# Lecture 4

# Physics 106 Spring 2007

# Review for Rotational dynamics

Q&A for the First Exam

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Lecture 4

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### Physics 106:

**TABLE 11-2** 

 $I=\tfrac{1}{12}\,ML^2$ 

360 degrees =  $2\pi$  radians = 1 revolution.  $s = r\theta$   $v_t = r\omega$   $a_t = r\alpha$   $a_c = a_r = v_t^2/r = \omega^2 r$   $a_{tot}^2 = a_r^2 + a_t^2 = a_t^2 +$ 

for rotation with constant angular acceleration:

$$\omega = \omega_0 + \alpha t \qquad \theta - \theta_0 = \omega_0 t + \sqrt{2} \alpha t^2 \qquad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0) \qquad \theta - \theta_0 = \frac{1}{2}(\omega + \omega_0)t \qquad KE_{rot} = \frac{1}{2}\log^2 \theta$$

 $I = \Sigma m_i r_i^2 \ l_{point} = m r^2 \ l_{hoop} = MR^2 \ l_{disk} = 1/2 \ MR^2 \ l_{sphere} = 2/5 \ MR^2 \ l_{shell} = 2/3 \ MR^2 \ l_{rod\ (center)} = 1/12 \ ML^2 \ l_{rod\ (enter)} = 1/3 \ ML^2$ 

 $\Sigma \mathbf{F} = \mathbf{ma}$   $\Sigma \mathbf{\tau} = \mathbf{I} \alpha$   $\mathbf{\tau} = \mathbf{r} \mathbf{x} \mathbf{F}$   $\mathbf{I}_{p} = \mathbf{I}_{cm} + \mathbf{Mh}^{2}$ 

 $\tau = \text{force}_x \text{moment arm} = \text{Frsin}(\phi)$   $\tau_{\text{net}} = \Sigma \tau = I \alpha$   $\tau = r \times F$   $\tau = r$ 

 $W_{tot} = \Delta K = K_f - K_l$   $W = \tau_{net} \Delta \Theta$   $K = K_{rot} + K_{cm}$   $E_{mech} = K + U$   $P_{average} = \Delta W/\Delta t$   $P_{instantaneous} = \tau.\omega$  ( $\tau$  constant)  $\Delta E_{mech} = 0$  (isolated system)  $v_{cm} = \omega r$  (rolling, no slipping)

 $\ell$  = rxp p = mv L =  $\Sigma$   $\ell$   $\tau_{net}$  = dL/dt L =  $l\omega$   $\ell_{point mass}$  = mrvsin( $\phi$ ) For isolated systems:  $\tau_{net}$  = 0 L is constant  $\Delta$ L = 0 L<sub>0</sub> =  $\Sigma$   $l_0\omega_0$  =  $L_f$  =  $\Sigma$   $l_f\omega$ 

 $\mathbf{a} \mathbf{x} \mathbf{b} = -\mathbf{b} \mathbf{x} \mathbf{a}$   $\mathbf{a} \mathbf{x} \mathbf{a} = 0$   $|\mathbf{a} \mathbf{x} \mathbf{b}| = \mathbf{a}.\mathbf{b}.\sin(\phi)$   $\mathbf{c} = \mathbf{a} \mathbf{x} \mathbf{b}$  is perpendicular to plane of  $\mathbf{a}$  and  $\mathbf{b}$   $\mathbf{c}_{\mathbf{x}} = \mathbf{a}_{\mathbf{y}}.\mathbf{b}_{\mathbf{z}} - \mathbf{a}_{\mathbf{z}}.\mathbf{b}_{\mathbf{y}}$   $\mathbf{c}_{\mathbf{y}} = -\mathbf{a}_{\mathbf{x}}.\mathbf{b}_{\mathbf{z}} + \mathbf{a}_{\mathbf{z}}.\mathbf{b}_{\mathbf{x}}$   $\mathbf{c}_{\mathbf{z}} = \mathbf{a}_{\mathbf{x}}.\mathbf{b}_{\mathbf{y}} - \mathbf{a}_{\mathbf{y}}.\mathbf{b}_{\mathbf{x}}$   $\mathbf{i} \mathbf{x} \mathbf{i} = \mathbf{j} \mathbf{x} \mathbf{j} = \mathbf{k} \mathbf{x} \mathbf{k} = 0$   $\mathbf{i} \mathbf{x} \mathbf{j} = \mathbf{k}$   $\mathbf{j} \mathbf{x} \mathbf{k} = \mathbf{i}$   $\mathbf{k} \mathbf{x} \mathbf{i} = \mathbf{j}$  etc.

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Rotational Inertia

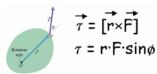
### Rotational Analogy to Linear Motion

	Translation	Rotation	
position	×	θ	
velocity	v = dx/dt	$\omega$ = d $\theta$ /dt	
acceleration	a = dv/dt	$\alpha$ = d $\omega$ /dt	

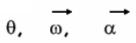
mass  $m = \sum m_i r_i^2$ 

Kinetic Energy  $K = \frac{1}{2} mv^2$   $K = \frac{1}{2} I \omega^2$ 

Force F = ma  $\tau_{net} = I \cdot \alpha$ 



Angular Displacement Angular Velocity Angular Acceleration



Linear Equation	Missing Variable		Angular Equation	
$v = v_0 + at$	$x - x_0$	$\theta$ - $\theta_0$	$\omega = \omega_0 + \alpha t$	
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$	
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	$\alpha$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	
$x - x_0 = vt - \frac{1}{2}at^2$	$v_0$	$\omega_0$	$\theta - \theta_0 = \omega t - \frac{1}{2} \cot^2$	

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#### Axis Thin Hoop about spherical shell Annular cylinder central axis about any (or ring) about diameter central axis $I = MR^2$ $I = \frac{1}{2}M(R_1^2 + R_2^2)$ $I = \frac{2}{3} MR^2$ (g) Solid cylinder Slab about perpendicular (or disk) about Solid cylinder axis through central diameter (or disk) about center central axis (d) $I = \frac{1}{12}M(a^2 + b^2)$ $I = \frac{1}{2}MR^2$ $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (i) Axis Axis Solid sphere Thin rod about Hoop about any about any axis through center diameter diameter perpendicular to length

(e)  $I = \frac{2}{5}MR^2$ 

 $I = \frac{1}{2}MR^2$ 

(h)

# Kinetic Energy of Rotation

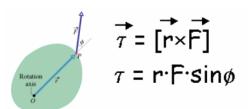
$$K = \frac{1}{2}I\omega^2$$
 (radian measure)

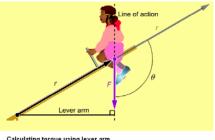
$$I = \sum m_i r_i^2$$
 (rotational inertia)

$$I = I_{com} + Mh^2$$
 (parallel – axis theorem).

## Calculate $I_{\text{com}}$ or see the Table in the Text Book

# Torque:





Calculating torque using lever arm

 $r \sin \theta = \text{lever arm}$ 

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### Chapter 11 Rotation

### **PROBLEM 55**

In Fig. 11-42, one block has mass M = 500 g, the other has mass m = 460 g, and the pulley, which is mounted in horizontal frictionless bearings, has a radius of 5.00 cm. When released from rest, the heavier block falls 75.0 cm in 5.00 s (without the cord slipping on the pulley). (a) What is the magnitude of the blocks' acceleration? What is the tension in the part of the cord that supports (b) the heavier block and (c) the lighter block? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

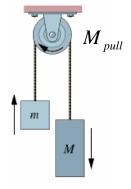


Fig. 11-42 Problem 55.

### Chapter 11 Rotation

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I of a pulley is equal to  $I_{pull} = \frac{1}{2} M_{pull} R^2$ 

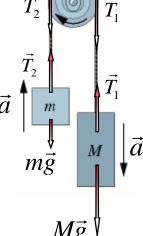
$$Mg > T_1 > T_2 > mg$$

$$\alpha < 0$$

$$\tau = I \cdot \alpha$$

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### PROBLEM 55

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$$I$$
 of a pulley is equal to  $I_{pull} = \frac{1}{2} M_{pull} R^2$   $Mg > T_1 > T_2 > mg$ 

$$m: \sum F_y = ma = T_2 - mg$$

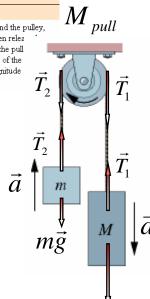
$$M: \sum F_y = Ma = Mg - T_1$$

$$M_{pull}: \sum \tau = (T_2 - T_1)R = I\alpha$$

$$\alpha = -\alpha R$$

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$$m: \qquad \sum F_{v} = ma = T_2 - mg$$

$$M: \sum F_{v} = Ma = Mg - T_{1}$$

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$$M_{pull}$$
:  $\sum \tau = (T_2 - T_1)R = I\alpha$  SOLUTION:

$$a = g \frac{M - m}{M + m + \frac{1}{2} M_{pull}}$$

$$\alpha = -\frac{g}{R} \frac{M - m}{M + m + \frac{1}{2} M_{pull}}$$

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