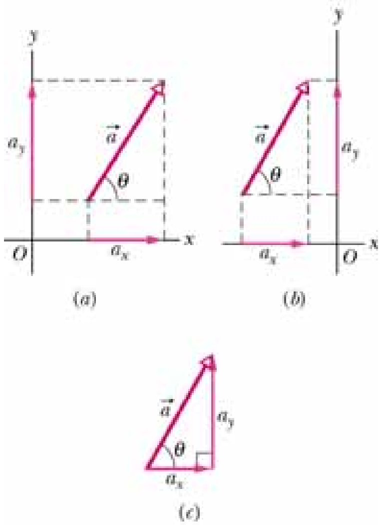
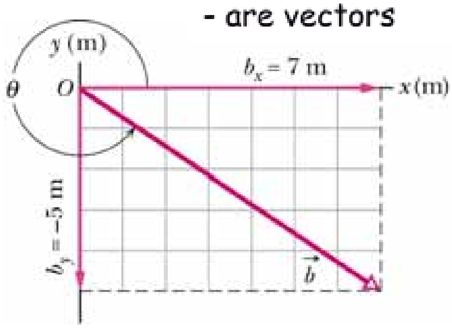
Components of Vectors:

- aligned along axis
- add to give vector
- are vectors





$a_x = a\cos\theta$ and $a_y = a\sin\theta$

$a = \sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \dfrac{a_y}{a_x}$

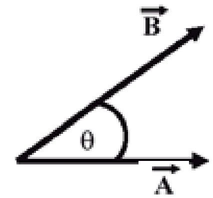
Length (Magnitude)

Vector Multiplication

Scalar product

$\vec{A}\bullet\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

θ is the angle between the vectors if you put their tails together



$\vec{A}\bullet\vec{B} = \vec{B}\bullet\vec{A}$

since $\cos(\theta) = \cos(-\theta)$

TABLE 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a] Make sure that the acceleration is indeed constant before using the equations in this table.

What does zero mean ?

- $t = 0$ beginning of the process
- $x = 0$ is arbitrary; can set where you want it
- $x_0 = x(t=0)$; position at $t=0$; do not mix with the origin

- $v(t) = 0$ x does not change $x(t) - x_0 = 0$
- $v_0 = 0$ $v(t) = at$; $x(t) - x_0 = at^2/2$
- $a = 0$ $v(t) = v_0$; $x(t) - x_0 = v_0 t$

- $a \neq 0$ $v(t) = v_0 + at$; $x(t) - x_0 = v_0 t + at^2/2$

help: $t = (v - v_0)/a$ $x - x_0 = ½(v^2 - v_0^2)/a$

$a = (v - v_0)/t$ $x - x_0 = ½(v + v_0)t$

- Acceleration and velocity are positive in the same direction as displacement is positive

Projectile Motion; General Case

Horizontal motion

$$\begin{aligned} x - x_0 &= v_{0x} t \\ &= (v_0 \cos\theta_0) t \end{aligned}$$

Trajectory and horizontal range

$$y = (\tan\theta_0) x - \frac{g x^2}{2(v_0 \cos\theta_0)^2}$$

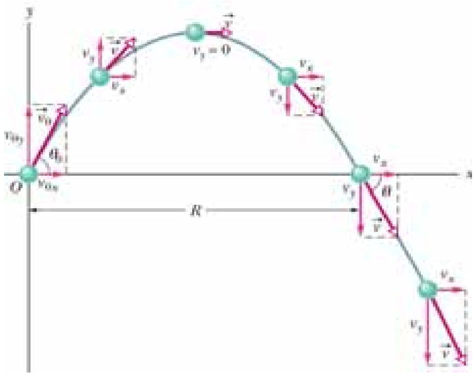
$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

Vertical motion

$$\begin{aligned} y - y_0 &= v_{0y} t - \frac{1}{2} g t^2 \\ &= (v_0 \sin\theta_0) t - \frac{1}{2} g t^2 \end{aligned}$$

$$v_y = v_0 \sin\theta_0 - g t$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$



Last Lecture: Projectile Motion

Horizontal motion

$a_x = 0$

Vertical motion

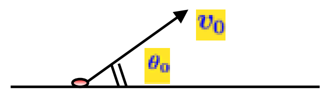
$a_y = -g$

In both directions the acceleration is constant

$v_x = v_{0x} \equiv$ constant

$x = x_0 + v_{0x} t$

$$\begin{aligned} x - x_0 &= v_{0x} t \\ &= (v_0 \cos\theta_0) t \end{aligned}$$



$v_y = v_{0y} - gt$

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$

$$\begin{aligned} y - y_0 &= v_{0y} t - \frac{1}{2} g t^2 \\ &= (v_0 \sin\theta_0) t - \frac{1}{2} g t^2 \end{aligned}$$

$$v_y = v_0 \sin\theta_0 - g t$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$$

Newton's Laws

I. If no net force acts on a body, then the body's velocity cannot change.

II. The net force on a body is equal to the product of the body's mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction (**F**$_{12}$ = -**F** $_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration;

$\vec{F}_{tot} = 0 \Leftrightarrow \vec{a} = 0$ (constant velocity)

$\vec{F}_{tot} = m\vec{a}$ for any object

$F_{tot,x} = ma_x \qquad F_{tot,y} = ma_y \qquad F_{tot,z} = ma_z$

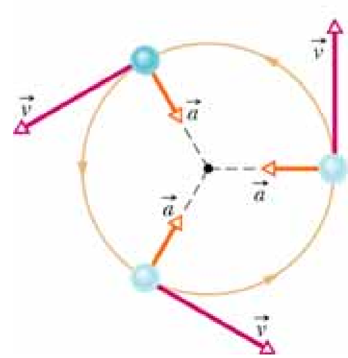
Uniform Circular Motion

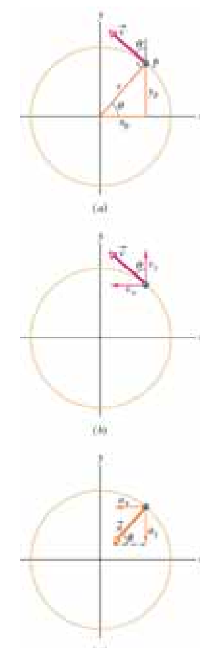
Centripetal acceleration

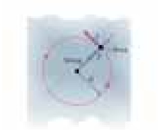
$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$



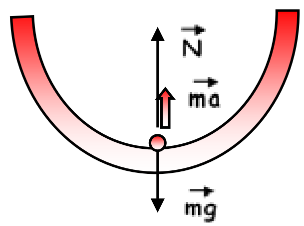




$ma_c = mv^2/R = \Sigma F$

(all forces along the direction towards the center)

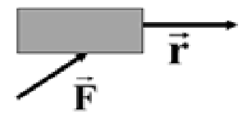
- Gravitational Force: $\vec{mg}$
  down to the ground
- Tension Force: $\vec{T}$
  along the string
- Normal Force: $\vec{N}$
  perpendicular to the support
- Static Friction Force
  maximum value $F_{fr}^{max} = \mu_{st} N$

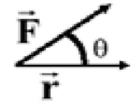


$ma = N - mg$

$ma = mv^2/R$

What does $W = \vec{F} \bullet \vec{r}$ mean?





$W = \vec{F} \bullet \vec{r}$
$= F_x r_x + F_y r_y$
$= Fr\cos\theta$

$W > 0$ if $\theta < 90°$ ⟶ force is adding energy to object

$W < 0$ if $\theta > 90°$ ⟶ force is reducing energy of object

$W = 0$ if $r = 0$ or $F = 0$ or $\vec{F} \perp \vec{r}$

Work Examples

Push on a wall

$W = 0$ since wall does not move ($\vec{r} = 0$)

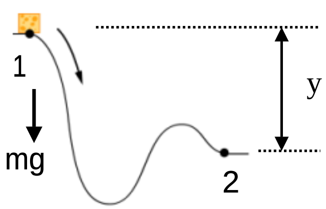
Kinetic Energy: $K = \frac{1}{2}mv^2$

Potential Energy:

$\Delta U = -W$

- Gravitation: $U = mgy$
- Elastic (due to spring force): $U = \frac{1}{2}kx^2$

$U = mgy$



$K = \frac{1}{2}mv^2$

U → K

$E_{mec} = K + U$

Conservation of Mechanical Energy

$K_2 + U_2 = K_1 + U_1$

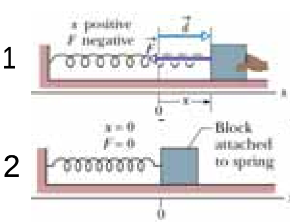
Kinetic Energy: $K = \frac{1}{2}mv^2$

Potential Energy:

$\Delta U = -W$

- Gravitation: $U = mgy$
- Elastic (due to spring force): $U = \frac{1}{2}kx^2$

1



2

$U = \frac{1}{2}kx^2$ K = 0

U = 0 $K = \frac{1}{2}mv^2$

U ← → K

$E_{mec} = K + U$

Conservation of Mechanical Energy

$K_2 + U_2 = K_1 + U_1$

Examples for Energy Conservation

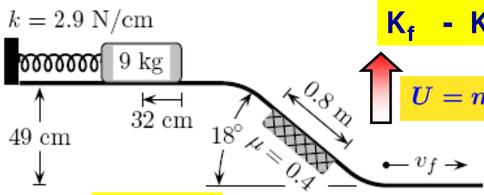
Kinetic Energy changes $K = \frac{1}{2}mv^2$

+ Gravitational Potential Energy $U = mgy$

+ Elastic Potential Energy $U = \frac{1}{2}kx^2$

Total Mechanical Energy = *Const.*

$U = \frac{1}{2}kx^2$ → $K = \frac{1}{2}mv^2$



$K_f - K_i = W = mgy - |W_{friction}|$

$U = mgy$

$K = \frac{1}{2}mv^2$

$W_{friction}$

$E_f - E_i = -|W_{friction}| = f_k \cdot d \cdot \cos 180° = -mg\,\mu \cdot d \cdot \cos 18°$

Linear Momentum

Particle: $\vec{p} = m\vec{v}$

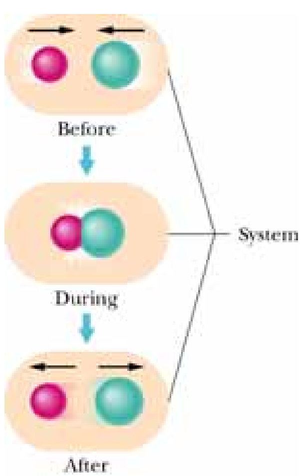
System of Particles: $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots$

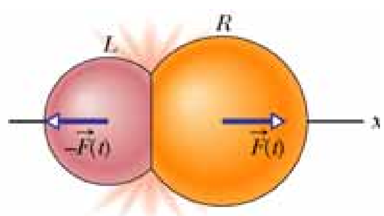
Extended objects: $\vec{P} = M\vec{v}_{com}$

Relation to Force: $\vec{F}_{tot} = m\vec{a}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\vec{F}_{net} = \frac{d\vec{P}}{dt}$

Collision of two particle-like bodies



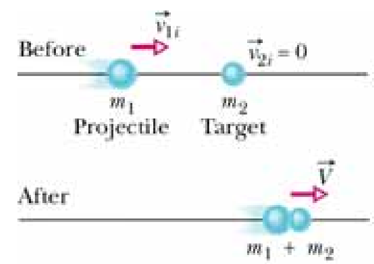


Completely Inelastic Collision
Collisions in 1D

Conservation of Linear Momentum works !

$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

$$m_1 v_{1i} = (m_1 + m_2)V$$
$$V = \frac{m_1}{m_1 + m_2} v_{1i}$$



Example: Two equal objects, one initially at rest

$mv_i = 2mv_f$ ⟶ $v_f = v_i/2$

Final Kinetic Energy $= \frac{1}{2}(2m)(v_i/2)^2$
$= \frac{1}{4}m(v_i)^2$

Half the original Kinetic Energy