# Lecture 7

## Physics 106

Spring 2006

**Review 2 for**

**$2^{nd}$ CQZ**

**Rolling and Kinetic Energy**

**Conservation of Angular Momentum**

http://web.njit.edu/~sirenko/



**Physics 106:**

360 degrees = $2\pi$ radians = 1 revolution. $s = r\theta$ $v_t = r\omega$ $a_t = r\alpha$ $a_c = a_r = v_t^2/r = \omega^2 r$ $a_{tot}^2 = a_r^2 + a_t^2$

for rotation with constant angular acceleration:

$\omega = \omega_o + \alpha t$ $\theta - \theta_o = \omega_o t + ½\alpha t^2$ $\omega^2 - \omega_o^2 = 2\alpha(\theta - \theta_o)$ $\theta - \theta_o = ½(\omega + \omega_o)t$ $KE_{rot} = 1/2 I\omega^2$

$I = \Sigma m_i r_i^2$ $I_{point} = mr^2$ $I_{hoop} = MR^2$ $I_{disk} = 1/2\ MR^2$ $I_{sphere} = 2/5\ MR^2$ $I_{shell} = 2/3\ MR^2$ $I_{rod\ (center)} = 1/12\ ML^2$ $I_{rod\ (end)} = 1/3\ ML^2$

$\Sigma \mathbf{F} = m\mathbf{a}$ $\Sigma\tau = I\alpha$ $\tau = \mathbf{r} \times \mathbf{F}$ $I_p = I_{cm} + Mh^2$

$\tau$ = force$_x$moment arm = $Fr\sin(\phi)$ $\tau_{net} = \Sigma\tau = I\alpha$ $\mathbf{F}_{net} = \Sigma\mathbf{F} = m\mathbf{a}$ $\tau = \mathbf{r} \times \mathbf{F}$ $I_p = I_{cm} + Mh^2$

$W_{tot} = \Delta K = K_f - K_I$ $W = \tau_{net}\Delta\theta$ $K = K_{rot} + K_{cm}$ $\boxed{E_{mech} = K + U}$ $P_{average} = \Delta W/\Delta t$

$P_{instantaneous} = \tau.\omega$ ($\tau$ constant) $\Delta E_{mech} = 0$ (isolated system) $v_{cm} = \omega r$ (rolling, no slipping)

$\boldsymbol{\ell} = \mathbf{r} \times \mathbf{p}$ $\mathbf{p} = m\mathbf{v}$ $\mathbf{L} = \Sigma\, \boldsymbol{\ell}_i$ $\tau_{net} = d\mathbf{L}/dt$ $\boxed{L = I\omega \quad \ell_{point\ mass} = mrv\sin(\phi)}$

For isolated systems: $\tau_{net} = 0$ $\mathbf{L}$ is constant $\Delta\mathbf{L} = 0$ $L_0 = \Sigma\, I_0\omega_0 = L_f = \Sigma\, I_f\omega_f$

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ $\mathbf{a} \times \mathbf{a} = 0$ $|\mathbf{a} \times \mathbf{b}| = a.b.\sin(\phi)$ $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is perpendicular to plane of **a** and **b**

$c_x = a_y{\cdot}b_z - a_z{\cdot}b_y$ $c_y = -a_x{\cdot}b_z + a_z{\cdot}b_x$ $\boxed{c_z = a_x{\cdot}b_y - a_y{\cdot}b_x}$

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$ $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$ etc.



# Vector Product:

$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$ $\mathbf{a} \times \mathbf{a} = 0$ $|\mathbf{a} \times \mathbf{b}| = a{\cdot}b{\cdot}\sin(\phi)$

$\mathbf{c} = \mathbf{a} \times \mathbf{b}$ is perpendicular to plane of **a** and **b**

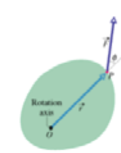
$c_x = a_y{\cdot}b_z - a_z{\cdot}b_y$ $c_y = -a_x{\cdot}b_z + a_z{\cdot}b_x$ $c_z = a_x{\cdot}b_y - a_y{\cdot}b_x$

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$ $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$



## Rotational Analogy to Linear Motion

| | *Translation* | *Rotation* |
|---|---|---|
| position | $x$ | $\theta$ |
| velocity | $v = dx/dt$ | $\omega = d\theta/dt$ |
| acceleration | $a = dv/dt$ | $\alpha = d\omega/dt$ |
| mass | $m$ | $I = \Sigma m_i r_i^2$ |
| Kinetic Energy | $K = \frac{1}{2}mv^2$ | $K = \frac{1}{2}I\,\omega^2$ |
| Force | $F = ma$ | $\tau_{net} = I \cdot \alpha$ |




$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$

Angular Displacement

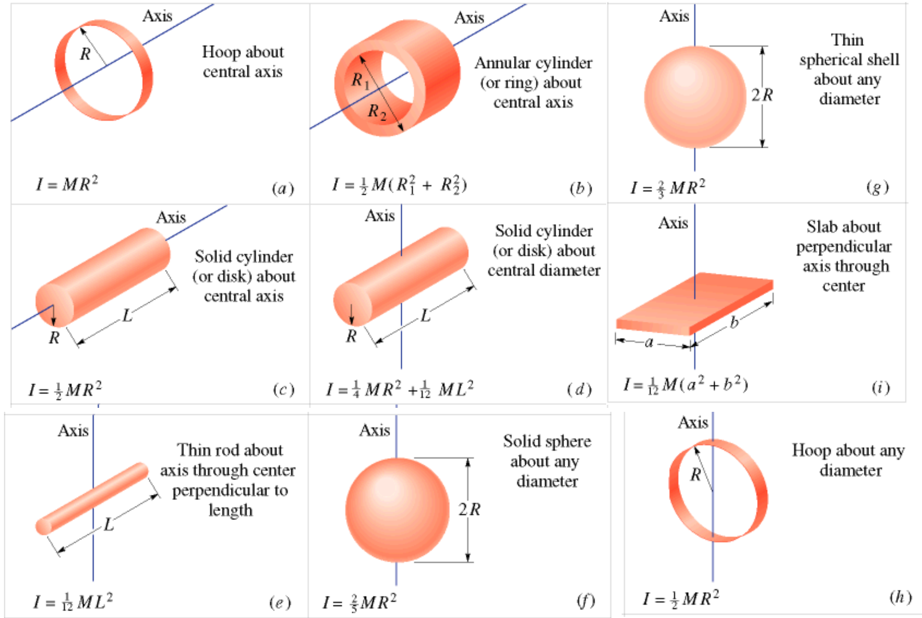
Angular Velocity

Angular Acceleration

$\theta,\ \vec{\omega},\ \vec{\alpha}$

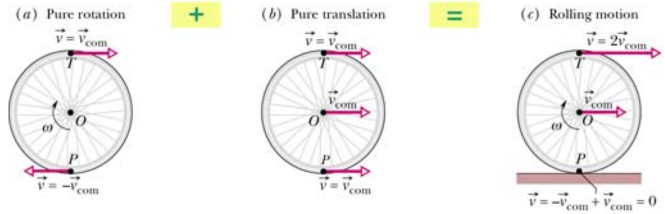
| Linear Equation | Missing Variable | | Angular Equation |
|---|---|---|---|
| $v = v_0 + at$ | $x - x_0$ | $\theta - \theta_0$ | $\omega = \omega_0 + \alpha t$ |
| $x - x_0 = v_0 t + \frac{1}{2}at^2$ | $v$ | $\omega$ | $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $v^2 = v_0^2 + 2a(x - x_0)$ | $t$ | $t$ | $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ |
| $x - x_0 = \frac{1}{2}(v_0 + v)t$ | $a$ | $\alpha$ | $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$ |
| $x - x_0 = vt - \frac{1}{2}at^2$ | $v_0$ | $\omega_0$ | $\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$ |

**TABLE 11-2** Rotational Inertia

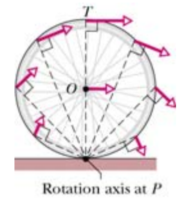



# Smooth rolling motion

## Rotation and Translation




## Reference frame




## Kinetic Energy of Rolling

$$K = \frac{1}{2} I_C \,\omega^2 + \frac{1}{2} M v_C^2$$

$$v_{com} = \omega R$$

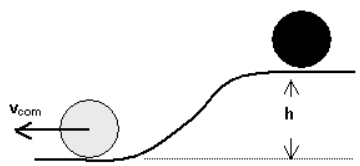
$$\boxed{\mathbf{K = \frac{1}{2}\left(\frac{I_C}{R^2} + M\right) v_C^2}}$$



# Example 1

Kinetic Energy of Rolling

$$\boxed{\mathbf{K = \frac{1}{2}\left(\frac{I_C}{R^2} + M\right) v_C^2}}$$




+ Energy conservation !!!

Kinetic Energy ←→ Potential Energy

$\Delta U + \Delta K = 0 \rightarrow U_{initial} = K_{final} \rightarrow Mgh = \frac{1}{2}\left(\frac{I_C}{R^2} + M\right) v_{com}^2$

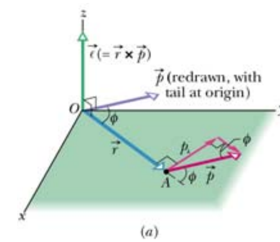
| Disk: | Hoop: | Sphere: |
|---|---|---|
| $I_{com} = ½ MR^2$ | $I_{com} = MR^2$ | $I_{com} = 2/5\ MR^2$ |

For disk: $Mgh = ½(1/2M + M)\, v^2_{com}$; $\mathbf{v_{com} = (4/3\ gh)^{½}}$



# Angular Momentum

$\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ [kg m²/s]

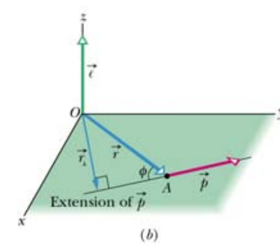



System of particles

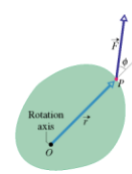
$\vec{L} = \vec{l}_1 + \vec{l}_2 + \ldots + \vec{l}_n = \sum_{i=1}^{n} \vec{l}_i$

L = m·r·v·sinϕ

For rotating body: $\mathbf{L = I\omega}$




# Torque:




$\vec{\tau} = [\vec{r} \times \vec{F}]$

$\tau = r \cdot F \cdot \sin\phi$

**Linear Momentum**

$\vec{p} = m\vec{v}$

[kg m/s]

**Angular Momentum**

$\vec{L} = \vec{r} \times \vec{p}$ [kg m²/s]

$\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$

$L = m \cdot r \cdot v \cdot \sin\phi$

**Both are vectors**

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\frac{d}{dt}(\vec{L}) = \vec{\tau} = I\vec{\alpha}$

For rotating body:

$L = I\omega$

$m \leftrightarrow I$

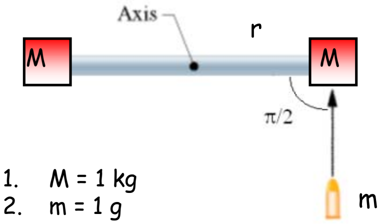
$v \leftrightarrow \omega$

**FOR ISOLATED SYSTEM: L IS CONSERVED**



## Angular Momentum Conservation:

"If the external torque is equal to zero, L is conserved"




1. $M = 1$ kg
2. $m = 1$ g
3. $r = 1$ m
4. $\omega_i = 0$  $\omega_f = 1$ rad/s
5. $v_{bullet} = ?$  $K_f/K_i = ?$

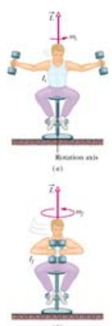
1. Define a rotational axis and the origin
2. Calculate **L** before interaction or any changes in I
3. Compare with **L** after the interaction or any change in I

1. $L_i = L_{bullet} = m \cdot v \cdot r \cdot \sin(\pi/2) = ???$
2. $L_f = I \cdot \omega = (Mr^2 + Mr^2 + mr^2)\,\omega_f = 2$ kg·m²/s
3. $L_i = L_f$ (angular momentum conserv.)
4. $v_{bullet} = \omega_f \cdot (2Mr^2 + mr^2)/mr = 2000$ m/s
5. $K_i = \frac{1}{2} m v^2_{bullet} = 2000$ J
6. $K_f = \frac{1}{2} I \omega^2 = 1$ J
7. $K_f/K_i = 1/2000$



## A Example:

A horizontal disc of rotational inertia **I = 1 kg.m²** and radius **100 cm** is rotating about a vertical axis through its center with an angular speed of **1 rad/s**. A wad of wet putty of mass **100 grams** drops vertically onto the disc from above and sticks to the edge of the disk. What is the angular speed of the disk right after the putty sticks to it?





## A Example:

A horizontal disc of rotational inertia **I = 1 kg.m²** and radius **100 cm** is rotating about a vertical axis through its center with an angular speed of **1 rad/s**. A wad of wet putty of mass **100 grams** drops vertically onto the disc from above and sticks to the edge of the disk. What is the angular speed of the disk right after the putty sticks to it?

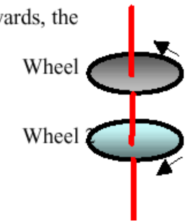
1. Define a rotational axis and the origin
2. Calculate **L** before interaction or any change in I
3. Compare with **L** after the interaction or any change in I

1. $L_i = I_i \cdot \omega_i = 1$ kg.m² · 1 rad/s = 1 kg·m²/s
2. $I_f = (I_i + mr^2) = (1$ kg.m² + 0.1 kg.m²)
3. $L_i = L_f$ (angular momentum conserv.)
4. $\omega_f = \cdot\omega_i\, I_i / I_f = 1$ rad/s · (1/1.1) = 0.91 rad/s

# More Examples:

2. One wheel of rotational inertia $I_1 = 2\ \mathrm{kgm}^2$ is rotating freely at 20 rad/sec in counterclockwise direction on a shaft whose rotational inertia is negligible. A second wheel of rotational inertia $I_2 = 5\ \mathrm{kgm}^2$, rotating freely at 15 rad/sec in the opposite direction, is suddenly coupled along the same shaft to the first wheel. Afterwards, the coupled wheel system rotates at

a. 1.00 rad/s, counterclockwise
b. 2.25 rad/s , clockwise
c. 4.50 rad/s , clockwise
d. 5.00 rad/s , counterclockwise
e. 5.00 rad/s , clockwise





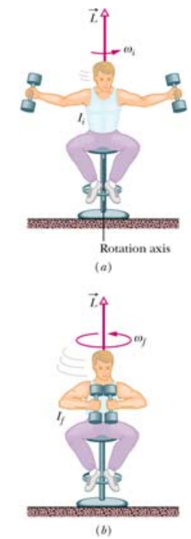

# More Examples:

3. A student, with arms at her sides, is spinning on a frictionless turntable. When the student extends her arms,
a. her angular velocity increases.
b. her angular velocity remains the same.
c. her rotational inertia decreases.
d. her rotational kinetic energy increases.
e. her angular momentum remains the same.

4. When a man on a frictionless rotating turntable extends his arms out horizontally, his angular momentum

A) must increase
B) must remain the same
C) must increase
D) may increase or decrease depending on his initial angular velocity
E) none of the above

5. A large bug walks from the center of a rotating turntable to its edge and stops. The angular velocity of the turntable
a. stays the same.
b. increases.
c. decreases.
d. can not be determined unless the mass of the bug and radius and rotational inertia of the turntable are given.
e. can not be determined even if the mass of the bug and radius and rotational inertia of the turntable are given.




$$L = I\omega$$

$$\vec{L} = \text{const.} \quad \Rightarrow \quad \vec{L}_i = \vec{L}_f$$



6. A wheel of moment of inertia of 5 kg m$^2$ starts from rest and accelerates under a constant torque of 3.0 N m for 8.0 seconds. What is the wheel's rotational kinetic energy at the end of 8 seconds?
**a. 57.6 J**
b. 64.0 J
c. 78.8 J
d. 122 J
e. 154 J

$$K = \frac{1}{2} I_C\, \omega^2$$

7. A 32-kg wheel, essentially a thin hoop, with moment of inertia I = 3 kg·m$^2$ is rotating at 280 rev/min. It must be brought to stop in 15 seconds. The required work to stop it is:
a. 1000 J
b. 1100 J
**c.** 1200 J
d. **1300** J

Work, constant torque

$$W = \tau(\theta_f - \theta_i)$$

8. A 10-kg disk with radius 30 cm must reach a final velocity of 300 rev/min in 10 sec. What is the required average power?

A) 10 W
B) 22 W
C) 45 W
D) 60 W
E) 72 W

Power, rotation about fixed axis

$$P = \frac{dW}{dt} = \tau\omega$$