Instructions:

1. The drawing shows the blade of a chain saw. The rotating sprocket tip at the end of the guide bar has a radius of 25 cm. The angular speed of the sprocket tip is 12 rad/s. Find the linear speed of a chain link at point A.

A) 4 m/s  
B) 5 m/s  
C) 1 m/s  
D) 2 m/s  
E) 3 m/s  

2. A disk with radius R = 3.0 m. is spinning about its center. Initially the disc has an angular velocity of 150 rev/min, and is slowing down uniformly at a rate of 6.0 rad/s^2. By the time it stops spinning, the total number of revolutions the disk will make is:

A) 7.1  
B) 9.3  
C) 1.1  
D) 3.3  
E) 5.5  

3. Two identical wheels are spinning, but wheel B is spinning with twice the angular velocity of wheel A and its radius is also twice as large as the radius of wheel A. The ratio of the radial acceleration of a point on the rim of B to the radial acceleration of a point on the rim of A is:

A) 1/8  
B) 1/4  
C) 1  
D) 2  
E) 8
4. Five equal 2.0-kg point masses are arranged in the x-y plane as shown. They are connected by massless sticks to form a rigid body. The distance $a$ is 4 m. Find the rotational inertia in kg.m$^2$ about an axis parallel to the z-axis through point P. The result is:

A) 128  
B) 512  
C) **288**  
D) 256  
E) 192  

5. A uniform rod of mass $M = 4$ kg and length $L = 3.0$ m is pivoted about an axis perpendicular to the rod and 50 cm from its left end. Find the rotational inertia about this axis (in kg.m$^2$):

A) **7.0**  
B) 3.0  
C) 36.0  
D) 5.0  
E) 11.0  

6. A force $F$ is applied to the edge of the wheel as shown at point P. The torque that it produces about an axis of rotation through the center of the wheel is:

A) $FR$  
B) $FR\sin(\phi)$  
C) $F\sin(\phi)$  
D) $R\sin(\phi)$  
E) $FR\cos(\phi)$  

7. The dumbbell in the figure consists of a uniform rod fastened to two masses attached to each end. The rotational inertia of the rod about an axis perpendicular to the plane of the figure through point "O" is $I_{ROD} = 8.0$ kg.m$^2$. Each mass is 2.0 kg located 2.0 meters from the center point "O" of the rod. What force $F$ must be applied to one end (perpendicular to the rod) to give the system an instantaneous angular acceleration of 4.0 rad/s$^2$ about the center?

A. 32 N  
B. **48 N**  
C. 64 N  
D. 27 N  
E. 18 N
8. A disk drive with rotational inertia $4.0 \times 10^{-3} \text{ kg.m}^2$ is to accelerate uniformly from rest to an angular velocity of 7200 rev/min in 10 sec. The torque that its motor must provide to cause this acceleration is:

A) 0.30 N.m  
B) 0.72 N.m  
C) 3.30 N.m  
D) 180 N.m  
E) 2.9 N.m  

9. What value of the angle $\phi$ would cause the angular acceleration to be zero in the diagram below. The forces $F_1$ and $F_2$ act on a thin rigid rod pivoted at the rotation axis shown. The rotation axis is perpendicular to the page.

A. 1.0 rad  
B. 0.0 rad  
C. $90^\circ$  
D. $45^\circ$  
E. none of these

10. A 12 kg block hangs on a cord that is wrapped around the rim of a flywheel of radius 0.25 m. The rotational inertia of the flywheel about a horizontal axis is $0.60 \text{ kg.m}^2$. When the block is released, the cord unwinds with no slipping, the system accelerates, and the tension in the cord is no longer equal to the weight of the block hanging from it. Find the acceleration of the block:

A) 49 m/s$^2$  
B) 9.8 m/s$^2$  
C) 5.4 m/s$^2$  
D) 4.9 m/s$^2$  
E) 0.02 m/s$^2$
Show all work for the following problem

A. (25 points) A small ball of mass 2.5 kg is attached to one end of a 4.0-m-long massless rod and the other end of the rod is attached to a pivot as shown. The bar is released from rest when it is horizontal at $t = 0$, after which it swings down due to gravity. The sketch shows the bar at a time when it makes an angle $\theta$ with the vertical and also when $t = 0$.

a) Calculate the net torque and angular acceleration of the bar at the instant after it is released.
b) Find the net torque on the bar and its angular acceleration when $\theta = \pi/6$. Show the moment arm you are using on the sketch.
c) Find the angular velocity $\omega$ when $\theta = 0$; that is, when the bar has swung so that it is momentarily vertical. There is no friction in the pivot. Note that the angular acceleration is not constant in this problem.
d) Find the angular acceleration when $\theta = 0$. Describe what happens to the torque and angular acceleration as the bar swings past this point.

Answers:

a) $\tau = -98 \text{ nm}$, $\alpha = -2.5 \text{ rad/s}^2$
b) $\tau = -49 \text{ nm}$, $\alpha = -1.2 \text{ rad/s}^2$
c) $\omega = -2.2 \text{ rad/s}$
d) $\alpha = 0$ since the moment arm is proportional to $\sin(\theta)$, which vanishes for $\theta = 0$. Afterwards, both $\alpha$ and $\tau$ reverse direction and become counter-clockwise. The angular velocity remains negative (CW) until the bar has swung to its most leftmost position.
Show all work for the following problem:

B. (25 points) Two masses are attached by a cord to a pulley as shown. The cord does not stretch and it cannot slip where it touches the pulley, but there is no other frictional force. The rotational inertia of the pulley is 5.0 kg.m$^2$.

The radius $R = .75 \text{ m}$ as shown.

a) Draw free-body diagrams and use Newton’s second law to get the equations that describe the motion of mass A and mass B.

b) Draw the free-body diagram and get the rotational second law equation for the pulley. Include the forces and torques acting on it.

c) You need 2 more equations to solve for the 5 unknowns in your equations above. What are they and why are they justified?

d) Calculate the angular acceleration of the pulley. \textbf{Ans - 1.1 rad/s}^2 \text{ CW}

e) Calculate the acceleration of mass B. \textbf{Ans: - 0.86 m/s}^2 (down)
Physics 106:

360 degrees = 2π radians = 1 revolution.  s = rθ  v = rω  a = rα  a_c = rω^2  \vec{a}_{tot} = \vec{a}_c + \vec{a}_r  
\omega = \omega_0 + \omega t  \theta_0 - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2  \omega^2 - \omega_0^2 = 2 \alpha (t - \theta_0)  \theta - \theta_0 = \frac{1}{2} (\omega + \omega_0) t  
\vec{I}_{point} = m \vec{r}^2  \\vec{I}_{hoop} = \vec{MR}^2  \vec{I}_{disk} = \frac{1}{2} M \vec{R}^2  \vec{I}_{sphere} = \frac{2}{5} M \vec{R}^2  \vec{I}_{shell} = \frac{2}{3} M \vec{R}^2  \vec{I}_{rod (center)} = \frac{1}{12} M \vec{L}^2  

\tau = \text{force \times moment arm} = F r \sin(\phi) = r \times F  \quad \tau_{net} = \sum \tau = I \alpha  \quad \vec{F}_{net} = \sum \vec{F} = m \vec{a}  
W_{tot} = \Delta K = K_f - K_i  \quad W = \tau_{net} \Delta \theta  \quad P = \tau \omega \ (\tau \text{ constant})  \quad K = K_{rot} + K_{cm}  \quad E_{mech} = K + U  

\Delta E_{mech} = 0 \ (\text{isolated system})  \quad \vec{v}_{cm} = \omega \vec{r} \ (\text{rolling, no slipping})  
I = r \times p = m \vec{v}  \quad L = \sum I \quad \tau_{net} = dL/dt  \quad L = I \omega  \quad I_{point mass} = m r^2 v \sin(\phi)  

For isolated systems:  \quad \tau_{net} = 0  \quad \Delta L = 0  \quad L_0 = \sum I_0 \omega_0 = L_f = \sum I_f \omega_f  

Equilibrium:  \Sigma \text{forces} = 0  \quad \Sigma \text{torques} = 0.  \quad \text{If net force on a system is zero, then the net torque is the same for any chosen rotation axis.}  \quad \text{CG definition: point about which torques due to gravity alone add to zero.}  

Physics 105:

F_g = mg  \quad g = 9.8 \text{ m/s}^2  \quad 1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}  \quad 1 \text{ kg} = 1000 \text{ g}  

v = v_o + at  \quad x - x_o = v_o t + \frac{1}{2} a t^2  \quad v^2 - v_o^2 = 2a(x - x_o)  \quad x - x_o = \frac{1}{2}(v + v_o) t  
F_{net} = m a  \quad \Sigma \vec{F} = m \vec{a}  \quad F_{s,max} = \mu_s N  \quad F_k = \mu_k N  \quad \text{incline:} \quad W_{mgx} = m g \sin(\theta)  \quad W_{mgy} = m g \cos(\theta)  
F_r = m a_r = m v^2/r  \quad a_r = v^2/r  \quad f = 1/T  \quad T = (2\pi/v)  \quad \text{Impulse:}  \quad F_{av} \Delta t = m \vec{v}_f - m \vec{v}_i  

Momentum conserved if net impulse = 0.  \quad \text{Then}  \ (\Sigma m \vec{v})_{initial} = (\Sigma m \vec{v})_{final}  

Work:  \quad W = F d \cos(\theta)  \quad W_{grav} = -mg(y-y_0)  \quad W_{spring} = -k(x^2-x_0^2)  \quad W_{frict} = -F_k d  \quad W_{tot} = K_f - K_i  

U_g = mg(y-y_0)  \quad \text{spring:} \quad F = -kx  \quad U_s = \frac{1}{2} k x^2  \quad \text{KE} = \frac{1}{2} m v^2  
W_{nc} = K_f - K_i + U_g + U_{si} + W_{frict} = K_f + U_g + U_{si}  

Mass center:  \quad x_{cm} = \Sigma (x_i m_i) / \Sigma m_i, \quad \text{similarly for} \ Y_{cm}, \ Z_{cm}  

Vectors:

Components:  \quad a_x = a \cos(\theta)  \quad a_y = a \sin(\theta)  \quad a = a_i + a_j  \quad a = \sqrt{a_x^2 + a_y^2}  \quad \theta = \tan^{-1}(a_y/a_x)  
Addition:  \quad \vec{a} + \vec{b} = \vec{c}  \quad \text{implies}  \quad c_x = a_x + b_x  \quad c_y = a_y + b_y  

Dot product:  \quad \vec{a} \cdot \vec{b} = a_x b_x + a_y b_y  \quad \text{unit vectors:} \quad \vec{i}_0 i = j  \quad j = k  \quad k = \vec{i}  \quad i = j = k  \quad \vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0  

Cross product:  \quad | \vec{a} \times \vec{b} | = |a| \sin(\theta)  \quad \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}  \quad \vec{a} \times \vec{a} = 0  \quad \text{always.}  \quad \vec{c} = \vec{a} \times \vec{b} \quad \text{is perpendicular to} \ \vec{a} - \vec{b} \ \text{plane}  
\vec{i} \times \vec{j} = \vec{k}  \quad \vec{j} \times \vec{k} = \vec{i}  \quad \vec{k} \times \vec{i} = \vec{j}  \quad \text{etc.}