Lecture 3

Physics 106

Spring 2006

Rotational dynamics:

- Newton's Second Law and examples

http://web.njit.edu/~sirenko/

Kinetic Energy of Rotation

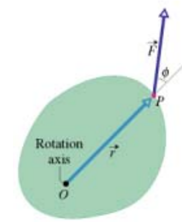
$K = \frac{1}{2} I \omega^2$ (radian measure)

$I = \sum m_i r_i^2$ (rotational inertia)

$I = I_{com} + Mh^2$ (parallel – axis theorem)

Do a calculation or see the Text Book

Torque: $\vec{\tau}$



$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$

Rotational Analogy to Linear Motion

	Translation	*Rotation*
position	x	θ
velocity	$v = dx/dt$	$\omega = d\theta/dt$
acceleration	$a = dv/dt$	$\alpha = d\omega/dt$
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2} mv^2$	$K = \frac{1}{2} I \omega^2$
Force	$F = ma$	$\tau_{net} = I \cdot \alpha$

Work and Rotational Kinetic Energy

Work-kinetic energy theorem

$$\Delta K = K_f - K_i = \tfrac{1}{2} I \omega_f^2 - \tfrac{1}{2} I \omega_i^2 = W$$

Work, rotation about fixed axis

$$W = \int_{\theta_i}^{\theta_f} \tau \, d\theta$$

Work, constant torque

$$W = \tau(\theta_f - \theta_i)$$

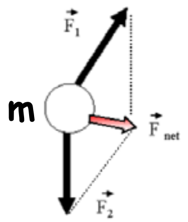
Power, rotation about fixed axis

$$P = \frac{dW}{dt} = \tau \omega$$

Newton's Second Law for Rotation

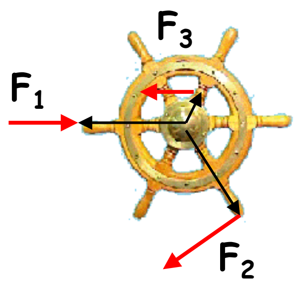
Force $F = ma$

Net Force (or Total Force)



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$\vec{F}_{net} = m\vec{a}$$

$$\tau_{net} = I \cdot \alpha$$



$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = I \cdot \vec{\alpha}$$
$$\vec{\tau}_{net} = [\vec{r}_1 \times \vec{F}_1] + [\vec{r}_2 \times \vec{F}_2] + [\vec{r}_3 \times \vec{F}_3] = I \cdot \vec{\alpha}$$

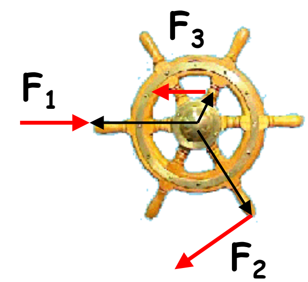
Newton's Second Law for Rotation

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = I \cdot \vec{\alpha}$$
$$\vec{\tau}_{net} = [\vec{r}_1 \times \vec{F}_1] + [\vec{r}_2 \times \vec{F}_2] + [\vec{r}_3 \times \vec{F}_3] = I \cdot \vec{\alpha}$$

When torque is positive ?

τ is positive if it rotates the body to positive direction (CCW)

"clock is negative".



$\tau_1 = 0$ $\quad \tau_3 > 0$ $\quad \tau_2 < 0$

$\odot$ $\quad \otimes$

Newton's Second Law for Rotation

$\tau_1 = 0$ $\quad \tau_3 > 0$ $\quad \tau_2 < 0$

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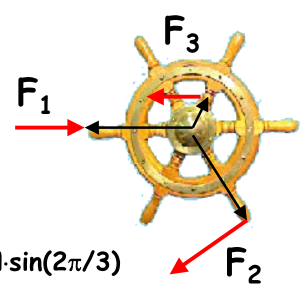


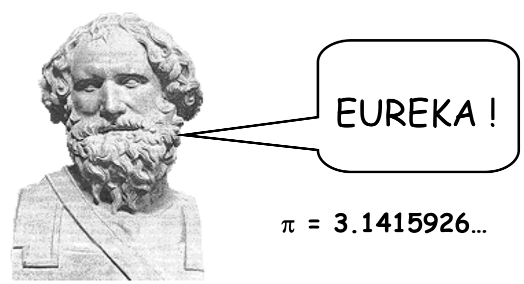
$r_1 = 1.0m$	$r_2 = 1.0m$	$r_3 = 0.5m$
$F_1 = 2.0N$	$F_2 = 3.0N$	$F_3 = 2.0N$
$\phi_1 = \pi$	$\phi_2 = -\pi/2$	$\phi_3 = 2\pi/3$

$I = 10\ kg \cdot m^2$

$$\tau_{net} = 1m \cdot 2N \cdot \sin(\pi) + 1m \cdot 3N \cdot \sin(-\pi/2) + 0.5m \cdot 2N \cdot \sin(2\pi/3)$$
$$= 0 + (-3)m \cdot N + (-3)m \cdot N + 0.87\ m \cdot N = -2.13\ m \cdot N$$

$$\alpha = \tau_{net} / I = -2.13\ m \cdot N / 10\ kg \cdot m^2 = -0.21\ rad/s^2$$

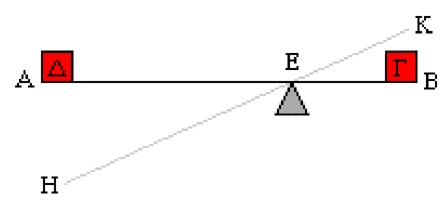
This Angular acceleration speeds up CW rotation

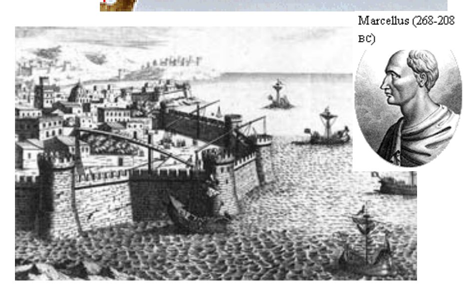


$\pi = 3.1415926...$

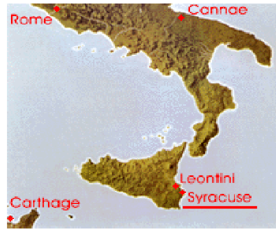
Archimedes

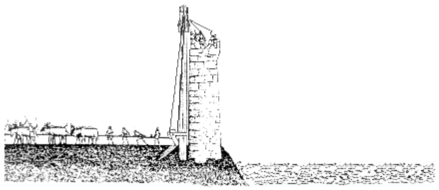
(287 BC - 211 BC)

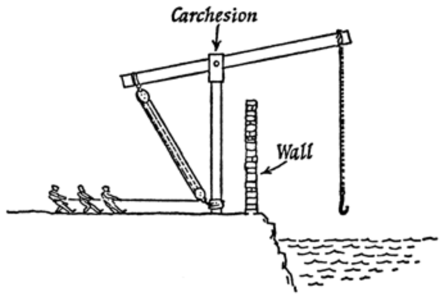




Archimedes' Claw

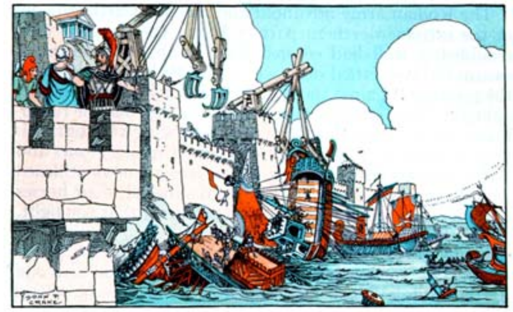


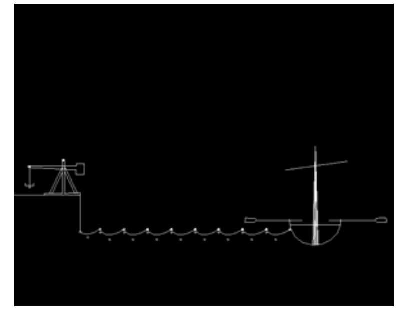




http://www.mcs.drexel.edu/~crorres/Archimedes/Claw/illustrations.html

Archimedes' Claw





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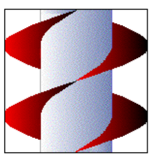
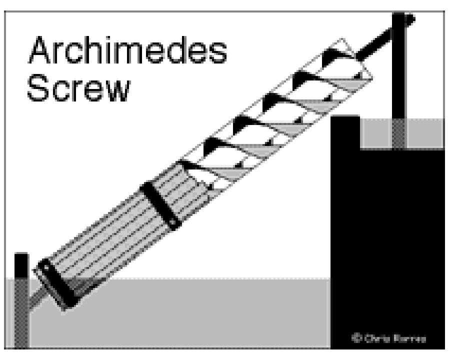
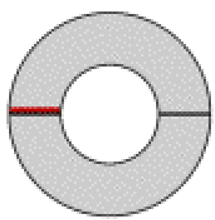


Archimedes Screw





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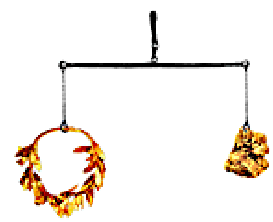
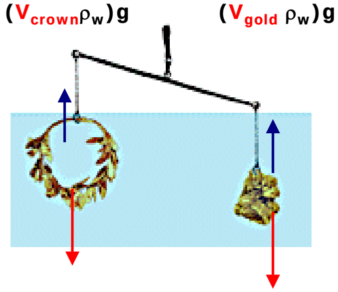
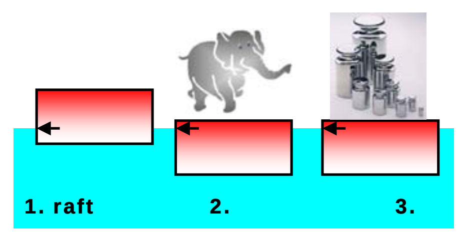
Hiero: "Is it really 100% gold ?"

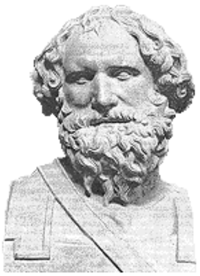


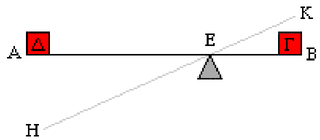
$(V_{crown}\rho_w)g$ $(V_{gold}\,\rho_w)g$





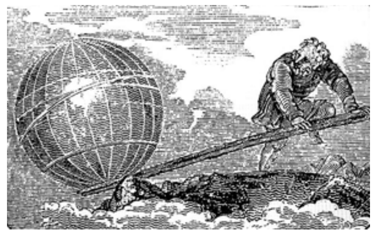
mg mg





Archimedes

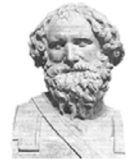
(287 BC - 211 BC)



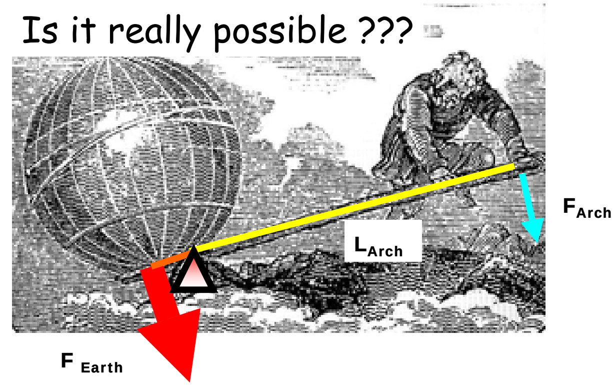
GIVE ME A PLACE TO STAND AND I WILL MOVE THE EARTH

Give me a lever long enough and a place to stand, and I will move the world

Is it really possible ???

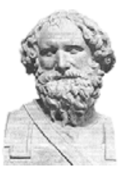


Is it really possible ???

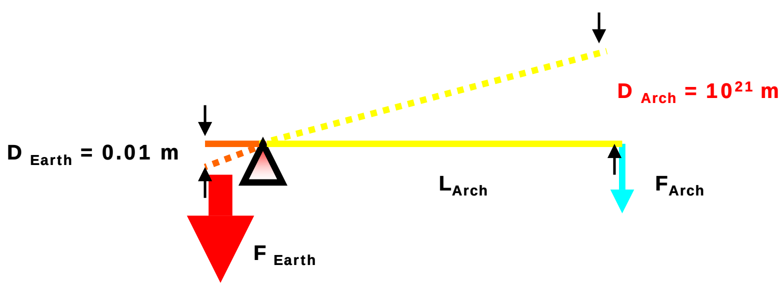


$F_{Earth} = 6\times10^{25}$ N **$F_{Arch} = 600$ N**

($M_{Earth} = 6\times10^{24}$ kg) **(60 kg)**



Is it really possible ???



$F_{Arch} = 600$ N (60 kg)

$F_{Earth} = 6\times10^{25}$ N ($M_{Earth} = 6\times10^{24}$ kg)

$L_{Arch} / L_{Earth} = 1\times10^{23}$; ➔ $D_{Arch} / D_{Earth} = 1\times10^{23}$

$D_{Arch} = 10^{21}$ m With the power of $P=600 N\cdot m/s$, $t = 10^{21} s \cong 5\times10^{14}$ years

(people do not live that long)

If Archimedes moves his arm with the speed of light, then

$D_{Arch} = 50$ m during 5×10^{9} years (life time of the Earth)

QZ Problem

A rigid sculpture consists of a thin hoop (of mass $m=1\ kg$ and radius $R = 1$ m) and a thin radial rod (of mass $M=2\ kg$ and length $L = 2$ m). The sculpture can pivot around a horizontal axis in the plane of the hoop, passing through its center.

a) What is the sculpture's rotational inertia ***I*** about the rotation axis?

b) Starting from rest, the sculpture rotates around the rotation axis from the initial upright position. What is the change of the sculpture's Potential Energy ΔU when it is inverted?

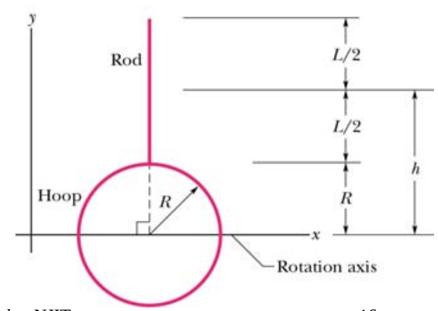
c) What is the Kinetic Energy of rotation when it is inverted?

d) What is the angular speed ω around the horizontal axis ?

Rod: $I_{com} = (1/12)\ ML^2$

Hoop: $I_{com} = (1/2)\ mR^2$

$I = I_{com} + Mh^2$; $I_{tot} = I_1 + I_2$; $K = ½ I \omega^2$



Homework

See the Physics 106 Course Syllabus

U of Texas HW is required

http://web.njit.edu/~sirenko/