# Lecture 9(+10)

## Physics 106

Fall 2006

### Gravitation

HW&R

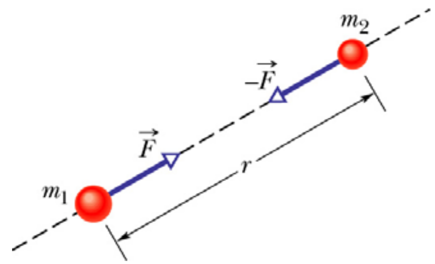
http://web.njit.edu/~sirenko/

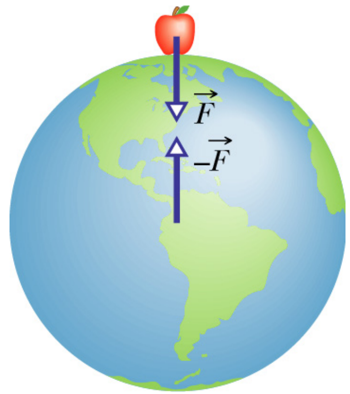


## Newton's Law of Gravitation (known since 1665)

$$F = G\frac{m_1 m_2}{r^2}$$

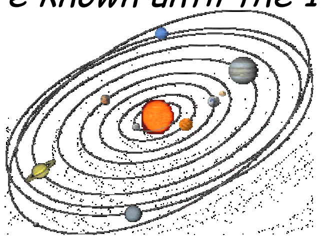
$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$

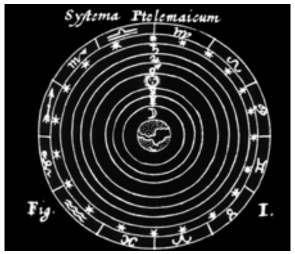


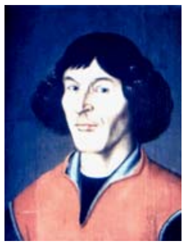





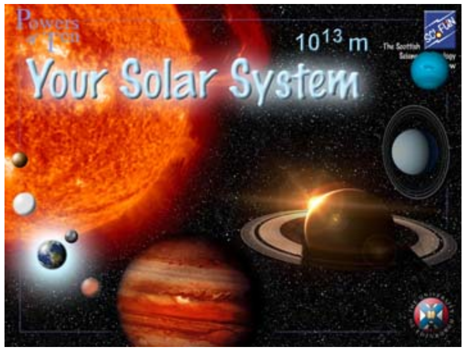

*Only six planets, including the Earth, were known until the 18th Century*





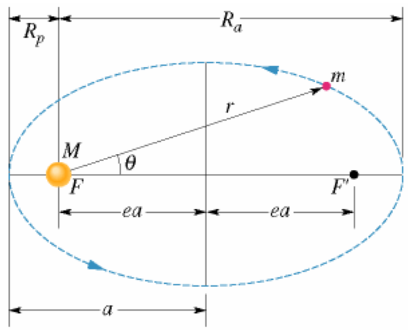
**Copernicus** 1510






## Planets and Satellites: Kepler's Laws: 1609

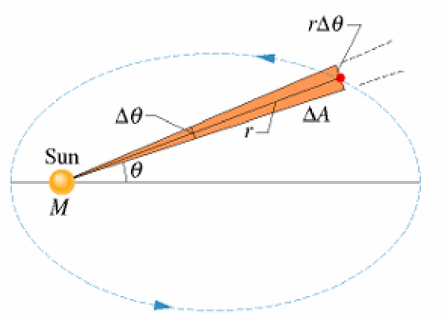
**THE LAW OF ORBITS:** All planets move in elliptical orbits, with the Sun at one focus.




A planet of mass $m$ moving in an elliptical orbit around the Sun. The Sun, of mass $M$, is at one focus $F$ of the ellipse. The other focus is $F'$, which is located in empty space. Each focus is a distance $ea$ from the ellipse's center, with $e$ being the eccentricity of the ellipse. The semimajor axis $a$ of the ellipse, the **perihelion** (nearest the Sun) distance $R_p$, and the **aphelion** (farthest from the Sun) distance $R_a$ are also shown.

# Planets and Satellites: Kepler's Laws

**THE LAW OF AREAS:** A line that connects a planet to the Sun sweeps out equal areas in the plane of the planet's orbit in equal times; that is, the rate $dA/dt$ at which it sweeps out area $A$ is constant.

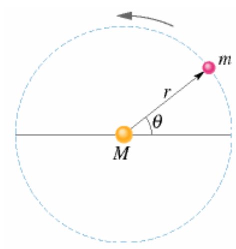



In time $\Delta t$, the line $r$ connecting the planet to the Sun (of mass $M$) sweeps through an angle $\Delta\theta$, sweeping out an area $\Delta A$ (shaded).



# Planets and Satellites: Kepler's Laws

**THE LAW OF PERIODS:** The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.




$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

| Planet | Semimajor Axis $a$ ($10^{10}$ m) | Period $T$ (y) | $T^2/a^3$ ($10^{-34}$ y$^2$/m$^3$) |
|---|---|---|---|
| Mercury | 5.79 | 0.241 | 2.99 |
| Venus | 10.8 | 0.615 | 3.00 |
| Earth | 15.0 | 1.00 | 2.96 |
| Mars | 22.8 | 1.88 | 2.98 |
| Jupiter | 77.8 | 11.9 | 3.01 |
| Saturn | 143 | 29.5 | 2.98 |
| Uranus | 287 | 84.0 | 2.98 |
| Neptune | 450 | 165 | 2.99 |
| Pluto | 590 | 248 | 2.99 |



Exploring The Planets
Uranus

## Uranus Facts

| | |
|---|---|
| Discoverer: | Sir William Herschel (1781) |
| Spacecraft Encounter(s): | Voyager 2 (1986) |
| Mean distance from Sun: | 19.19 AU (2.871 billion km/1.784 billion mi) |
| Length of year: | 84.01 Earth years |
| Rotation period: | 17.24 hours |
| Mean orbital velocity: | 6.81 km/s (4.2 m/s) |
| Inclination of axis: | 97.92° |
| Diameter: | 51,118 km (31,765 mi) |
| Number of Observed Satellites: | >20 |

**Comparisons with Earth:**

| | |
|---|---|
| Diameter: | 4.0 x Earth's |
| Mean Distance from Sun: | 19.2 x Earth's |
| Mass: | 14.5 x Earth's |
| Density: | 0.22 x Earth's |

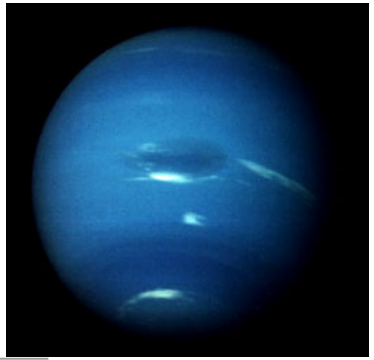


Exploring The Planets
Neptune

## Neptune Facts

- Neptune is the outermost of the four gas giants: Jupiter, Saturn, Uranus, Neptune.
- Because of its distance from the Sun, Neptune's atmosphere is a frigid -225° C (-373° F).
- The blue-green color of the planet is due to the presence of methane in the atmosphere. The atmosphere consists mostly of hydrogen, helium and methane.
- Until the Voyager encounter in 1989, the rings surrounding Neptune were thought to be arcs. We now know that the rings completely circle the planet, but the thickness of each ring varies along its length.

| | |
|---|---|
| Discoverer(s): | Galle, Challis, Adams, & Le Verrier (1846) |
| Spacecraft Encounter(s): | Voyager 2 |
| Mean Distance from the Sun: | 30.06 AU (4.497 billion km/2.794 billion mi) |
| Length of year: | 165 years |
| Rotation period: | 16.11 hours |
| Mean orbital velocity: | 5.43 km/s (3.3 mi/s) |
| Diameter: | 49,528 km/30,775 mi |
| Inclination of axis: | 29.6° |
| Number of observed satellites: | 8 |



**Comparisons with Earth:**

| | |
|---|---|
| Diameter: | 3.883 x Earth's |
| Average distance from Sun: | 30.06 x Earth's |
| Mass: | 17.14 x Earth's |
| Density: | 0.31 x Earth's |

## Pluto Facts

- Pluto is too faint to be seen with the naked eye. When viewed through a telescope, it looks like a star.
- Stellar occultations have revealed a tenuous atmosphere on Pluto composed primarily of nitrogen and methane gas.
- Pluto is cold: -233° C (-390 ° F), just 40° C (72 ° F) above absolute zero. At this temperature, all elements would be frozen but neon, hydrogen, and helium.
- Measurements indicate Pluto is the smallest planet, 2320 kilometers (1440 miles) in diameter. It is smaller than Earth's Moon.

| | |
|---|---|
| Discoverer: | Clyde Tombaugh - January 23, 1930 |
| Spacecraft Encounter: | none |
| Average distance from Sun | 39.48 AU 5.906 billion km (3.670 billion mi) |
| Length of year | 248.5 years |
| Rotation period | 6.4 days |
| Average orbital velocity | 4.7 km/sec (3 mi/sec) |
| Inclination of axis | 122.46 degrees |
| Diameter | 2340 km (1454 mi) |
| Number of satellites | 1 |

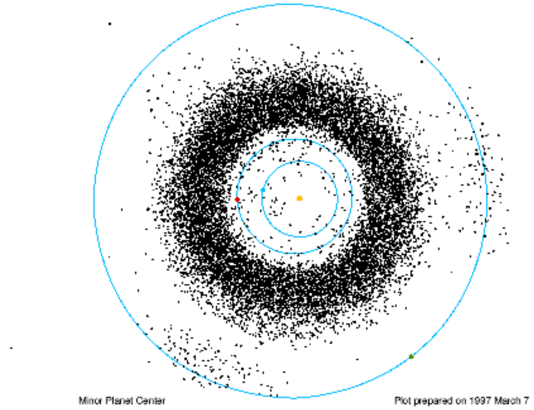
**Comparisons with Earth**

| | |
|---|---|
| Average distance from Sun | 39.5 x Earth |
| Diameter | 0.18 x Earth |
| Mass | 0.002 x Earth |
| Density | 0.36 x Earth |

### Pluto and Charon



## Asteroids





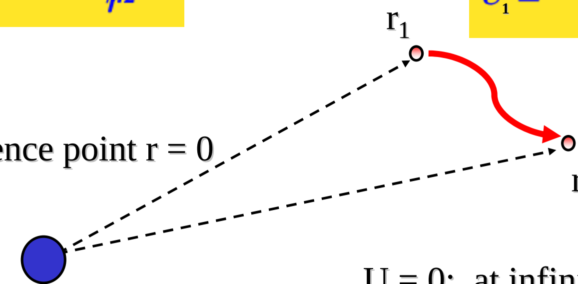

## Potential Energy :

$\Delta U$ between $r_1$ and $r_2$ is the work done by the Gravitation Force during the move from $r_1$ to $r_2$ :

$$F = G\frac{m_1 m_2}{r^2}$$

$$U_1 = -\frac{GMm}{r_1}$$

$$U_2 = -\frac{GMm}{r_2}$$

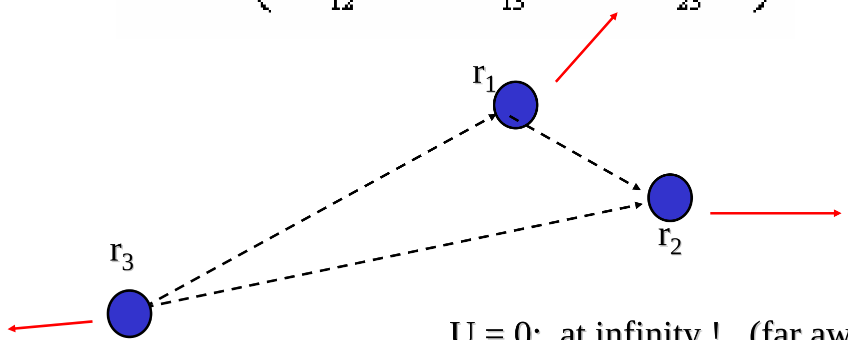



U = 0; at infinity ! (far away)



## Potential Energy of a System :

$$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_1m_3}{r_{13}} + \frac{Gm_2m_3}{r_{23}}\right).$$



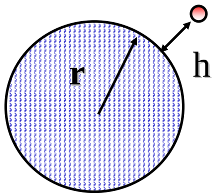

U = 0; at infinity ! (far away)

# Potential Energy :

Is it $\Delta U = mgh$ or $U = -\frac{GMm}{r}$, anyway ?

It is the same thing, just different zero levels.

$U = -\frac{GMm}{r}$ is more universal (always correct)




$\Delta U = mgh$ works for $h \ll r$,
zero at the Earth surface

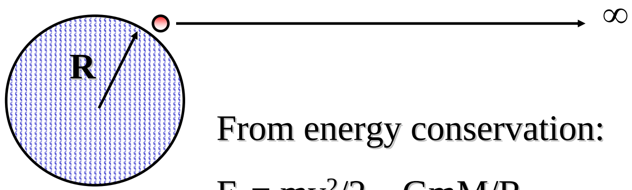
$U = -\frac{GMm}{r}$ always works, zero at $\infty$

$\Delta U = GMm/r - GMm/(r+h) = GMm(r+h-r)/(r\cdot(r+h))$
$= mh\cdot[GM/(r\cdot(r+h))] \approx mgh$



# Escape Speed:

$F = G\frac{m_1 m_2}{r^2}$




From energy conservation:

$E_1 = mv^2/2 - GmM/R$

$E_2 = 0$ (velocity is small)

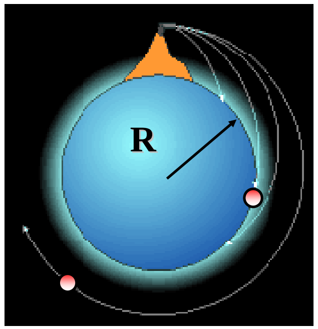
$v^2 = 2GM/R = 2gR$

$v = (2GM/R)^{1/2} \approx 11{,}200$ m/s



# First Satellite Speed:

$F = G\frac{m_1 m_2}{r^2}$




"Newton's cannon"

in 1687 in "*Principia Mathematica*"

$v_{satellite} \approx (gR)^{1/2}$

$v_{satellite} \approx 8{,}000$ m/s

$g \approx 8.70$ m/s$^2$

An object in orbit is weightless not because 'it is beyond the earth's gravity' but because it is in 'free-fall' - just like a skydiver.



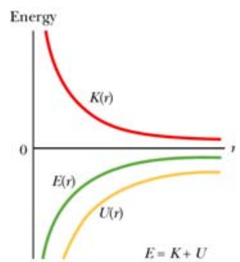
# Potential and Kinetic Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

Kinetic Energy for the orbital motion

$$F = G\frac{Mm}{r^2} = m\frac{v^2}{r} \Rightarrow K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$



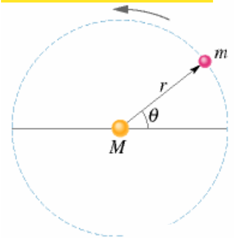

Total Energy

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

# Satellites: Orbits and Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

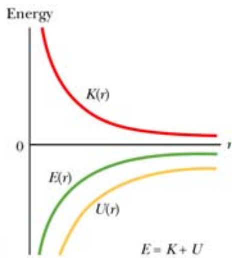



$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Kinetic Energy for the orbital motion

$$K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$

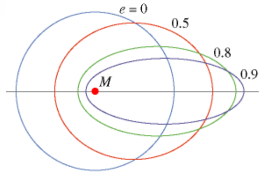
$$K = -\frac{U}{2} \quad \text{(circular orbit)}$$




Total Energy: $E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ (circular orbit)

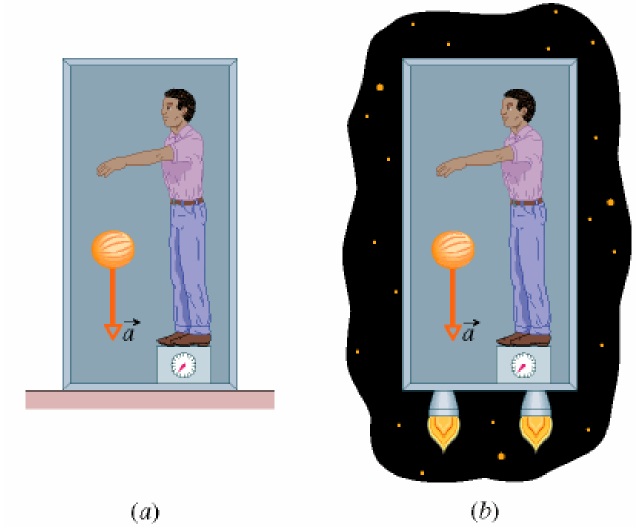
$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit)}$$

semimajor axis $a$






## Einstein and Gravitation





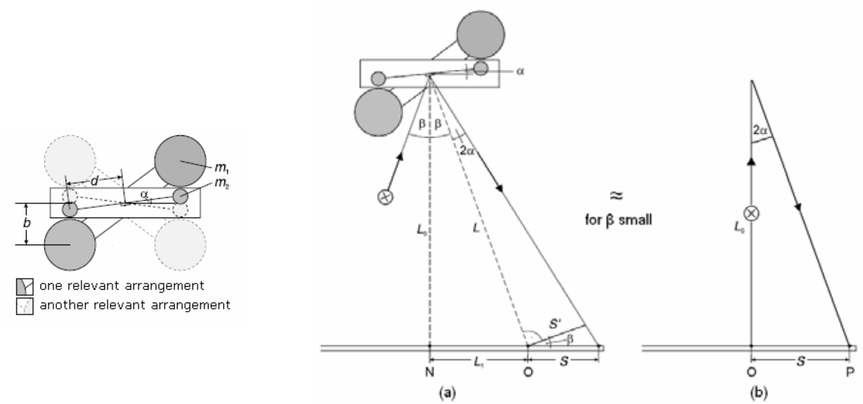

# Gravitation

- **On Earth: the Earth gravitation dominates** (F =mg) $F = G\frac{m_1 m_2}{r^2}$
- **In the Solar System: attraction to the Sun is the main effect**
- **In the Galaxy (Milky Way): Attraction to the center of the Galaxy determines everything.**
- **At the edge of the Universe: the conceptual problems begin … Accelerating expansion of the visible Universe is known since 1998. From that time this problem became one of the frontiers of the modern Physics**

The Andromeda galaxy. Located 2.3 $10^6$ light-years from us

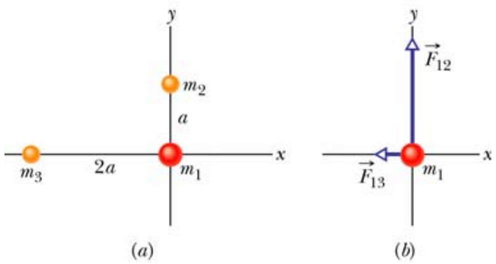


# Measuring the Gravitational constant G using the Cavendish method




$$G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$$

# QZ9:




$$F = G\frac{m_1 m_2}{r^2}$$

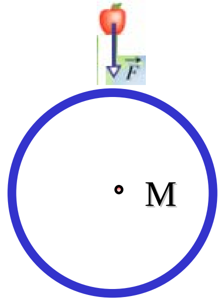
$$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$

Sample Problem from HW&R:

$m_1$ = 6 kg, $m_2 = m_3$ = 4 kg, $d_{12} = a$, and $d_{13} = 2a$. What is the net gravitational force $F_1$ that acts on the particle "1" due to the other particles? Use $a$ = 0.1 m.



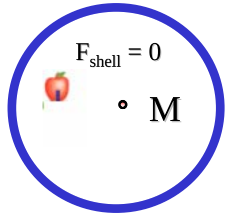
# Newton's Law of Gravitation

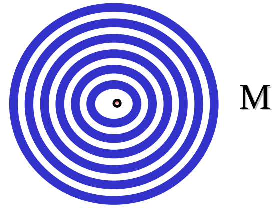



$$F = G\frac{m_1 m_2}{r^2}$$

A uniform spherical shell of matter attracts a particles that is outside the shell as if all the shell's mass is concentrated at its center!

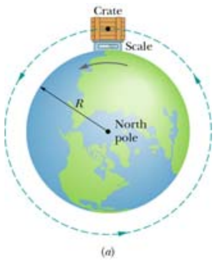
Solid sphere is a combination of spherical shells:



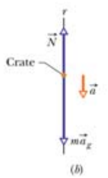


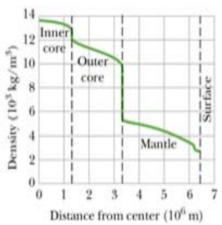

# Gravitation Near Earth's Surface




Gravitational acceleration:

$$F = G\frac{Mm}{r^2} = ma_g \quad \Rightarrow \quad a_g = \frac{GM}{r^2}$$



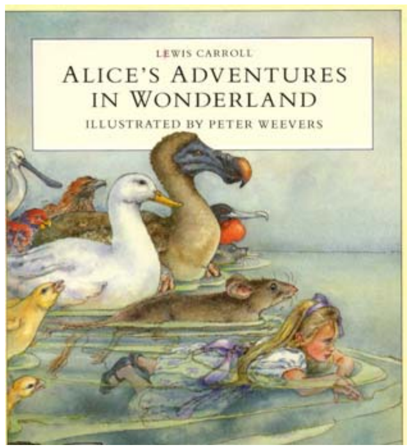




1. Earth is not uniform.
2. Earth is not a sphere.
3. Earth is rotating.

Mean Earth surface (0 km, 9.83 m/s$^2$), Mt. Everest (8.8 km, 9.80 m/s$^2$), highest manned balloon ( 36.6 km, 9.71 m/s$^2$), Space Shuttle orbit (400 km, 8.70 m/s$^2$), and communications satellite (35,700 km, 0.225 m/s$^2$)



# Gravitation inside the Earth "Alice in Wonderland"

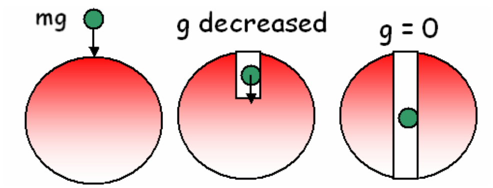



# Gravitation inside the Earth "Alice in Wonderland"

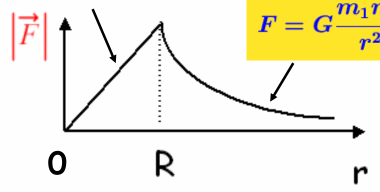
Outside the Earth
(r > R)

$$F = G\frac{m_1 m_2}{r^2}$$


$a_g \approx g = 9.8\ m/s^2$



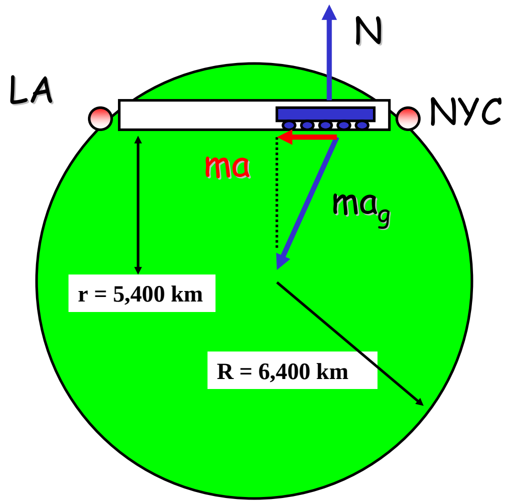

$F \approx K \cdot r \approx (G\, m_1 m_2 / R^3) \cdot r$




$$F = G\frac{m_1 m_2}{r^2}$$



# Gravitational Train "Alice"




**(a)** Will this train move at all?

**(b)** What is the total force on the train in the middle of the tunnel ?

**(c)** Where will the train stop? LA or in the middle of the tunnel

**(c)** What is the speed of the train in the middle of the tunnel ?

**Answers:**

**(a)** v ≈ 6000 m/s

**(b)** v ≈ 600 m/s

**(c)** v ≈ 60 m/s

**(d)** v ≈ 6 m/s

**(e)** v ≈ 0