Lecture 10

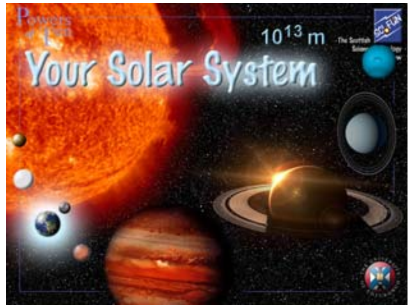
Physics 106

Spring 2006

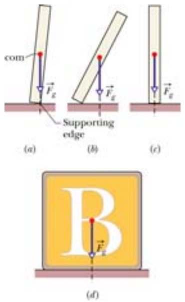
Review for the 3rd Common QZ

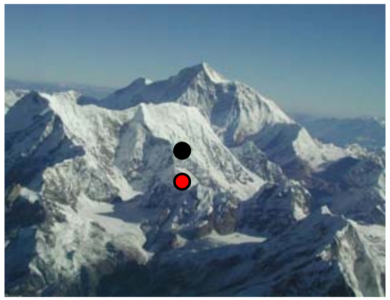
http://web.njit.edu/~sirenko/

Equilibrium and Gravity



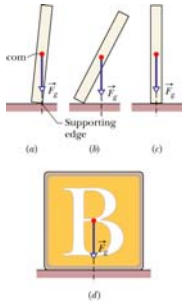
COG vs. COM

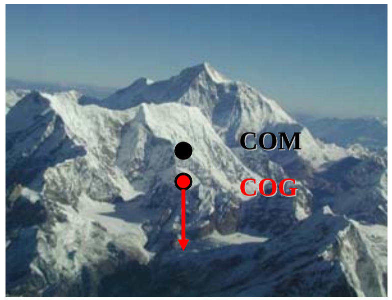




The gravitational force F_g on a body effectively acts on a single point, called the center of gravity (COG) of the body. If g is the same for all elements of the body, then the body's cog is coincident with the body's center of mass (COM).

COG vs. COM





The gravitational force F_g on a body effectively acts on a single point, called the center of gravity (COG) of the body. If g is the same for all elements of the body, then the body's cog is coincident with the body's center of mass (COM).

Equilibrium

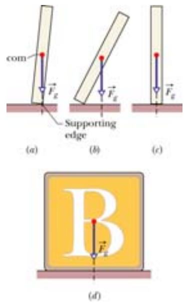
Balance of Forces:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

Balance of Torques:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{P} = 0$$

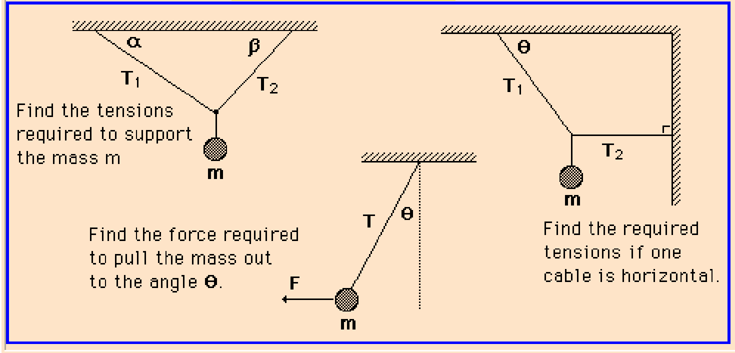


1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all the external torques that act on the body, measured about any possible point, must be zero.
3. The linear momentum $\underline{P}$ of the body must be zero.
4. The gravitational force $\underline{F}_g$ on a body effectively acts on a single point, called the center of gravity (cog) of the body. If g is the same for all elements of the body, then the body's cog is coincident with the body's center of mass.

Equilibrium

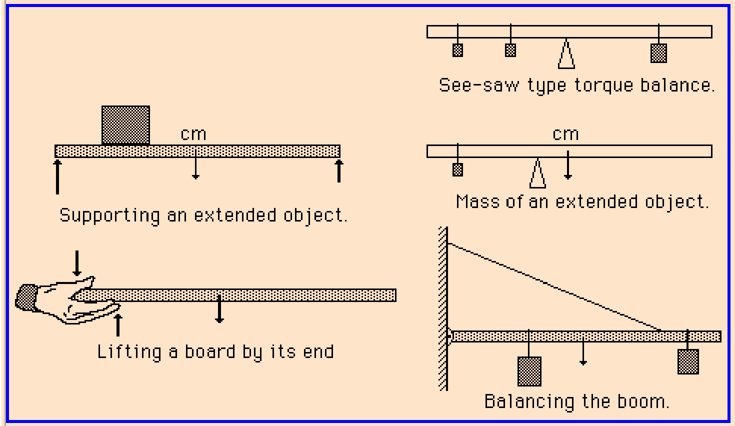
Force Equilibrium Examples

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

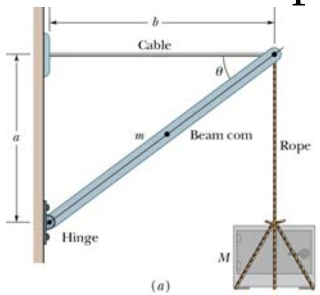


Torque Equilibrium Examples

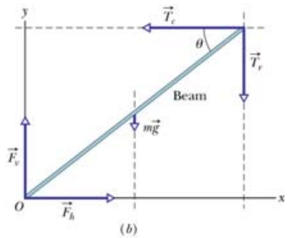
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$



Sample Problem XIII – 3



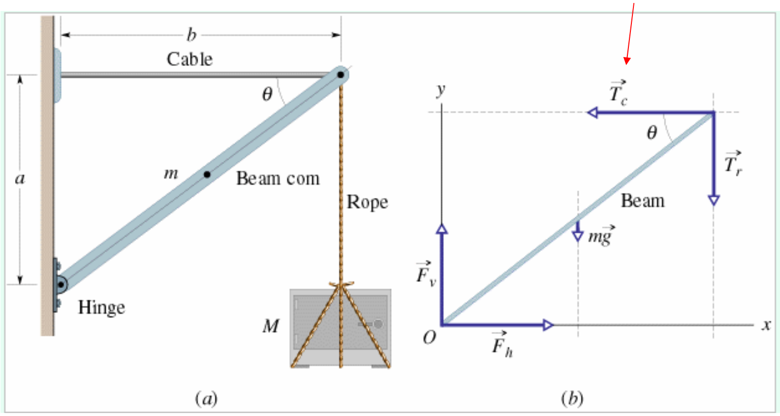
A safe of mass M = 430 kg is hanging by a rope from a boom with dimensions a = 1.9 m and b = 2.5 m. The boom consists of a hinged beam and a horizontal cable that connects the beam to a wall. The uniform beam has a mass m = 85 kg. The masses of the cable and the rope are negligible.



a) What are the tension T_c in the cable? In other words, what is the magnitude of the force $\underline{T}_c$ on the beam from the cable?

b) Find the magnitude F of the net force on the beam from the hinge.

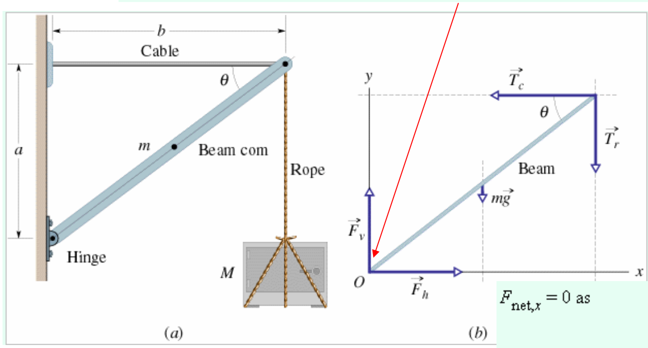
What is the tension T_c in the cable?



$$(a)(T_c) - (b)(T_r) - (\tfrac{1}{2}b)(mg) = 0.$$

$$T_c = \frac{gb(M + \frac{1}{2}m)}{a}$$
$$= \frac{(9.8\ \mathrm{m/s^2})(2.5\ \mathrm{m})(430\ \mathrm{kg} + 85/2\ \mathrm{kg})}{1.9\ \mathrm{m}}$$
$$= 6093\ \mathrm{N} \approx 6100\ \mathrm{N}.$$

Find the magnitude F of the net force on the beam from the hinge.



$F_{\mathrm{net},x} = 0$ as

$$F_h - T_c = 0.$$

$$F_h = T_c = 6093\ \mathrm{N}.$$

$$F_v - mg - T_r = 0.$$

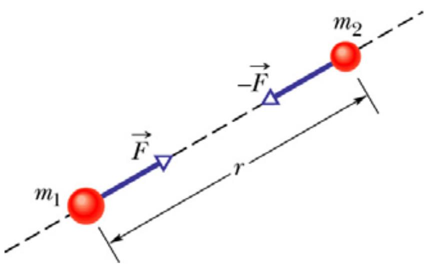
$$F_v = (m + M)g = (85\ \mathrm{kg} + 430\ \mathrm{kg})(9.8\ \mathrm{m/s^2})$$
$$= 5047\ \mathrm{N}.$$

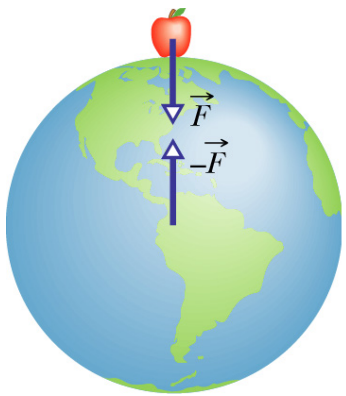
$$F = \sqrt{F_h^2 + F_v^2}$$
$$= \sqrt{(6093\ \mathrm{N})^2 + (5047\ \mathrm{N})^2} \approx 7900\ \mathrm{N}.$$

Newton's Law of Gravitation (known since 1665)

$$F = G\frac{m_1 m_2}{r^2}$$

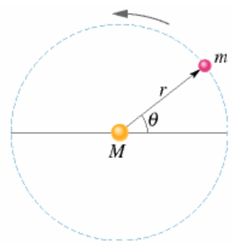
$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$





Planets and Satellites: Kepler's Laws

THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

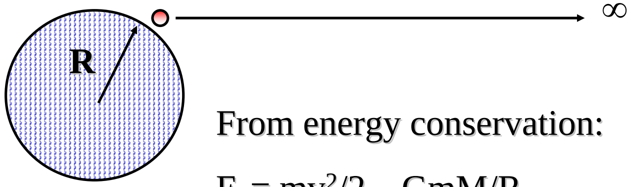


$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Escape Speed:

$$F = G\frac{m_1 m_2}{r^2}$$



From energy conservation:

$E_1 = mv^2/2 - GmM/R$

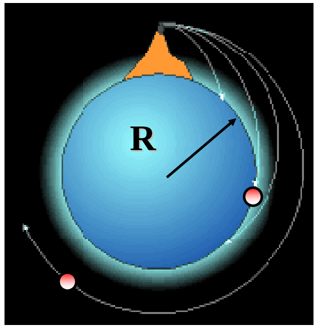
$E_2 = 0$ (velocity is small)

$v^2 = 2GM/R = 2gR$

$v = (2GM/R)^{1/2} \approx 11{,}200$ m/s

First Satellite Speed:

$$F = G\frac{m_1 m_2}{r^2}$$



"Newton's cannon"

in 1687 in "*Principia Mathematica*"

$v_{\text{satellite}} \approx (gR)^{1/2}$

$v_{\text{satellite}} \approx 8{,}000$ m/s

Potential and Kinetic Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

Kinetic Energy for the orbital motion

$$F = G\frac{Mm}{r^2} = m\frac{v^2}{r} \quad \Rightarrow \quad K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$

Total Energy

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$