Lecture 13

REVIEW

Physics 106
Spring 2006

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What should we know ?

- **Vectors**
  addition, subtraction, scalar and vector multiplication
- **Trigonometric functions**
  $\sin\theta$, $\cos\theta$, $\tan\theta$, $\theta = \tan^{-1}(a/b)$, *etc.*
- **Integration and Derivatives (basic concepts)**
  $2x = (x^2)'$
- **SI Units**
- **Newton's Laws**
  $\mathbf{F} = m\mathbf{a}$ $\mathbf{F}_{12} = -\mathbf{F}_{21}$
- **Energy Conservation**
  Kinetic Energy, Potential Energy, and Work
- **Circular motion and Centripetal Force**
  $a^c = v^2/R$

What should we know ?

- ***Angular variables***
  angular velocity, angular acceleration, etc.
- **Rotational Inertia**
- **Kinetic Energy of Rotation**
- ***Angular Momentum and Torque***
- **Newton's law of Gravitation and planetary motion**
- **Satellite orbits, Potential and Kinetic Energy of a satellite**
- **Oscillations and Pendulums**

Newton's Laws

I. If no net force acts on a body, then the body's velocity cannot change.

II. The net force on a body is equal to the product of the body's mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction ($\mathbf{F}_{12} = -\mathbf{F}_{21}$)

Force is a vector
Force has direction and magnitude
Mass connects Force and acceleration;

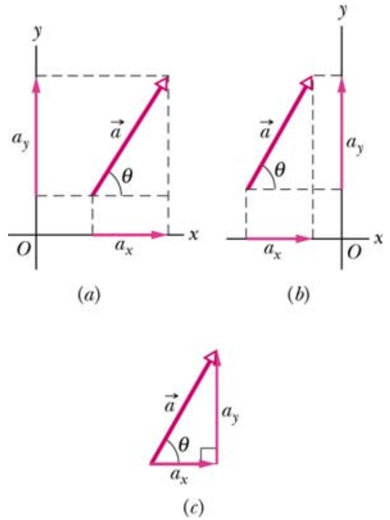
$$\vec{F}_{tot} = 0 \Leftrightarrow \vec{a} = 0 \text{ (constant velocity)}$$

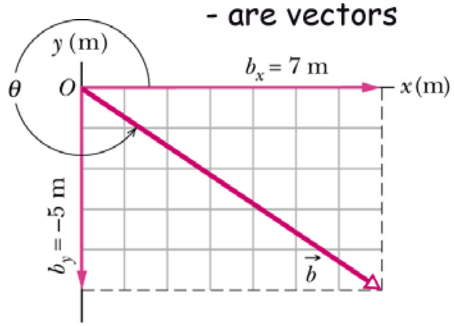
$$\vec{F}_{tot} = m\vec{a} \text{ for any object}$$

$$F_{tot,x} = ma_x \qquad F_{tot,y} = ma_y \qquad F_{tot,z} = ma_z$$

Components of Vectors:

- aligned along axis
- add to give vector
- are vectors





$a_x = a\cos\theta$ and $a_y = a\sin\theta$

$a = \sqrt{a_x^2 + a_y^2}$ and $\tan\theta = \dfrac{a_y}{a_x}$

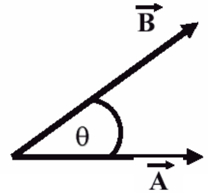
Length (Magnitude)

Vector Multiplication

Dot product

$\vec{A} \bullet \vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

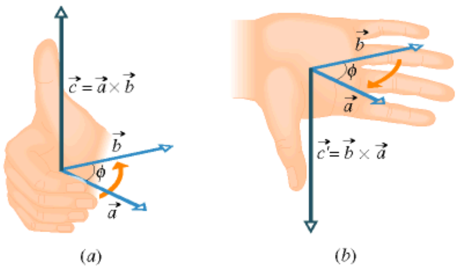
θ is the angle between the vectors if you put their tails together



$\vec{A} \bullet \vec{B} = \vec{B} \bullet \vec{A}$

since cos(θ) =cos(-θ)

Vector Cross Product



The value of *cross product:*

c = a·b·sinϕ

ϕ=0 ➔ c = 0

ϕ=π/2 ➔ c = a·b (max)

Cross product is maximized when vectors are perpendicular

$$\vec{a} \times \vec{b} = (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - b_za_x)\hat{j} + (a_xb_y - b_xa_y)\hat{k}$$

Order is important: $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

TABLE 2-1 Equations for Motion with Constant Acceleration[a]

Equation Number	Equation	Missing Quantity
2-11	$v = v_0 + at$	$x - x_0$
2-15	$x - x_0 = v_0t + \frac{1}{2}at^2$	v
2-16	$v^2 = v_0^2 + 2a(x - x_0)$	t
2-17	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
2-18	$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a] Make sure that the acceleration is indeed constant before using the equations in this table.

What does zero mean ?

- $t = 0$ beginning of the process
- $x = 0$ is arbitrary; can set where you want it
- $x_0 = x(t=0)$; position at t=0; do not mix with the origin

- $v(t) = 0$ x does not change $x(t) - x_0 = 0$
- $v_0 = 0$ $v(t) = at$; $x(t) - x_0 = at^2/2$
- $a = 0$ $v(t) = v_0$; $x(t) - x_0 = v_0 t$

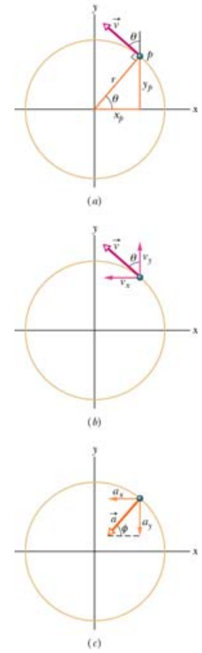
- $a \neq 0$ $v(t) = v_0 + at$; $x(t) - x_0 = v_0 t + at^2/2$

help: $t = (v - v_0)/a$ $x - x_0 = \frac{1}{2}(v^2 - v_0^2)/a$

$a = (v - v_0)/t$ $x - x_0 = \frac{1}{2}(v + v_0)t$

- Acceleration and velocity are positive in the same direction as displacement is positive

Uniform Circular Motion

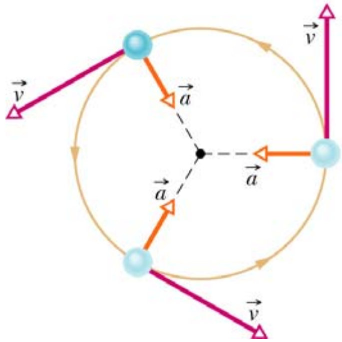


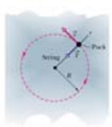
Centripetal acceleration

$$a = \frac{v^2}{r}$$

Period

$$T = \frac{2\pi r}{v}$$

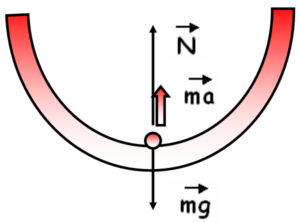




$ma_c = mv^2/R = \Sigma F$

(all forces along the direction towards the center)

- Gravitational Force: $\vec{mg}$
  down to the ground $F = G\frac{m_1 m_2}{r^2}$
- Tension Force: $\vec{T}$
  along the string
- Normal Force: $\vec{N}$
  perpendicular to the support
- Static Friction Force
  maximum value $F_{fr}^{max} = \mu_{st} N$



$ma = N - mg$

$ma = mv^2/R$

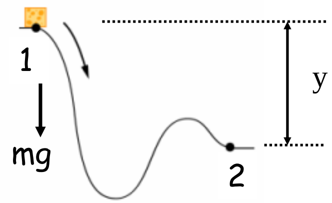
Kinetic Energy: $K = \frac{1}{2}mv^2$

Potential Energy:

$\Delta U = -W$

- Gravitation: $U = mgy$
- Elastic (due to spring force): $U = \frac{1}{2}kx^2$

$U = mgy$



$K = \frac{1}{2}mv^2$

$E_{mec} = K + U$

U → K

Conservation of Mechanical Energy

$K_2 + U_2 = K_1 + U_1$

Kinetic Energy: $K = \frac{1}{2}mv^2$

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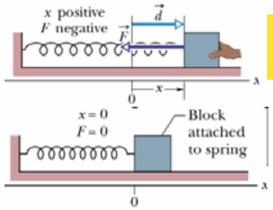


1 $U = \frac{1}{2}kx^2$ K = 0

2 U = 0 $K = \frac{1}{2}mv^2$

U ← → K

$E_{mec} = K + U$

Conservation of Mechanical Energy

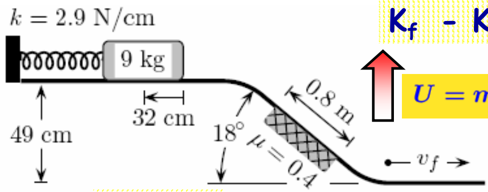
$K_2 + U_2 = K_1 + U_1$

Examples for Energy Conservation

Kinetic Energy changes $K = \frac{1}{2}mv^2$
+ Gravitational Potential Energy $U = mgy$
+ Elastic Potential Energy $U = \frac{1}{2}kx^2$

Total Mechanical Energy = *Const.*

$U = \frac{1}{2}kx^2$ → $K = \frac{1}{2}mv^2$



$K_f - K_i = W = mgy - |W_{friction}|$

$U = mgy$

$K = \frac{1}{2}mv^2$

$W_{friction}$

$E_f - E_i = -|W_{friction}| = f_k \cdot d \cdot \cos 180° = -mg\,\mu \cdot d \cdot \cos 18°$

Linear Momentum

Particle: $\vec{p} = m\vec{v}$

System of Particles: $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots$

Extended objects: $\vec{P} = M\vec{v}_{com}$

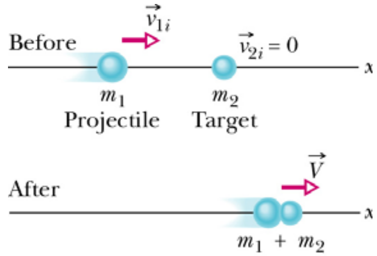
Relation to Force: $\vec{F}_{tot} = m\vec{a}$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\vec{F}_{net} = \frac{d\vec{P}}{dt}$

Completely Inelastic Collision
Collisions in 1D

Conservation of Linear Momentum works !

$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$



$$m_1 v_{1i} = (m_1 + m_2)V$$
$$V = \frac{m_1}{m_1 + m_2} v_{1i}$$

Example: Two equal objects, one initially at rest

$mv_i = 2mv_f$ ⟶ $v_f = v_i/2$

Final Kinetic Energy $= \frac{1}{2}(2m)(v_i/2)^2$
$= \frac{1}{4}m(v_i)^2$

Half the original Kinetic Energy

Rotational Kinematics:

Linear Displacement	←→	Angular Displacement
Linear Velocity	←→	Angular Velocity
Linear Acceleration	←→	Angular Acceleration

$\vec{x}, \vec{v}, \vec{a} \Leftrightarrow \vec{\theta}, \vec{\omega}, \vec{\alpha}$

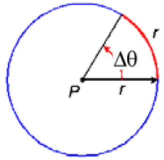
If α is constant:

$$\theta(t) = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega = \frac{d\theta}{dt} \rightarrow \quad \omega(t) = \omega_0 + \alpha t$$

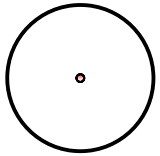
$$\text{combine: } 2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$$

Radian



1 Radian = 180° / π ≈ 57.3°

The arc length is equal to the radius $\Delta s = r\Delta\theta$

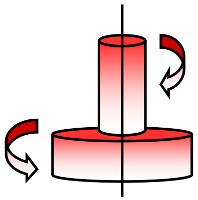


Circle: 360° = 2π radians ≈ 6.283 radians

½ Circle: 180° = π radians ≈ 3.1415 radians

- Radians = degrees × (π /180)
- 1 degree = π /180=0.0174532925 radians.
- 180°= 3.14156 radians
- 90° = 1.5708 radians
- 45° = 0.7854 radians

Rotation:



Angular Displacement
Angular Velocity
Angular Acceleration

$\theta, \vec{\omega}, \vec{\alpha}$

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Kinetic Energy of Rotation

$K = \frac{1}{2} mv^2$ Point mass (no rotation); v of the COM

$$K = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 + \tfrac{1}{2}m_3 v_3^2 + \cdots = \sum \tfrac{1}{2} m_i v_i^2,$$

System of particles or an object

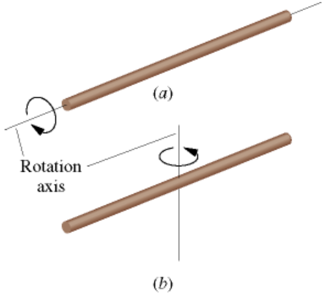
$v = \omega r$

$$K = \sum \tfrac{1}{2} m_i (\omega r_i)^2 = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

$I = \sum m_i r_i^2$ (rotational inertia)

$K = \frac{1}{2} I\omega^2$ (radian measure)

Rotational Inertia



$K = \frac{1}{2} I \omega^2$ (radian measure)

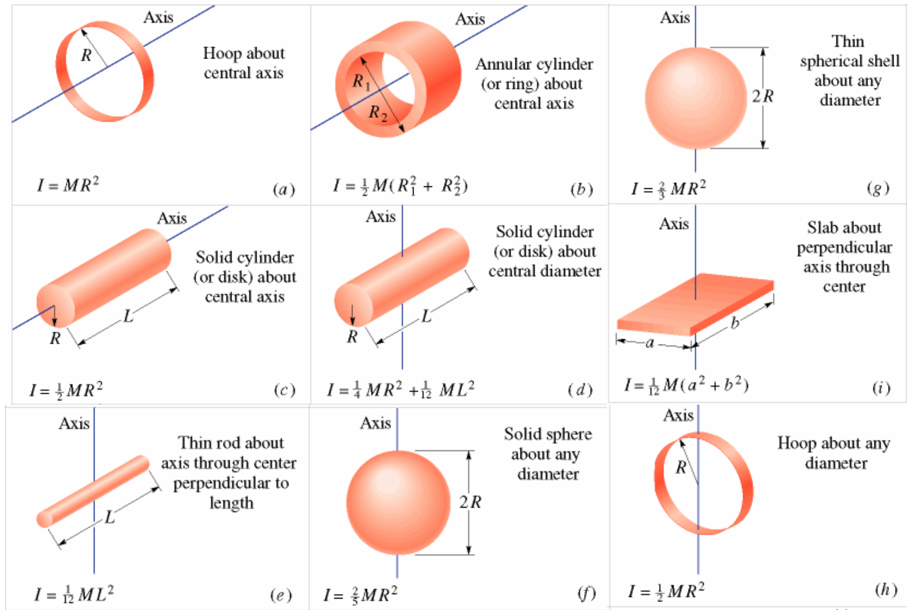
$I^{(a)} \neq I^{(b)}$

I – rotational equivalent of mass m

Main difference between m and I: *Rotational Inertia* depends on the direction of rotation !

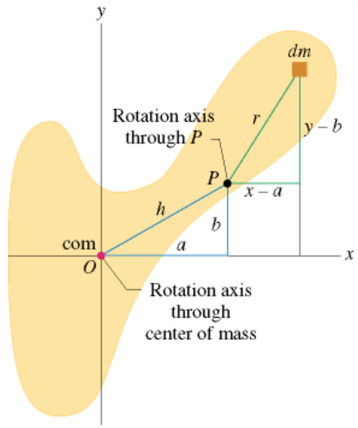
For a rigid body, I depends on how the mass is distributed in an object relative to the axis of rotation

TABLE 11-2 Rotational Inertia



Parallel-Axis Theorem for Rotational Inertia

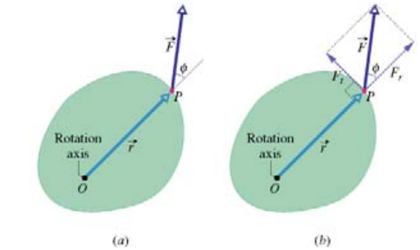
$$I = I_{com} + Mh^2 \quad \text{(parallel-axis theorem)}.$$

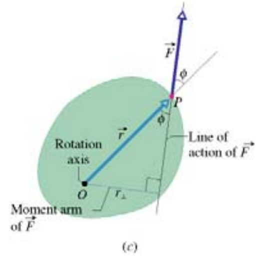


Calculate I_{com} for the axis going through the COM

Use *Parallel-Axis Theorem* to calculate I

Torque: $\vec{\tau}$





The value of *torque:*

τ = r·F·sinϕ

ϕ=0 ➔ τ=0

ϕ=π/2 ➔ τ= r·F (max)

In vector notation form:

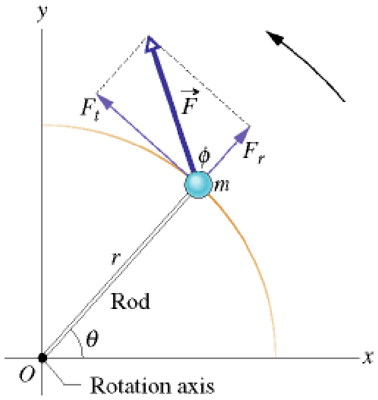
$\vec{\tau} = [\vec{r} \times \vec{F}]$

Newton's Second Law for Rotation

Torque causes the change in ω

$$\tau_{net} = I \cdot \alpha$$

Rotational equivalent of F = ma



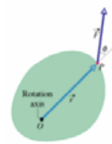
$$F_t = ma_t$$

$$\tau = F_t r = ma_t r$$

$$\tau = m(\alpha r)r = (mr^2)\alpha$$

Rotational Analogy to Linear Motion

	Translation	Rotation
position	x	θ
velocity	$v = dx/dt$	$\omega = d\theta/dt$
acceleration	$a = dv/dt$	$\alpha = d\omega/dt$
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$
Force	$F = ma$	$\tau_{net} = I \cdot \alpha$



$$\vec{\tau} = [\vec{r} \times \vec{F}]$$

$$\tau = r \cdot F \cdot \sin\phi$$

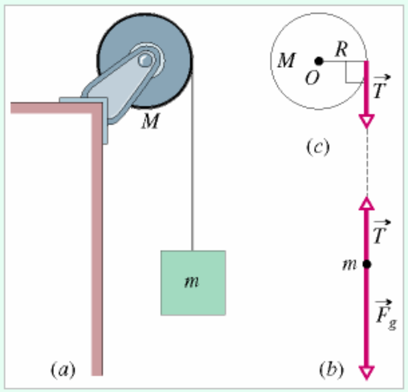
Angular Displacement
Angular Velocity
Angular Acceleration

θ, $\vec{\omega}$, $\vec{\alpha}$

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Sample Problem 11-7

Figure 11-17*a* shows a uniform disk, with mass $M = 2.5$ kg and radius $R = 20$ cm, mounted on a fixed horizontal axle. A block with mass $m = 1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. The cord does not slip, and there is no friction at the axle.



I of the disk is $\frac{1}{2}MR^2$.

Fig. 11-17 Sample Problems 11-7 and 11-9. (*a*) The falling block causes the disk to rotate. (*b*) A free-body diagram for the block. (*c*) An incomplete free-body diagram for the disk.

PROBLEM 55

In Fig. 11-42, one block has mass $M = 500$ g, the other has mass $m = 460$ g, and the pulley, which is mounted in horizontal frictionless bearings, has a radius of 5.00 cm. When released from rest, the heavier block falls 75.0 cm in 5.00 s (without the cord slipping on the pulley). (a) What is the magnitude of the blocks' acceleration? What is the tension in the part of the cord that supports (b) the heavier block and (c) the lighter block? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

I of the disk is $\frac{1}{2}MR^2$.

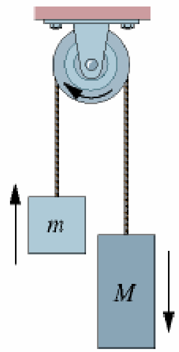


Fig. 11-42 Problem 55.

Work and Rotational Kinetic Energy

Work-kinetic energy theorem

$$\Delta K = K_f - K_i = \tfrac{1}{2}I\omega_f^2 - \tfrac{1}{2}I\omega_i^2 = W$$

Work, rotation about fixed axis

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$

Work, constant torque

$$W = \tau(\theta_f - \theta_i)$$

Power, rotation about fixed axis

$$P = \frac{dW}{dt} = \tau\omega$$

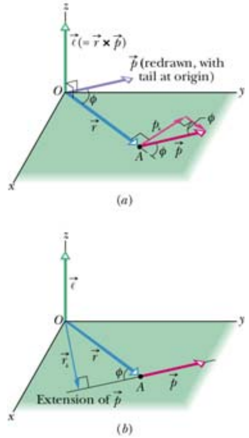
Angular Momentum:

Definition: $\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ [kg m^2/s]

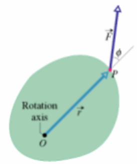
$$l = r \cdot m \cdot v \cdot \sin\phi$$

Angular Momentum for rotation $l = I \cdot \omega$

System of particles: $\vec{L} = \vec{l}_1 + \vec{l}_2 + \ldots + \vec{l}_n = \sum_{i=1}^{n} \vec{l}_i$



Torque:

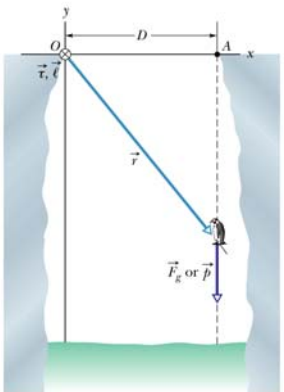


$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$

$$\frac{d}{dt}(\vec{L}) = \vec{\tau} = I\vec{\alpha}$$

Sample Problem XII–5



A penguin of mass *m* falls from rest at point *A*, a horizontal distance *D* from the origin *O* of an *xyz* coordinate system.

a) What is the angular momentum of $\underline{l}$ of the penguin about *O*?

b) About the origin *O*, what is the torque $\underline{\tau}$ on the penguin due to the gravitational force $\underline{F}_g$?

Newton's 2nd Law

Angular Momentum of a particle:

$$\frac{d}{dt}(\vec{L}) = \frac{d}{dt}(\vec{r} \times \vec{p}) = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\boxed{\frac{d}{dt}(\vec{L}) = \vec{\tau}}$$

Linear Momentum

$$\vec{p} = m\vec{v}$$

[kg m/s]

Angular Momentum

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{[kg m}^2\text{/s]}$$

$$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$$

$$L = m \cdot r \cdot v \cdot \sin\phi$$

Both are vectors

$$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$$

$$\frac{d}{dt}\left(\vec{L}\right) = \vec{\tau} = I\vec{\alpha}$$

For rotating body:

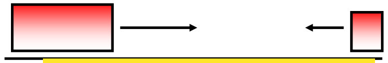
$$L = I\omega$$

m ←→ I

v ←→ ω

FOR ISOLATED SYSTEM: L IS CONSERVED

Linear Momentum Conservation:



$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

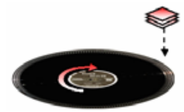
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

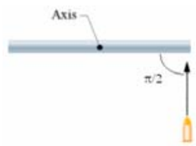
Both, elastic and Inelastic collisions

1. Define a reference frame
2. Calculate **P** before the collision
3. Compare with **P** after the collision

Angular Momentum Conservation:

"If the external torque is equal to zero, L is conserved"





1. Define a rotational axis and the origin
2. Calculate **L** before interaction or any changes in I
3. Compare with **L** after the interaction or any change in I

Conservation of Angular Momentum

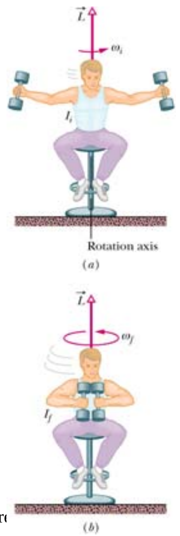
Angular momentum of a solid body about a fixed axis

$$L = I\omega$$

Law of conservation of angular momentum

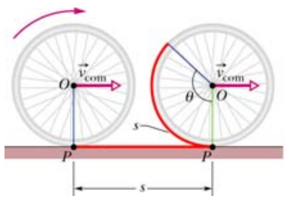
$$\vec{L} = \text{const.} \quad \Rightarrow \quad \vec{L}_i = \vec{L}_f$$

(Valid from microscopic to macroscopic scales!)



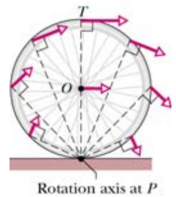
If the net external torque τ_{net} acting on a system is zero, the angular momentum L of the system remains constant, no matter what changes take place within the system

Rolling



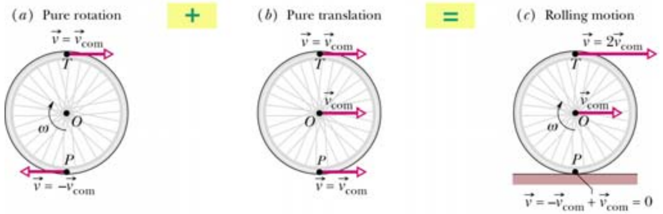
Smooth rolling motion

$$v_{com} = \omega R$$



Rotation and Translation

Reference frame



Kinetic Energy

$$
\begin{aligned}
K &= \tfrac{1}{2} I_P\, \omega^2 \\
I_P &= I_{com} + MR^2 \\
K &= \tfrac{1}{2} I_{com}\, \omega^2 + \tfrac{1}{2} MR^2 \omega^2 \\
v_{com} &= \omega R \\
K &= \tfrac{1}{2} I_{com}\, \omega^2 + \tfrac{1}{2} M v_{com}^2
\end{aligned}
$$

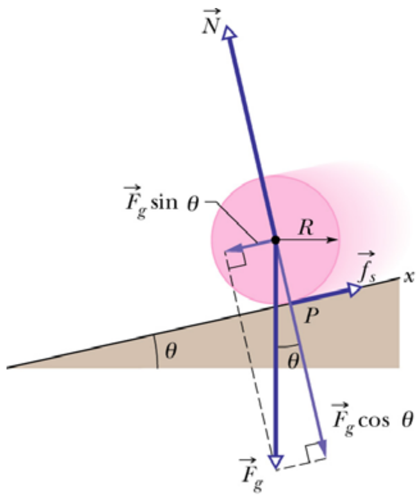
Stationary observer

Parallel–axis theorem

A rolling object has two types of kinetic energy: a rotational kinetic energy due to its rotation about its center of mass and a translational kinetic energy due to translation of its center of mass.

<u>Sample Problem X12–1:</u> A uniform solid cylindrical disk (M = 1.4 kg, r = 8.5 cm) roll smoothly across a horizontal table with a speed of 15 cm/s. What is its kinetic energy K?

Forces



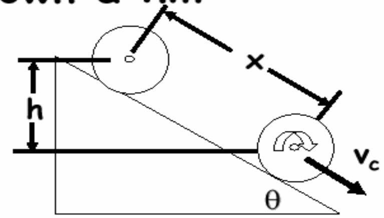
The acceleration tends to make the wheel slide.

A static frictional force f_s acts on the wheel to oppose that tendency.

Rolling down a hill

Conservation of Energy

$$\frac{1}{2}\left(\frac{I_C}{R^2} + M\right) v_C^2 = Mgh$$



$$v_C = \sqrt{\frac{2gh}{1 + I_C/MR^2}}$$

For a particle: $v_C = \sqrt{2gh}$

A Disc

$$I_C = \frac{1}{2} MR^2$$

A Ring

$$I_C = MR^2$$

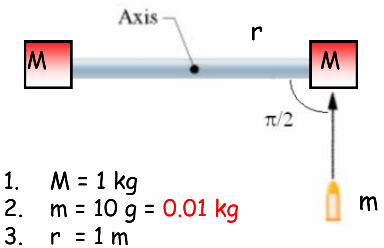
$$v_C = \sqrt{\frac{2gh}{1 + \frac{1}{2}MR^2/MR^2}} = \sqrt{\frac{2}{3}2gh}$$

$$v_C = \sqrt{\frac{2gh}{1 + MR^2/MR^2}} = \sqrt{\frac{1}{2}2gh}$$

Free falling / sliding without friction: $V_C = \sqrt{2gh}$

<u>Angular Momentum Conservation:</u>

"If the external torque is equal to zero, L is conserved"



1. M = 1 kg
2. m = 10 g = 0.01 kg
3. r = 1 m
4. $\omega_i = 0$ ω_f = 1 rad/s
5. v_{bullet} = ? K_f/K_i = ?

1. Define a rotational axis and the origin
2. Calculate **L** before interaction or any changes in I
3. Compare with **L** after the interaction or any change in I

1. $L_i = L_{bullet} = m \cdot v \cdot r \cdot \sin(\pi/2)$ = ???
2. $L_f = I \cdot \omega = (Mr^2 + Mr^2 + mr^2)\,\omega_f$ = =2 kg·m²/s
3. $L_i = L_f$ (angular momentum conserv.)
4. $v_{bullet} = \omega_f \cdot (2Mr^2 + mr^2)/mr$ = 200 m/s
5. $K_i = \frac{1}{2} m v^2_{bullet}$ = 200 J
6. $K_f = \frac{1}{2} I \omega^2$ = 1 J
7. K_f/K_i = 1/200

Equilibrium

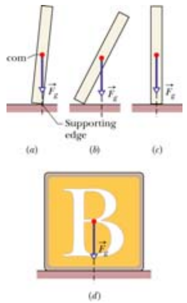
Balance of Forces:

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} = 0$$

Balance of Torques:

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt} = 0$$

$$\vec{P} = 0$$

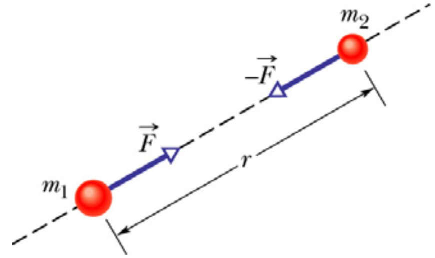


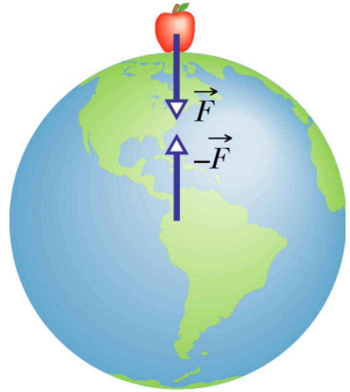
1. The vector sum of all the external forces that act on the body must be zero.
2. The vector sum of all the external torques that act on the body, measured about any possible point, must be zero.
3. The linear momentum $\underline{P}$ of the body must be zero.
4. The gravitational force $\underline{F}_g$ on a body effectively acts on a single point, called the center of gravity (cog) of the body. If g is the same for all elements of the body, then the body's cog is coincident with the body's center of mass.

Newton's Law of Gravitation (known since 1665)

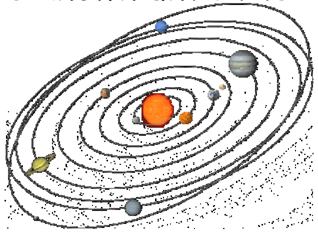
$$F = G\frac{m_1 m_2}{r^2}$$

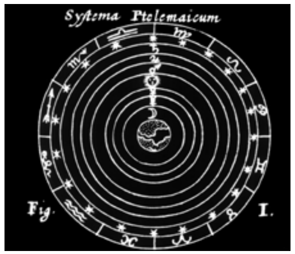
$G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$

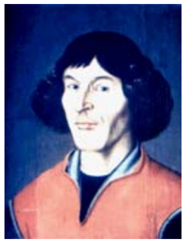




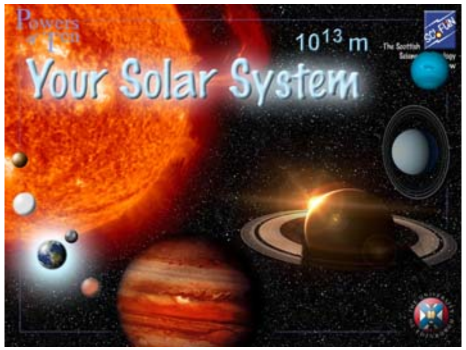
Only six planets, including the Earth, were known until the 18th Century





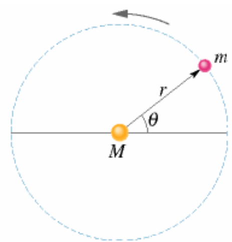


Copernicus 1510



Planets and Satellites: Kepler's Laws

THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.



$$\frac{GMm}{r^2} = (m)(\omega^2 r).$$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

Potential Energy :

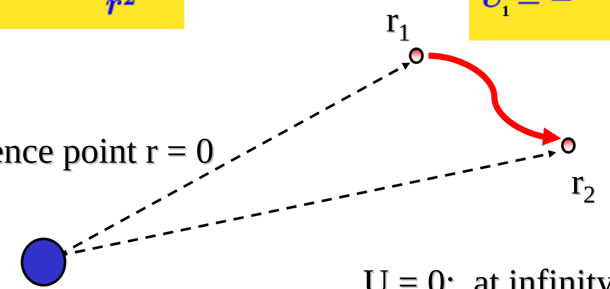
ΔU between r_1 and r_2 is the work done by the Gravitation Force during the move from r_1 to r_2 :

$$F = G\frac{m_1 m_2}{r^2}$$

$$U_1 = -\frac{GMm}{r_1}$$

$$U_2 = -\frac{GMm}{r_2}$$

Reference point r = 0



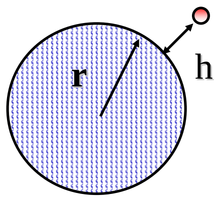
U = 0; at infinity !   (far away)

Potential Energy :

Is it $\Delta\, U = mgh$ or $U = -\frac{GMm}{r}$, anyway ?

It is the same thing, just different zero levels.

$U = -\frac{GMm}{r}$ is more universal (always correct)



$\Delta\, U = mgh$ works for **h** << **r**, zero at the Earth surface

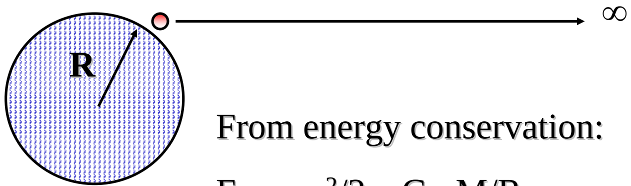
$U = -\frac{GMm}{r}$ always works, zero at ∞

Δ U = GMm/r – GMm/(r+h) = GMm(r+h-r)/(r· (r+h))

= mh·[GM/(r· (r+h))] ≈ mgh

Escape Speed:

$$F = G\frac{m_1 m_2}{r^2}$$



From energy conservation:

$E_1 = mv^2/2 – GmM/R$

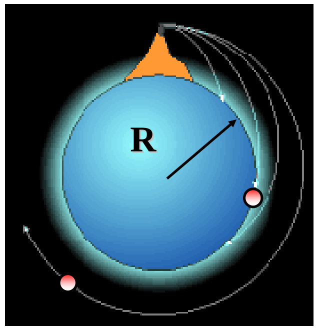
E_2 = 0 (velocity is small)

$v^2 = 2GM/R = 2gR$

v = (2GM/R) ½ ≈ 11,200 m/s

First Satellite Speed:

$$F = G\frac{m_1 m_2}{r^2}$$



"Newton's cannon"

in 1687 in "*Principia Mathematica*"

$v_{satellite} \approx (gR)^{½}$

$v_{satellite} \approx$ 8,000 m/s

g ≈ 8.70 m/s^2

An object in orbit is weightless not because 'it is beyond the earth's gravity' but because it is in 'free-fall' - just like a skydiver.

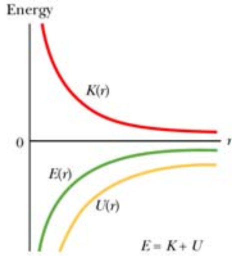
Potential and Kinetic Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

Kinetic Energy for the orbital motion

$$F = G\frac{Mm}{r^2} = m\frac{v^2}{r} \quad \Rightarrow \quad K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$



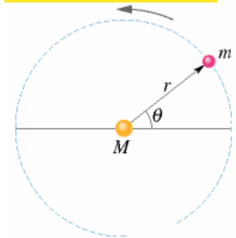
Total Energy

$$E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Satellites: Orbits and Energy

Potential Energy

$$U = -\frac{GMm}{r}$$

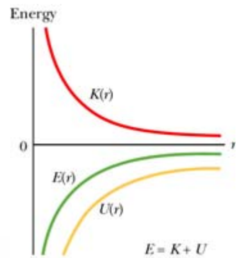


$$\frac{GMm}{r^2} = m\frac{v^2}{r}$$

Kinetic Energy for the orbital motion

$$K = \tfrac{1}{2}mv^2 = \frac{GMm}{2r}$$

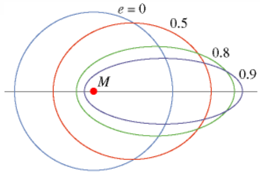
$$K = -\frac{U}{2} \quad \text{(circular orbit)}$$



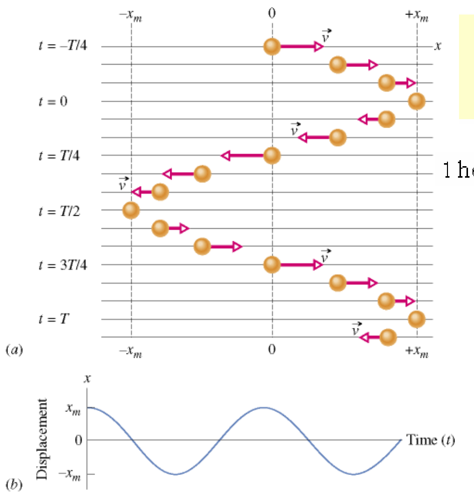
Total Energy: $E = K + U = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ (circular orbit)

$$E = -\frac{GMm}{2a} \quad \text{(elliptical orbit)}$$

semimajor axis a



Simple Harmonic Motion



$$T = \frac{1}{f}.$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1}

Displacemnt at time t

Phase

$$x(t) = x_m \cos(\omega t + \phi)$$

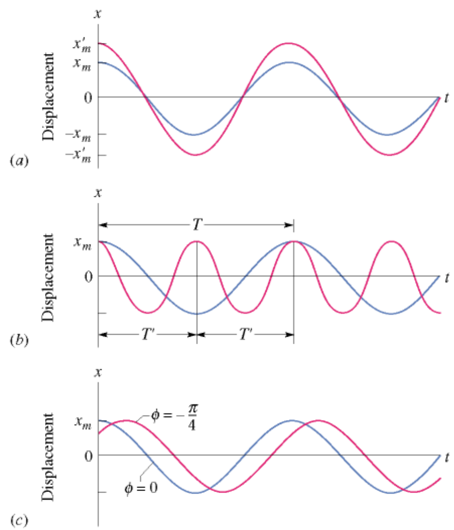
Amplitude

Time

Angular frequency

Phase constant or phase angle

Simple Harmonic Motion (SHM)



1. Amplitude is different
2. Period (or frequency) is different.
3. Phase is different.

Displacemnt at time t

Phase

$$x(t) = x_m \cos(\omega t + \phi)$$

Amplitude

Time

Angular frequency

Phase constant or phase angle

Displacement, Velocity, and Acceleration of SHM

$x(t) = x_m \cos(\omega t + \phi)$ (displacement),

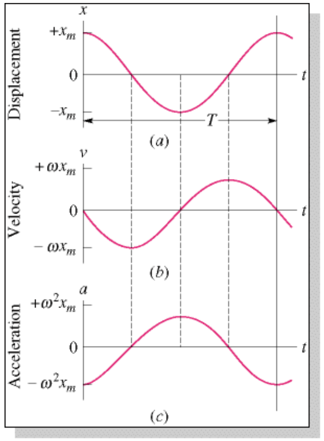
$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[x_m \cos(\omega t + \phi)\right]$$

$v(t) = -\omega x_m \sin(\omega t + \phi)$ (velocity).

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega x_m \sin(\omega t + \phi)\right]$$

$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$ (acceleration).

$a(t) = -\omega^2 x(t)$



Click on the image to start the simulation

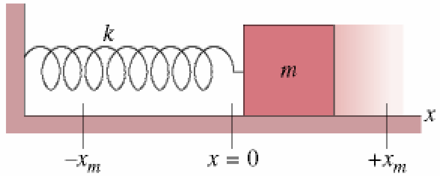
Displacement, Velocity, and Acceleration of Simple Harmonic Motion

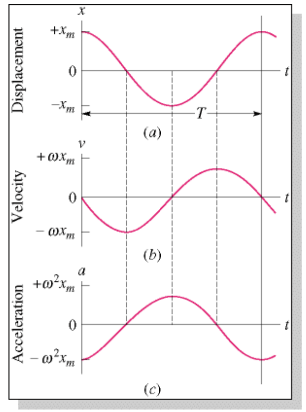
$x(t) = x_m \cos(\omega t + \phi)$ (displacement),

$v(t) = -\omega x_m \sin(\omega t + \phi)$ (velocity).

$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$ (acceleration).

$$a(t) = -\omega^2 x(t)$$





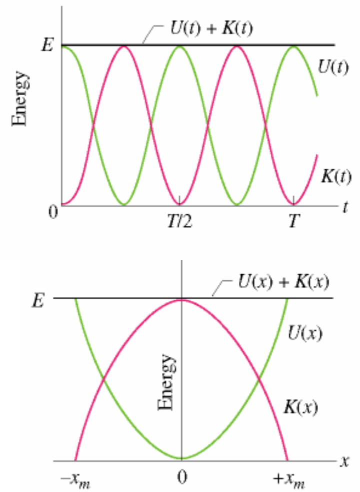
Click on the image to start the simulation

$F = -kx$

$\omega = \sqrt{\frac{k}{m}}$ (angular frequency)

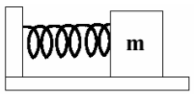
$T = 2\pi\sqrt{\frac{m}{k}}$ (period)

Energy of SHM



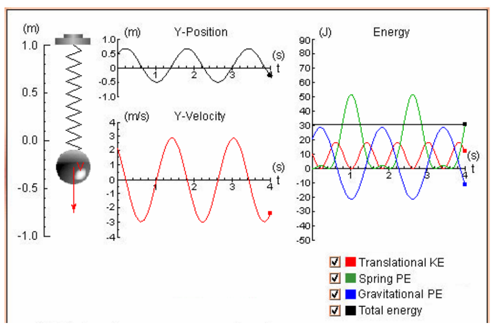
Total Energy is a constant

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

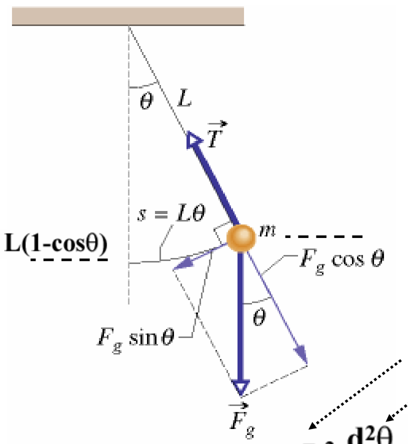


$E = \frac{1}{2}k(x_m)^2$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgx$$



Simple Pendulum



$h = L(1-\cos\theta)$

$$I = mL^2$$

$$F_T = -mg\sin(\theta)$$

$$\tau = -mgL\sin(\theta)$$

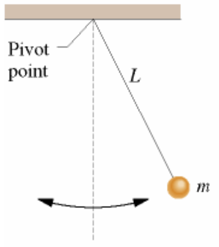
$$\cong -mgL\theta \quad \text{(for small } \theta\text{)}$$

$$I \cdot \alpha = \tau$$

$$mL^2 \frac{d^2\theta}{dt^2} = -mgL\theta \longrightarrow \frac{d^2\theta}{dt^2} = -(g/L)\theta$$

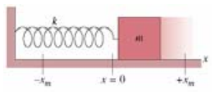
Simple Pendulum

Simple pendulum follows SHM



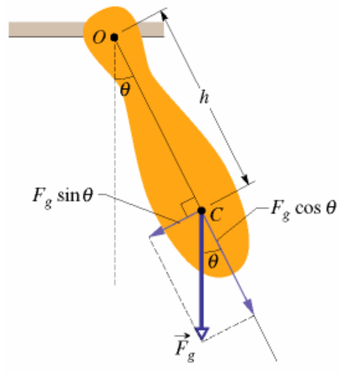
$$\frac{d^2\theta}{dt^2} = -(g/L)\theta \quad \text{Looks like spring} \quad \frac{d^2x}{dt^2} = -(k/m)x$$

Solution by analogy



Spring	Pendulum
$x = x_m\cos(\omega t+\phi)$	$\theta = \theta_m\cos(\omega t+\phi)$
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{L}}$
$T = 2\pi\sqrt{\frac{m}{k}}$	$T = 2\pi\sqrt{\frac{L}{g}}$

The Physical Pendulum



Any rigid body behaves like SHO close to stable equilibrium

$$\tau = I\alpha$$

$$\tau = -mgh\sin(\theta) \cong -mgh\theta$$

$$I\frac{d^2\theta}{dt^2} = -mgh\theta$$

We know the solution

$$\omega = \sqrt{\frac{mgh}{I}}$$

$$\theta = \theta_m\cos(\omega t + \phi)$$

$T = 2\pi\sqrt{\frac{I}{mgh}}$ (physical pendulum, small amplitude).

Compare to: $T = 2\pi\sqrt{\frac{L}{g}}$ $I = mL^2$

for SHO