Lecture 2
Physics 106
Spring 2006

Rotational dynamics:

- Kinetic Energy of rotation,
- Rotational inertia,
- Torque,
- Cross product.

http://web.njit.edu/~sirenko/

Required Home Work

Specific information for the UT homework system:
 UT Guest ID Registration:
 https://utdirect.utexas.edu/nlogon/eid_suite/essentials/create_eid.WBX?portal_role=O
 UT HW Student Instructions:
 https://hw.utexas.edu/bar/studentGuestEID.html
 Student Login Page (Univ. of Texas):
 https://utdirect.utexas.edu/security-443/UTEIDLogon.wb
 UT EID Home Page (Forgotten Password):
 https://utdirect.utexas.edu/nlogon/eid_suite/general/

Your instructor will announce the 5 digit course number you need to use when you register for Physics 106 in the UT system.

If you already have a UT Guest login ID and password, you can continue to use it.

Fill out the following for your own future reference, and keep it someplace where you can find it:

§ Unique course number to be announced by instructors: 10615
§ Your Login ID on the UT system (generated when you register with UT; case sensitive!): ___________
§ Your own password (selected upon registration with UT; confidential!): ____________

Note that NJIT instructors can not access your password.

Rotation:

Angular Displacement
Angular Velocity
Angular Acceleration

\[
\begin{align*}
\theta, & \quad \omega, \quad \alpha \\
\text{Linear Equation} & \quad \text{Missing Variable} & \quad \text{Angular Equation} \\
\omega & = \omega_0 + at & \omega & = \omega_0 + at \\
\theta - \theta_0 & = \omega t & \theta - \theta_0 & = \omega t + \frac{1}{2} \alpha t^2 \\
\frac{x - x_0}{v} & = \frac{\omega}{\omega_0} & \frac{x - x_0}{v_0} & = \frac{\omega}{\omega_0} + \frac{1}{2} \alpha t^2 \\
\frac{x - x_0}{t} & = \frac{v_0}{t} & \frac{x - x_0}{t} & = \frac{v}{t} + \frac{1}{2} \alpha t^2 \\
\frac{x - x_0}{\frac{1}{2}(v_0 + v)t} & = \frac{\alpha}{\omega_0} & \frac{x - x_0}{\frac{1}{2}(v + v_0)t} & = \frac{\alpha}{\omega} \\
\frac{x - x_0}{\frac{1}{2}at^2} & = \frac{v}{v_0} & \frac{x - x_0}{\frac{1}{2}at^2} & = \frac{v}{v_0} + \frac{1}{2} \alpha t^2 \\
\end{align*}
\]

Acceleration in Circular Motion:

\( \omega \) increases with time \( \alpha > 0 \)

\[
\begin{align*}
a_T & = r \alpha & a_c & = \frac{v^2}{r} = r \omega^2 \\
a & = a_c + a_T & a & = (a_c^2 + a_T^2)^{1/2}, \tan \phi = \frac{a_T}{a_c}
\end{align*}
\]

\( \omega \) decreases with time \( \alpha < 0 \)
Our linear velocity with respect to the Sun

\[ R_{\text{E-S}} = 1.5 \times 10^{11} \text{ m} \quad T = 1 \text{ year} \]

\[ v = \omega R \]
\[ \omega = \frac{2 \pi}{T} \]

When do we move faster?
(a) Day
(b) Night

What is the velocity difference between Day and Night at the Equator line?

\[ |V_{\text{day}} - V_{\text{night}}| / V_{\text{average}} \]

(a) 0.00008
(b) 0.015
(c) 0.03
(d) 0.3
(e) 100 %

Show work!

Kinetic Energy of Rotation

\[ K = \frac{1}{2} m v^2 \quad \text{Point mass (no rotation); } v \text{ of the COM} \]

\[ K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 + \cdots \]

System of particles or an object

\[ K = \sum \frac{1}{2} m_i v_i^2, \quad v = \omega r \]

\[ K = \sum \frac{1}{2} m_i (\omega r_i)^2 = \frac{1}{2} \left( \sum m_i r_i^2 \right) \omega^2 \]

\[ I = \sum m_i r_i^2 \quad \text{(rotational inertia)} \]
\[ K = \frac{1}{2} I \omega^2 \quad \text{(radian measure)} \]

Rotational Inertia

\[ K = \frac{1}{2} I \omega^2 \quad \text{(radian measure)} \]

\[ I(a) \neq I(b) \]

\[ I - \text{rotational equivalent of mass } m \]

Main difference between \( m \) and \( I \):
Rotational Inertia depends on the direction of rotation!

Rotational Inertia Of Point Mass

For a single particle \( I = mr^2 \)
(all mass at same \( r \))

A single particle

The same particle farther out

\[ I = (4 \text{ kg})(1 \text{ m})^2 = 4 \text{ kg m}^2 \]
\[ I = (4 \text{ kg})(2 \text{ m})^2 = 16 \text{ kg m}^2 \]

Four times the rotational “mass”
### Parallel-Axis Theorem for Rotational Inertia

$$I = I_{\text{com}} + Mh^2 \quad \text{(parallel-axis theorem)}.$$

Calculate $I_{\text{com}}$ for the axis going through the COM.

Use Parallel-Axis Theorem to calculate $I$.

### Example: Rotational Inertia

$$I = \sum m_i r_i^2 = (m)\left(\frac{L}{2}\right)^2 + (m)\left(\frac{L}{2}\right)^2 = \frac{1}{2}mL^2.$$

$$I = m(0)^2 + mL^2 = mL^2$$

$$I = I_{\text{com}} + Mh^2 = \frac{1}{2}mL^2 + (2m)\left(\frac{L}{2}\right)^2 = mL^2.$$

Not only the force is important, but how you apply it!
**Torque:** \( \tau \)

The value of torque:
\[ \tau = r \cdot F \cdot \sin \phi \]
\[ \phi = 0 \Rightarrow \tau = 0 \]
\[ \phi = \pi/2 \Rightarrow \tau = r \cdot F \text{ (max)} \]

In vector notation form:
\[ \tau = [r \times F] \]

**Vector Cross Product**

The value of cross product:
\[ c = a \cdot b \cdot \sin \phi \]
\[ \phi = 0 \Rightarrow c = 0 \]
\[ \phi = \pi/2 \Rightarrow c = a \cdot b \text{ (max)} \]

Cross product is maximized when vectors are perpendicular:
\[ \vec{a} \times \vec{b} = (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k} \]

Order is important:
\[ \vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \]

**Newton’s Second Law for Rotation**

Torque causes the change in \( \omega \)
\[ \tau_{\text{net}} = \mathbf{I} \cdot \alpha \]
Rotational equivalent of \( F = ma \)

\[ F_r = m \alpha_r \]
\[ \tau = F_r r = m \alpha r \]
\[ \tau = m (\alpha r) r = (mr^2) \alpha \]

**Rotational Analogy to Linear Motion**

Translation | Rotation
--- | ---
position | \( x \) | \( \theta \)
velocity | \( v = \frac{dx}{dt} \) | \( \omega = \frac{d\theta}{dt} \)
acceleration | \( a = \frac{dv}{dt} \) | \( \alpha = \frac{d\omega}{dt} \)
mass | \( m \) | \( I = \Sigma m_i r_i^2 \)
Kinetic Energy | \( K = \frac{1}{2} m v^2 \) | \( K = \frac{1}{2} I \omega^2 \)
Force | \( F = ma \) | \( \tau_{\text{net}} = \mathbf{I} \cdot \alpha \)
A Falling Stuntman

Stuntmen sometimes need to fall large distances
Without getting hurt!

But it has to look like they fall

Hollow Cylinder (R)
Spindle (r)

\[ I_{\text{cyl}} = MR^2 \]
\[ \tau = r \cdot T \]
\[ \alpha = \frac{r \cdot T}{MR^2} \]
\[ a = \alpha r \]

The rope is around the spindle

\[ ma = mg - T \]
\[ a = g/(1+MR^2/mr^2) \]

For \( m = 70 \text{ kg}, M = 10 \text{ kg}, \)
\( r = 0.1 \text{ m and } R = 0.5 \text{ m} \)
\( a = 9.8/(1+25/7) \approx 2 \text{ m/s}^2 \)

Homework

See the Physics 106 Course Syllabus

U of Texas HW is required

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