

Signature \_\_\_\_\_

Name (Print): \_\_\_\_\_

4 Digit ID: \_\_\_\_\_ Section: \_\_\_\_\_

**Instructions:**

- Questions 1 through 10 are multiple choice questions worth 5 points each. Answer each of them on the Scantron sheet using #2 pencil. Answer all the questions as there is no penalty for guessing. You will need to do calculations on the exam paper for most of the questions and you may use the back for extra space.
- Questions A and B are workout problems worth 25 points each. Answer them on the exam booklet and show ALL work, otherwise there is no way to give partial credit.
- Be sure your name and section number are on both the Scantron form and the exam booklet. Also write your name, Id., and section at the top of each page with long answer questions A and B on them.
- You may bring and use your own formula sheet, using both sides of an 8.5 x 11 sheet or two 5x8 cards. A default formula sheet is also provided (see final page of this booklet). Make sure to bring your own calculator: sharing of calculators is not permitted.
- As you know, NJIT has a zero-tolerance policy for ethics code violations. Students are not to communicate with each other once the exam has started. All cell phones, pagers, or similar electronic devices should be turned off.
- If you have questions or need something call your proctor or instructor.

1. When solving rigid body equilibrium problems, the vector sum of the **gravitational** forces acting on individual parts of a body can **always** be replaced by a single force acting at:

A) the geometrical center      B) the mass center      **C) the center of gravity**  
D) the rotation axis      E) any of the above

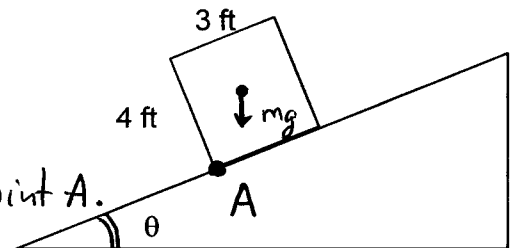
2. A rectangular box ~~4~~ feet high and ~~3~~ feet wide is stationary on a ramp as shown. There is enough friction so that it cannot slide and the density is uniform. At what **angle**  $\theta$  does the box begin to tip over?

A) **36.9°**      B) 30.9°      C) 41.4°      D) 48.6°  
E) Cannot be determined from the information given

the box tips over when the gravitational force  $mg$  has positive torque with respect to point A.

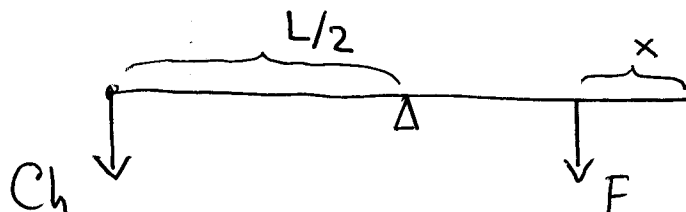
$$\tan \theta = 3/4 ; \quad \theta = \tan^{-1} 0.75$$

$$\theta = 36.9^\circ$$



3. A see saw is a playground toy consisting of a uniform beam pivoted about its center. A 25 kg child is sitting on the left hand end of see-saw that is 12 meters long. How far from the right hand end should his 75 kg father sit in order to have the see-saw balance (remain horizontal)?

A) 3 m      **B) 4 m**      C) 0 m      D) 6 m      E) 5 m



$$\sum \vec{\tau} = 0$$

$$W_{ch} \cdot \frac{L}{2} - W_F \cdot \left( \frac{L}{2} - x \right) = 0$$

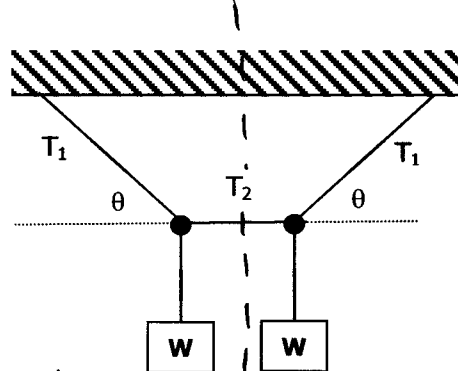
$$W_{ch} \cdot \frac{L}{2} = W_F \left( \frac{L}{2} - x \right)$$

$$\frac{W_{ch}}{W_F} \cdot \frac{L}{2} = \frac{L}{2} - x$$

$$x = \frac{L}{2} - \frac{W_{ch}}{W_F} \cdot \frac{L}{2} = 6\text{ m} - \frac{1}{3} \cdot 6 = 4\text{ m}$$

4. Find the tension  $T_2$  in the horizontal rope shown in the figure, where  $\theta = 50^\circ$  and  $W = 80 \text{ N}$ . The arrangement is symmetrical as indicated.

A) 51.4 N    **B) 67.1 N**    C) 39.4 N  
D) 28.6 N    E) 18.7 N



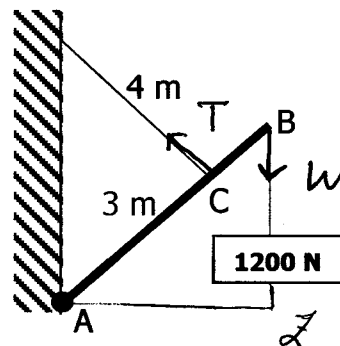
$$\sum \vec{F} = 0 \quad \left. \begin{array}{l} \textcircled{x}: T_1 \cdot \cos \theta = T_2 \\ \textcircled{y}: T_1 \cdot \sin \theta = W \end{array} \right\} \Rightarrow$$

$$\Rightarrow T_2 = \frac{W}{\tan \theta} = \frac{80 \text{ N}}{\tan 50^\circ} \approx \underline{67 \text{ N}}$$

5. A block that weighs 1200 N is suspended as shown from the end of beam AB, which is weightless and is hinged to the wall at A. The beam is 5 m long. The 4-m-long cable is attached to point C, which is 3 m from point A along the beam. The cable is perpendicular to the beam. The tension in the cable is closest to:

A) 2400 N    B) 320 N  
D) 900 N    E) 400 N

**C) 1600 N**



$$\sum \vec{\tau} = 0 \text{ for point A}$$

$$T \cdot L_{AC} - W \cdot L_{Ax} = 0$$

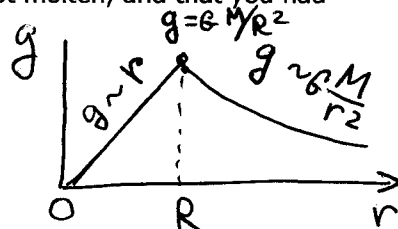
$$T = W \cdot \frac{L_{Ax}}{L_{AC}} = 1200 \text{ N} \cdot \frac{4 \text{ m}}{3 \text{ m}} = \underline{1600 \text{ N}}$$

6. If you drilled a hole from the Earth's surface straight down toward its center, what would the acceleration of gravity be at a point halfway there? Assume the Earth is a perfect sphere, that the core is not molten, and that you had investors willing to pay for this project.

A. Zero    **B.  $g/2$**     C.  $g/4$

~~D.  $g/8$~~

E.  $g$



7. A space mission to visit the Earth's moon passes a point along the way where the Moon's gravitational field becomes stronger than the Earth's gravity. How far from the Earth is that point, expressed as a fraction of the average radius of the Moon's orbit?

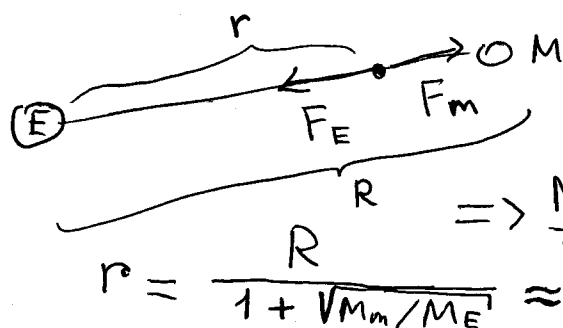
A) 0.75

B) 0.80

C) 0.85

**D) 0.90**

E) 0.95



$$\left. \begin{array}{l} F_E = G \frac{M_E \cdot m}{r^2} \\ F_M = G \frac{M_m \cdot m}{(R-r)^2} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \frac{M_E}{r^2} = \frac{M_m}{(R-r)^2} \Rightarrow \frac{(R-r)^2}{r^2} = \frac{M_m}{M_E}$$

$$r = \frac{R}{1 + \sqrt{M_m/M_E}} \approx \underline{0.9 R}$$

8. The acceleration due to gravity at the surface of the planet Krypton equals that on the surface of the Earth, but Krypton's radius is 4 times that of the Earth. What is the mass of Krypton in terms of the mass of the Earth,  $M_E$ ?

A) **16  $M_E$**

B)  $32 M_E$

C)  $64 M_E$

D)  $0.032 M_E$

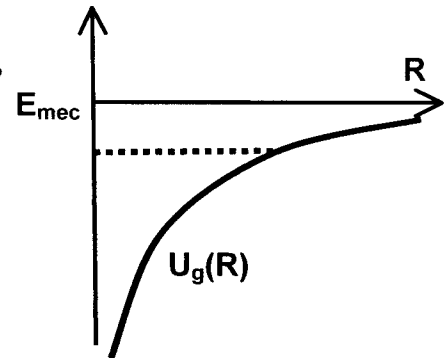
E)  $0.08 M_E$

$$g_E = G \frac{M_E}{R_E^2} ; \quad g_K = G \frac{M_K}{R_K^2}$$

$$g_E = g_K ; \Rightarrow G \frac{M_E}{R_E^2} = G \frac{M_K}{R_K^2} ; \Rightarrow M_K = M_E \cdot \frac{R_K^2}{R_E^2} = M_E \cdot 4^2 \cdot \frac{R_E^2}{R_E^2} = 16 M_E$$

9. The sketch at right shows the mechanical energy for a pair of masses that might be a planet with a smaller object moving near it. If the dashed line represents the total energy, which of the following statements are correct?

- ☒ 1. The masses can fly apart and never come together again.
- ☒ 2. There is just enough energy to reach orbit.
- ☒ 3. If there is a stable orbit, it will be an ellipse.
- ☒ 4. The potential energy is negative.
- ☒ 5. The total mechanical energy is positive.
- ☒ 6. The total mechanical energy is negative.
- ☒ 7. The orbit will be a hyperbola.
- ☒ 8. The particle's velocity is greatest at the right hand side of the sketch.
- ☒ 9. The vertical distance between horizontal lines represents escape velocity
- ☒ 10. There could be periodic or oscillating motion



A) 1, 7, 8

B) **3, 4, 6, 10**

C) 2, 9, 10

D) 1, 5, 7, 9

E) 4, 7, 9, 10

10. The Moon takes 27.2 days to revolve around the Earth in an orbit whose radius  $R_m$  is about 383,000 km. What would be the orbital radius for a satellite in geo-synchronous orbit, meaning its period equals 1 day and it therefore always appears at the same place in the sky?

A) 22, 000 km

B) 14,100 km

C) 2,700 km

D) **42,300 km**

E) 28,200 km

$$\frac{T_1^2}{R_1^3} = \frac{T_2^2}{R_2^3} ;$$

$$\frac{(27.2 \text{ days})^2}{(383,000 \text{ km})^3} = \frac{(1 \text{ day})^2}{R_2^3}$$

$$R_2 = 383,000 \cdot \frac{1}{\text{km} (27.2)^{2/3}} = 42 \times 10^3 \text{ km}$$

**Work-Out Problem A (25 pts, Show all equations and work):**

A certain rather dense planet has a radius  $R = 2000 \text{ km}$ . The gravitational acceleration on its surface is  $g_p = 5.0 \text{ m/s}^2$ .

(a) (6 pts) What is the **mass M** of this planet? **Answer:**

$$g = G \frac{M}{R^2} ; \Rightarrow M = g \cdot \frac{R^2}{G} = \frac{5 \frac{\text{m}}{\text{s}^2} \cdot (2 \cdot 10^6 \text{ m})^2}{6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}} = \frac{5 \cdot 4 \cdot 10^{12}}{6.67 \cdot 10^{-11}} \text{ kg} = \boxed{3.0 \cdot 10^{23} \text{ kg}}$$

(b) (6 pts) What is the **escape speed v** from the surface of the planet? **Answer:**

$$V_{\text{esc}} = \sqrt{2g \cdot R} = \sqrt{2 \cdot 5 \frac{\text{m}}{\text{s}^2} \cdot 2 \cdot 10^6 \text{ m}} = 4.5 \times 10^3 \text{ m/s}$$

another way to get it:

$$V_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 3.0 \cdot 10^{23}}{2 \cdot 10^6}} \frac{\text{m}}{\text{s}} = 4.5 \times 10^3 \text{ m/s}$$

(c) (7 pts) What is the **total mechanical energy E** of a 1 kg mass at the surface of the planet if it is given an upward speed of 2,000 m/s? **Answer:**

$$E = \frac{1}{2} m v^2 - G \frac{M m}{R} = \frac{1}{2} \cdot 1 \text{ kg} \cdot (2 \cdot 10^3 \text{ m/s})^2 - 6.67 \cdot 10^{-11} \cdot \frac{3 \cdot 10^{23} \text{ kg} \cdot 1 \text{ kg}}{2 \cdot 10^6 \text{ m}} = 2 \cdot 10^6 \text{ J} - 10 \cdot 10^6 \text{ J} = -8 \cdot 10^6 \text{ J}$$

(d) (6 pts) How far **from the surface** will the mass go if it leaves the planet with a speed as in part c) above?

**Answer:**

$$E_i = E_f$$

$$E_i = \frac{1}{2} m v^2 - G \frac{M m}{R} = -8 \cdot 10^6 \text{ J} \quad (\text{see part "c"})$$

$$E_f = -G \frac{M \cdot m}{R+h} ; -G \frac{M \cdot m}{R+h} = E_i = -8 \cdot 10^6 \text{ J}$$

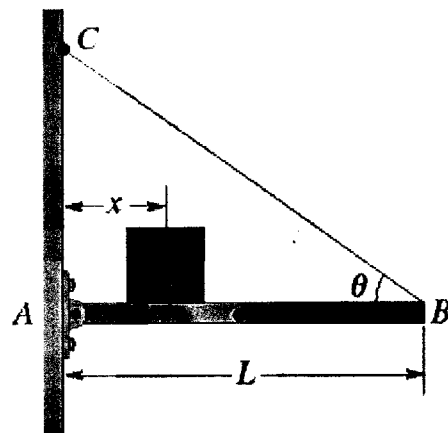
$$R+h = \frac{G \cdot M \cdot m}{|E_i|} ; h = \frac{G \cdot M \cdot m}{|E_i|} - R$$

$$h = \frac{6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot 3 \cdot 10^{23} \text{ kg} \cdot 1 \text{ kg}}{8 \cdot 10^6 \text{ J}} - 2 \cdot 10^6 \text{ m} =$$

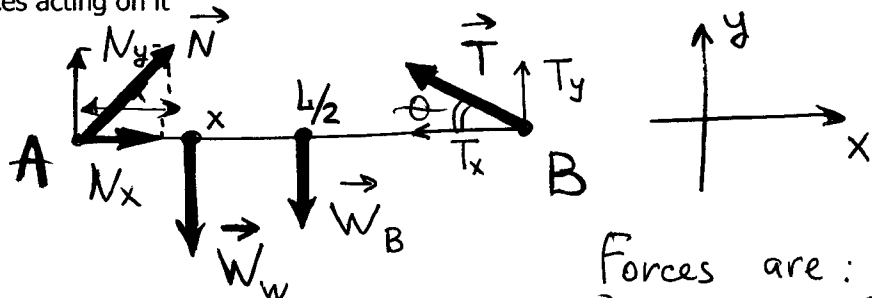
$$= 2.5 \cdot 10^6 \text{ m} - 2 \cdot 10^6 \text{ m} = \boxed{0.5 \cdot 10^6 \text{ m}} = \underline{\underline{500 \text{ km}}}$$

**Work-Out Problem B (25 pts, Show all equations and work):**

In the figure, a uniform horizontal bar AB of weight  $W_B = 250 \text{ N}$  and length  $L = 16 \text{ m}$  is hinged to a vertical wall at A and is supported at B by a thin wire BC that makes an angle  $\theta = 30^\circ$  with the horizontal. A load of weight  $W_w = 350 \text{ N}$  can be moved anywhere along the bar; its position is defined by the distance  $x$  from the wall to its center of mass.



- a) (5 pts) Draw and label the free-body diagram for the beam, showing all of the forces acting on it



Forces are:  $\vec{W}_w$ ,  $\vec{W}_B$ ,  $\vec{T}$ , and  $\vec{N}$   
 $\vec{W}_w$  and  $\vec{W}_B$  have only  $\odot$  comp.  
 $\vec{T}$  and  $\vec{N}$  have both  $\odot$  and  $\otimes$  comp.

- b) (10 pts) Set up the equations you will need using the First and Second conditions for equilibrium.

$$\begin{aligned} \otimes: & N_x - T_x = 0; & N_x - T \cdot \cos \theta &= 0 \\ \odot: & N_y + T_y - W_w - W_B = 0; & N_y + T \cdot \sin \theta - W_w - W_B &= 0 \\ \sum \vec{\tau}: & T \cdot \sin \theta \cdot L - W_w \cdot x - W_B \cdot L/2 = 0 \end{aligned}$$

point A

- c) (5 pts) Solve the equations for the tension in the wire as a function of  $x$ . As the weight moves outward from the wall the tension should grow. Find the tension when  $x = 8 \text{ m}$ .

$$\sum \vec{\tau}: \quad T \cdot \sin \theta \cdot L - W_w \cdot x - W_B \cdot L/2 = 0$$

point A

$$T = \frac{W_w \cdot x + W_B \cdot L/2}{\sin \theta \cdot L}$$

$$T = \frac{W_w}{\sin \theta \cdot L} \cdot x + \frac{W_B}{2 \cdot \sin \theta}$$

for  $x = 8 \text{ m}$   $T = \frac{350 \text{ N} \cdot 8 \text{ m}}{\sin(30^\circ) \cdot 16 \text{ m}} + \frac{250 \text{ N}}{2 \cdot \sin 30^\circ} = \boxed{600 \text{ N}}$