at which it was thrown?

A) Impossible to solve; need the mass of the football
B) 11 m;
C) 21 m
D) 31 m
E) 41 m

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

$$R = \frac{(20\ m/s)^2}{9.8\ m/s^2} \cdot \sin 50^\circ = 31\ m$$

Newton's Laws

I. If no net force acts on a body, then the body's velocity cannot change.
II. The net force on a body is equal to the product of the body's mass and acceleration.
III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction ($\mathbf{F}_{12} = -\mathbf{F}_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration;

$\vec{F}_{tot} = 0 \Leftrightarrow \vec{a} = 0$ (constant velocity)

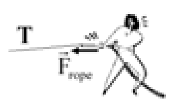
$\vec{F}_{tot} = m\vec{a}$ for any object

$F_{tot,x} = ma_x$ $F_{tot,y} = ma_y$ $F_{tot,z} = ma_z$

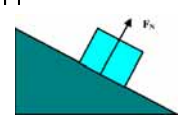
Forces:

- Gravitational Force: $\vec{F}_g = m\vec{g}$ down to the ground

- Tension Force: $\vec{T}$ along the string



- Normal Force: $\vec{N}$ perpendicular to the support



- Friction Force
  - Static; maximum value $\mathbf{f}_s = \mu_{st}\mathbf{N}$
  opposite to the component of other forces parallel to the support
  - Kinetic; value $\mathbf{f}_k = \mu_{kin}\mathbf{N}$
  opposite to the **velocity**, parallel to the support

$\mu_{st} > \mu_{kin}$

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2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a] Make sure that the acceleration is indeed constant before using the equations in this table.

Problem 1: Please mark the version of the exam you are taking

A) YOU ARE TAKING VERSION A
B)
C)
D)
E)

Problem 2: Find the mass of an object whose initial speed of 4 m/s is reduced to zero with a constant 4 N force in 2 seconds.

B) 0.5 kg
B) 2 kg
C) 4 kg
D) 8 kg
E) 16 kg

$F = ma$; $a = \frac{F}{m}$; $m = \frac{F}{a}$

$v = v_0 - a \cdot t \Rightarrow v_0 = a \cdot t \Rightarrow a = v_0 / t$

$m = \frac{F}{v_0} \cdot t = \frac{4N}{4 m/s} \cdot 2s = 2 kg$

Problem 3: Two forces acting on an object of mass 5.0 kg give rise to an acceleration $\mathbf{a} = (2.0\ m/s^2)\mathbf{i} + (3.0\ m/s^2)\mathbf{j}$. One of the forces is $\mathbf{F_1} = (10\ N)\mathbf{i} - (4\ N)\mathbf{j}$. The other must be

A) $\mathbf{F_2} = (10\ N)\mathbf{i} + (15\ N)\mathbf{j}$
B) $\mathbf{F_2} = (20\ N)\mathbf{i} + (11\ N)\mathbf{j}$
C) $\mathbf{F_2} = (10\ N)\mathbf{i}$
D) $\mathbf{F_2} = (12\ N)\mathbf{i} - (1\ N)\mathbf{j}$
E) $\mathbf{F_2} = (19\ N)\mathbf{j}$

$m = 5 kg$ $\vec{a} = 2\hat{i} + 3\hat{j}\ (m/s^2)$

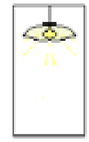
$\vec{F_1} = 10\hat{i} - 4\hat{j}\ (N)$

$\vec{F_2} = ?$ $\vec{F}_{net} = \vec{F_1} + \vec{F_2} \Rightarrow \vec{F_2} = \vec{F}_{net} - \vec{F_1}$

$\vec{F}_{net} = m \cdot \vec{a} = 10\hat{i} + 15\hat{j}\ (N)$

$\vec{F_2} = 10\hat{i} + 15\hat{j} - (10\hat{i} - 4\hat{j}) = 19\hat{j}\ (N)$

Problem 4: A 5 kg lamp is suspended by a string from the ceiling inside an elevator moving up with decreasing speed. If the magnitude of the elevator's acceleration is 3 m/s², what is the tension in the string?



A) 64 N
B) 49 N
C) 34 N
D) 15 N
E) 60 N

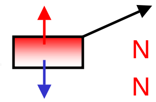
$\vec{a}$; T ; mg ; x

$mg - T = ma$

$T = mg - ma = m(g-a)$

$T = 5 kg (9.8 - 3) m/s^2 = 5 \cdot 6.8 = 34 N$

Problem 5: A 10 kg block is dragged along a horizontal frictionless surface with a 100 N force that makes an angle of 25° with the horizontal. The normal force exerted by the surface on the block is



A) 98 N
B) 140 N
C) 7.4 N
D) 56 N
E) 2 N

$N + F \sin\theta = mg$;

$N = - F \sin\theta + mg$

$N = -100 \sin 25^o + 10*9.8$ (Newton) = 56 N

Problem 6: A block initially moving at 4 m/s upwards on an incline comes to rest after traveling 5 m up the incline. What is the angle between the incline and the horizontal in degrees?

A) 9.4
B) 81
C) 45
D) 53
E) 6.7

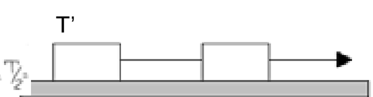
$v^2 - v_o^2 = -2ax$; ➔ $a = v_o^2 / 2x$

$F = mg \sin\theta$; ➔ $a = g \sin\theta$ ➔ $\sin\theta = v_o^2 / (2gx)$;

$\theta = \sin^{-1}(v_o^2 / (2gx)) = \sin^{-1}(0.16) = 9.4^o$

Problem 7: The tension in the string on the right of the right block is 36 N. Each block has a mass of 2 kg. The surface is frictionless. What is the tension in the string between the blocks?

A) 9 N
B) 36 N
C) 18 N
D) 12 N
E) 27 N



$m_1 + m_2 = M$; $Ma = T \Rightarrow a = \frac{T}{m_1+m_2}$

$T - T' = m_1 a$; $T' = T - m_1 a = T(1 - \frac{m_1}{m_1+m_2}) = T/2$

T' = 18 N

Problem 8: A 2000kg car slides on the ice and stops in 20m due to the frictional force between the car and the ice. If the initial speed of the car is 5 m/s, the coefficient of kinetic friction between the ice and car is:

A) 0.
B) 0.064
C) 0.013
D) 1.0
E) 9.8

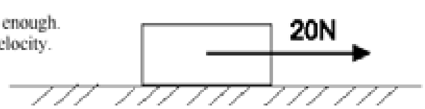
$v^2 - v_o^2 = -2ax$; ➔ $a = v_o^2 / 2x$

$F = mg\,\mu$; ➔ $a = g\,\mu$ ➔ $\mu = v_o^2 / (2gx)$;

$\mu = 25 / (19.6*20)) = 0.064$ (mass is not important !)

Problem 9: A block of mass 5kg is pulled along a horizontal floor by a force of 20N as shown in the figure. The coefficient of static friction is 0.4. The coefficient of dynamic friction is 0.2. the magnitude of the acceleration of the block is

A) The block does not accelerate. The 20N force is not strong enough.
B) The acceleration is zero, but the block moves at constant velocity.
C) 2.04 m/s²
D) 0.24 m/s²
E) 9.8 m/s²



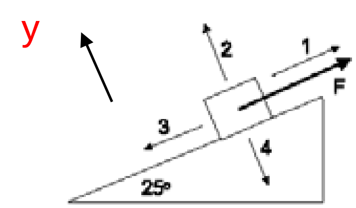
$F > F_{st}$ (20N > 19.6N) or $F \cong F_{st}$ (20N ≅ 19.6N)

$a = (F - F_{kin})/m = (20-9.8)/5 = 2.04\ m/s^2$ (too many sign. Figs.)

Problem 10: As shown in the Figure below, a sled is pulled up a snow covered hill by a force F. The angle of the slope is 25 degrees. The weight of the sled is 100N. Which of the labeled arrows below indicate the DIRECTION of the frictional force?

A) Arrow 1
B) Arrow 2
C) Arrow 3
D) Arrow 4
E) None of the above

ma = F - mg sin θ - f
ma = F – 42N – f
f is directed as "3"



Problem 11: Referring to the sled problem above, the coefficient of static friction is 0.25 and the coefficient of kinetic friction is 0.15. What value of F is required such that the sled moves at a constant velocity?

A) 56 N
B) 65 N
C) 42 N
D) 91 N
E) 100 N

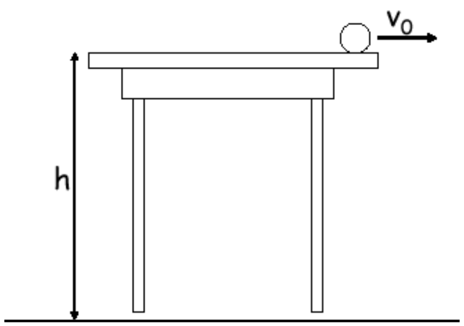
Y: $mg \cos\theta = N$; $f = \mu N$; ➔ $f = \mu\, mg \cos\theta$

$ma = 0 = F - mg \sin\theta - \mu\, mg \cos\theta$

$F = mg(\sin\theta + \mu \cos\theta)$

$F = 100(N)*(\sin 25^o + 0.15 \cos 25^o) = 55.8\ (N) \approx 56\ (N)$

QZ#7

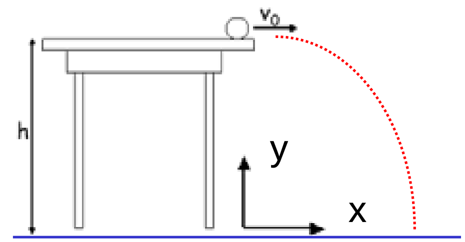


$v_0 = 5$ m/s;
h = 2 m

A ball rolls off a table of height h. The ball has horizontal velocity v0 when it leaves the table.

How far away does it strike the ground?

How long does it take to reach the ground?



Δx = ???

$$x - x_0 = v_{0x}t = (v_0 \cos\theta_0)t$$

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

$$v_y = v_0 \sin\theta_0 - gt$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

$v_{0x} = v_{0;}$ $x_0 = 0$

$v_{0y} = 0;$ $y_0 = h$

For **x** direction: $t = \Delta x / v_0$

For **y** direction: $y(t) = 0$

$y(t) - h = v_{0y}\, t - gt^2/2$

$\Delta x = v_0 \cdot (2h/g)^{½}$