Lecture 4

Physics 106

Spring 2007

Review for Rotational dynamics

Q&A for the First Exam

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Physics 106:

360 degrees = 2π radians = 1 revolution. $s = r\theta$ $v_t = r\omega$ $a_t = r\alpha$ $a_c = a_r = v_t^2/r = \omega^2 r$ $a_{tot}^2 = a_r^2 + a_t^2$

for rotation with constant angular acceleration:
$\omega = \omega_o + \alpha t$ $\theta - \theta_o = \omega_o t + ½\alpha t^2$ $\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$ $\theta - \theta_o = ½(\omega + \omega_o)t$ $KE_{rot} = 1/2 I\omega^2$

$I = \Sigma m_i r_i^2$ $I_{point} = mr^2$ $I_{hoop} = MR^2$ $I_{disk} = 1/2\, MR^2$ $I_{sphere} = 2/5\, MR^2$ $I_{shell} = 2/3\, MR^2$ $I_{rod\ (center)} = 1/12\, ML^2$
$I_{rod\ (end)} = 1/3\, ML^2$

$\Sigma \mathbf{F} = m\mathbf{a}$ $\Sigma\tau = I\alpha$ $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $I_p = I_{cm} + Mh^2$

τ = force$_x$moment arm = $F\sin(\phi)$ $\boldsymbol{\tau}_{net} = \Sigma\tau = I\,\alpha$ $\mathbf{F}_{net} = \Sigma\mathbf{F} = m\,\mathbf{a}$ $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ $I_p = I_{cm} + Mh^2$

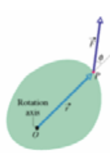
$W_{tot} = \Delta K = K_f - K_I$ $W = \tau_{net}\Delta\theta$ $K = K_{rot} + K_{cm}$ $E_{mech} = K + U$ $P_{average} = \Delta W/\Delta t$
$P_{instantaneous} = \tau.\omega$ (τ constant) $\Delta E_{mech} = 0$ (isolated system) $v_{cm} = \omega r$ (rolling, no slipping)

$\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$ $\mathbf{p} = m\mathbf{v}$ $\mathbf{L} = \Sigma\, \boldsymbol{\ell}_i$ $\boldsymbol{\tau}_{net} = d\mathbf{L}/dt$ $L = I\omega$ $\ell_{point\ mass} = mrv\sin(\phi)$
For isolated systems: $\tau_{net} = 0$ **L** is constant $\Delta\mathbf{L} = 0$ $L_0 = \Sigma\, I_0\omega_0 = L_f = \Sigma\, I_f\omega_f$

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ $\mathbf{a} \times \mathbf{a} = 0$ $|\mathbf{a} \times \mathbf{b}| = a.b.\sin(\phi)$ $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is perpendicular to plane of **a** and **b**
$c_x = a_y \cdot b_z - a_z \cdot b_y$ $c_y = -a_x \cdot b_z + a_z \cdot b_x$ $c_z = a_x \cdot b_y - a_y \cdot b_x$
$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$ $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$ etc.

Rotational Analogy to Linear Motion

	Translation	*Rotation*
position	x	θ
velocity	v = dx/dt	$\omega = d\theta/dt$
acceleration	a = dv/dt	$\alpha = d\omega/dt$
mass	m	$I = \Sigma m_i r_i^2$
Kinetic Energy	$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\,\omega^2$
Force	F = ma	$\tau_{net} = I \cdot \alpha$



$\vec{\tau} = [\vec{r} \times \vec{F}]$

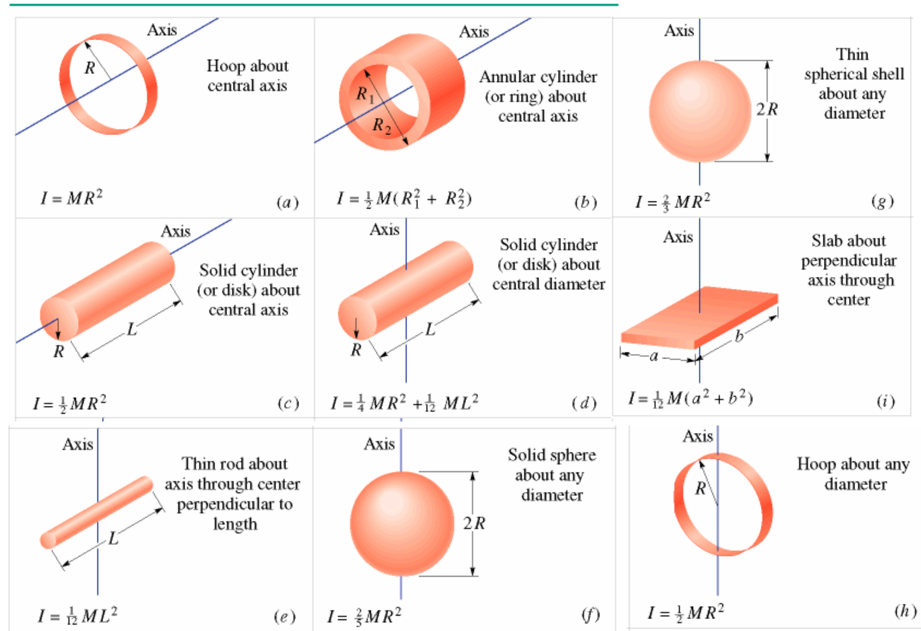
$\tau = r \cdot F \cdot \sin\phi$

Angular Displacement
Angular Velocity
Angular Acceleration

θ, $\vec{\omega}$, $\vec{\alpha}$

Linear Equation	Missing Variable		Angular Equation
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

TABLE 11-2 Rotational Inertia



Kinetic Energy of Rotation

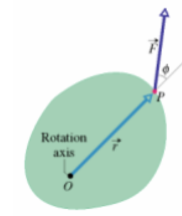
$K = \frac{1}{2} I\omega^2$ (radian measure)

$I = \sum m_i r_i^2$ (rotational inertia)

$I = I_{com} + Mh^2$ (parallel – axis theorem).

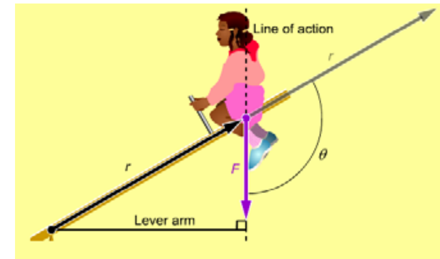
Calculate I_{com} or see the Table in the Text Book

Torque:



$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$



Calculating torque using lever arm

$\tau = F(r \sin\theta)$

$r \sin\theta =$ lever arm

Chapter 11 Rotation

PROBLEM 55

In Fig. 11-42 , one block has mass $M = 500$ g, the other has mass $m = 460$ g, and the pulley, which is mounted in horizontal frictionless bearings, has a radius of 5.00 cm. When released from rest, the heavier block falls 75.0 cm in 5.00 s (without the cord slipping on the pulley). (a) What is the magnitude of the blocks' acceleration? What is the tension in the part of the cord that supports (b) the heavier block and (c) the lighter block? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

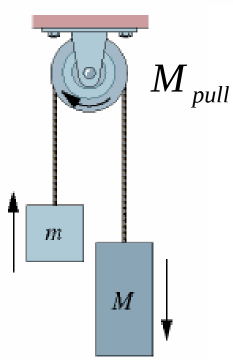


Fig. 11-42 Problem 55.

Chapter 11 Rotation

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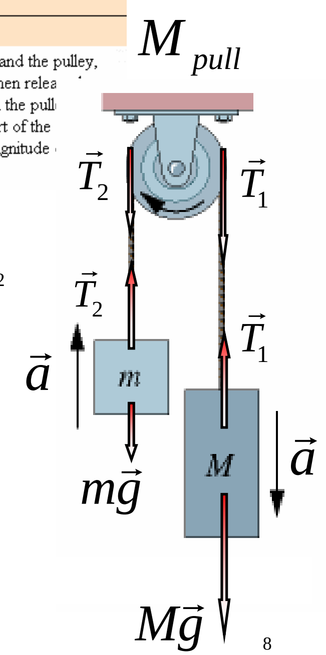

I of a pulley is equal to $I_{pull} = \frac{1}{2} M_{pull} R^2$

$$Mg > T_1 > T_2 > mg$$

$$\alpha < 0$$

$$\tau = I \cdot \alpha$$

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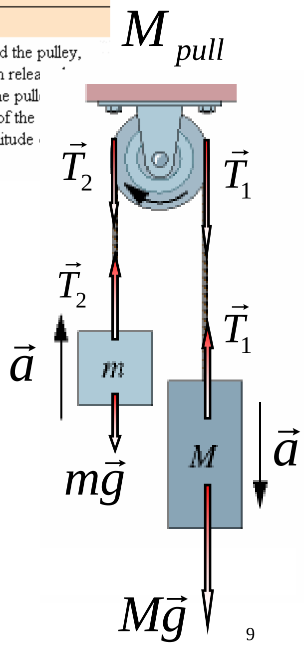
$$M_{pull}: \quad \sum \tau = (T_2 - T_1) R = I\alpha$$

$$a = -\alpha R$$



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SOLUTION:

$$a = g \frac{M - m}{M + m + \frac{1}{2} M_{pull}}$$

$$\alpha = -\frac{g}{R} \frac{M - m}{M + m + \frac{1}{2} M_{pull}}$$