

Electromagnetism II

Instructor: Andrei Sirenko

sirenko@njit.edu

Spring 2013

Thursdays 1 pm – 4 pm

PROBLEMS for CH. 6

<http://web.njit.edu/~sirenko/Phys433/PHYS433EandM2013.htm>

	week	Topic	Reading Materials	HW Assignment from the Textbook
1	Jan 24	Magnetostatics	Ch. 1 – 4 repeated Ch. 5	
2	Jan 31	Magnetostatics (cont.)	Ch. 5	HW1: 5.1 5.5 5.11 5.26 5.36
3	Feb 7	Magnetic Fields in Matter	Ch. 6	
4	Feb 14	Magnetic Fields in Matter	Ch. 6	HW2: 6.3 6.7 6.13 6.23 6.26
5	Feb 21	Electrodynamics	Ch. 7	
6	Feb 28	Electrodynamics / Maxwell Eqs.	Ch. 7	HW3:
7	March 7	CQZ1 on Ch 5 - 7 Conservation Laws	Ch. 8	
8	March 14	Conservation Laws	Ch. 8	HW4:

Classical consideration: diamagnetism

Can obtain the same formula classically:

Consider an electron rotating about the nucleus in a circular orbit; let a magnetic field be applied.

Before this field is applied, we have, according to Newton's second law,

$$F_0 = m\omega_0^2 r$$

F_0 is the attractive Coulomb force between the nucleus and the electron, and ω_0 is the angular velocity.

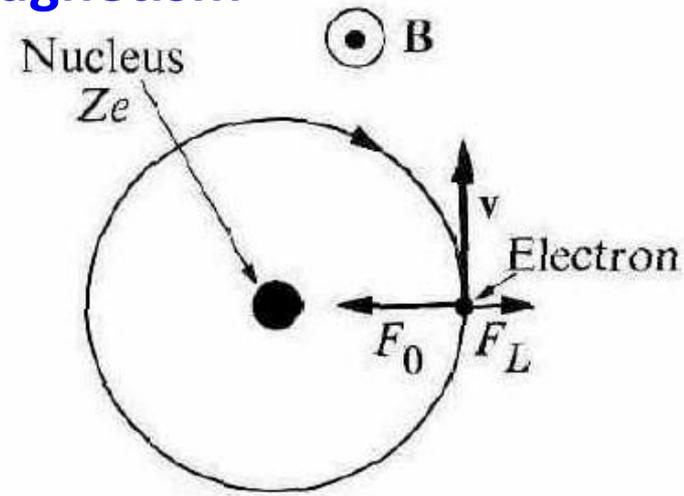
Applied field \rightarrow an additional force: the *Lorentz force*

$$F_L = -e(\mathbf{v} \times \mathbf{B})$$

$$F_L = -eB\omega r \Rightarrow F_0 - eB\omega r = m\omega^2 r \quad \omega = \omega_0 - \frac{eB}{2m}$$

Reduction in frequency \rightarrow corresponding change in the magnetic moment.

The change in the frequency of rotation is equivalent to the change in the current around the nucleus: $\Delta I = (\text{charge}) \times (\text{revolutions per unit time})$



$$\Delta I = -Ze \frac{1}{2\pi} \frac{eB}{2m}$$

The magnetic moment of a circular current is given by the product (current) x (area of orbit)

$$\Delta \mathbf{m} = -\frac{1}{2} e(\Delta v) R \hat{\mathbf{z}} = -\frac{e^2 R^2}{4m_e} \mathbf{B} \qquad \Delta \mu = -e \frac{1}{2\pi} \frac{eB}{2m} \pi \langle r_{xy}^2 \rangle = -\frac{e^2 \langle r_{xy}^2 \rangle}{4m} B$$

Here $\langle r_{xy}^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$. The mean square distance of the electrons from the nucleus is $\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$.

For a spherically symmetrical charge distribution $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = \langle r^2 \rangle / 3$

$$\Rightarrow \langle r_{xy}^2 \rangle = \frac{2}{3} \langle r^2 \rangle \quad \Rightarrow \quad \Delta \mu = -\frac{e^2 \langle r^2 \rangle}{6m} B \quad \Rightarrow \quad \chi = -\frac{\mu_0 e^2 N Z \langle r^2 \rangle}{6m}$$

Diamagnetism in ionic crystals and crystals composed of inert gas atoms: they have atoms or ions with complete electronic shells.

Another class of diamagnetics is noble metals, which will be discussed later.

$$\chi^{\text{molar}} = -Z_i N_A \frac{e^2}{6mc^2} \langle r^2 \rangle = -Z_i \left(\frac{e^2}{\hbar c} \right)^2 \frac{N_A a_0^3}{6} \langle (r/a_0)^2 \rangle.$$

Since $a_0 = 0.529 \text{ \AA}$, $e^2/\hbar c = 1/137$, and $N_A = 0.6022 \times 10^{24}$,

$$\chi^{\text{molar}} = -0.79 Z_i \times 10^{-6} \langle (r/a_0)^2 \rangle \text{ cm}^3/\text{mole}.$$

The quantity $\langle (r/a_0)^2 \rangle$ is of order unity, as is the number of moles per cubic centimeter (by which the molar susceptibility must be multiplied to get the dimensionless susceptibility

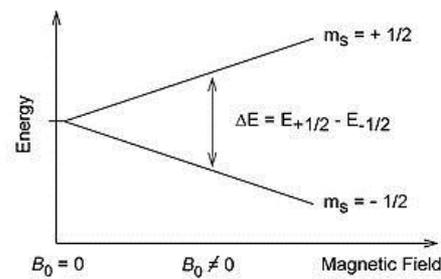
We conclude that diamagnetic susceptibilities are typically of order 10^{-5} ; i.e., M is minute compared with H .

MOLAR SUSCEPTIBILITIES OF NOBLE GAS ATOMS AND ALKALI HALIDE IONS^a

ELEMENT SUSCEPTIBILITY	ELEMENT SUSCEPTIBILITY	ELEMENT SUSCEPTIBILITY
	He -1.9	Li ⁺ -0.7
F ⁻ -9.4	Ne -7.2	Na ⁺ -6.1
Cl ⁻ -24.2	Ar -19.4	K ⁺ -14.6
Br ⁻ -34.5	Kr -28	Rb ⁺ -22.0
I ⁻ -50.6	Xe -43	Cs ⁺ -35.1

^a In units of $10^{-6} \text{ cm}^3/\text{mole}$. Ions in each row have the same electronic configuration. Source: R. Kubo and T. Nagamiya, eds., *Solid State Physics*, McGraw-Hill, New York, 1969, p. 439.

Quantum theory of paramagnetism



$$\Delta H = H - T_0 = \mu_B (\mathbf{L} + 2\mathbf{S}) \cdot \mathbf{B} + \frac{e^2}{8m} B^2 \sum_i (x_i^2 + y_i^2)$$

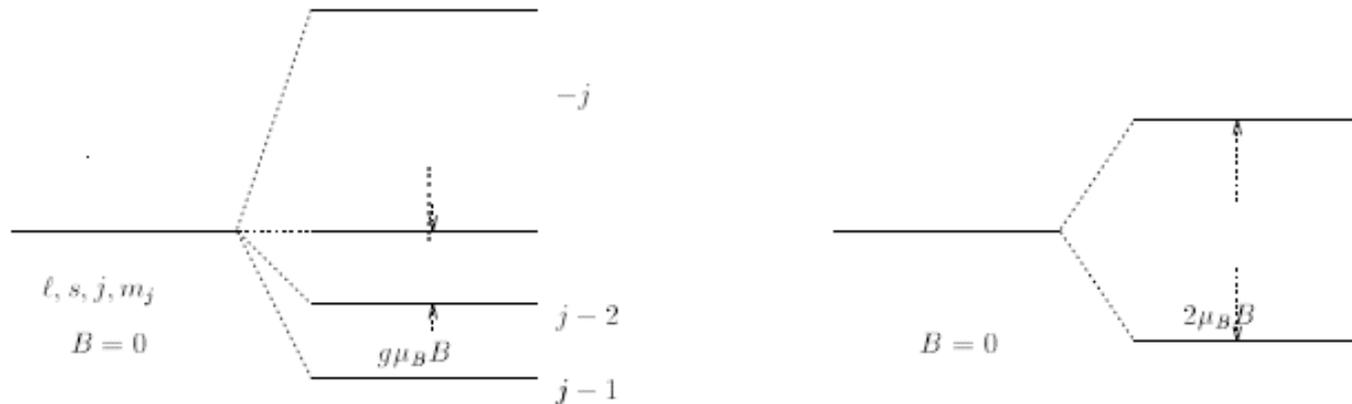
CGS:

$$\mathcal{H}'_{\text{para}} = \sum_i - \left(\frac{e}{2mc} \right) (\vec{L}_i + 2\vec{S}_i) \cdot \vec{B} = -\vec{\mu}_{\text{para}} \cdot \vec{B}$$

$$\vec{\mu}_{\text{para}} = \frac{e}{2mc} \sum_i (\vec{L}_i + 2\vec{S}_i).$$

1. determination of the moment $\vec{\mu}_{\text{para}}$
2. a statistical calculation of the average of $\vec{\mu}_{\text{para}}$.

Quantum theory of paramagnetism



(a)

- (a) Equally spaced levels for different m_j values $-j \leq m_j \leq j$ in a magnetic field where the Zeeman splitting between adjacent levels is $g\mu_B B$ where g is the Landé g -factor.
- (b) For a 2-level, spin up and spin down system, the Zeeman splitting is $2\mu_B B$.

1. determination of the moment $\vec{\mu}_{\text{para}}$
2. a statistical calculation of the average of $\vec{\mu}_{\text{para}}$.

Quantum theory of paramagnetism

If atoms in a solid have non-filled electronic shells than we have to take into account the first term in the Hamiltonian.

Its contribution is much larger than the contribution from the second term → can ignore it.

Consider the effect of the first term on an ion in a ground state which can be described by quantum numbers L, S, J and J_z , where \mathbf{J} is the total angular momentum, $\mathbf{J} = \mathbf{L} + \mathbf{S}$; and J_z is the projection of this momentum into a quantization axis.

It can be shown that

$$\langle LSJJ_z | \mathbf{L} + 2\mathbf{S} | LSJJ'_z \rangle = g \langle LSJJ_z | \mathbf{J} | LSJJ'_z \rangle$$
$$\mathbf{L} + g_0\mathbf{S} = g(JLS)\mathbf{J}.$$

where g is the g-factor, given by

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

This relation is valid only within the $(2J+1)$ -dimensional set of states that make up the degenerate atomic ground state in zero field, i.e. only for matrix elements taken between states that are diagonal in J, L , and S .

If the splitting between the zero-field atomic ground state multiplet and the first excited multiplet is large compared with $k_B T$ (as is frequently the case), then only the $(2J + 1)$ states in the ground-state multiplet will contribute appreciably to the energy.

In that case the first term in the Hamiltonian leads to the energy $E = -\boldsymbol{\mu} \cdot \mathbf{B}$

Magnetic moment of an ion is proportional to the total angular momentum of the ion:

$$\boldsymbol{\mu} = -g(JLS)\mu_B \mathbf{J}$$

The applied magnetic field lifts degeneracy of the ground state multiplet and splits it into $2J+1$ equidistant levels - *Zeeman splitting*.

The energies of these levels are given by

$$E_{J_z} = g\mu_B B J_z$$

J_z is an quantized and has values from $-J$ to J .

If thermal energy is less or comparable with the Zeeman splitting, these levels will be populated differently and give a different contribution to the magnetic moment of the ion.

Magnetization is determined by the average value of the magnetic moment:

$\mathbf{M} = N\langle\boldsymbol{\mu}\rangle$, where N is the concentration of ions in the solid;

$\langle\boldsymbol{\mu}\rangle$ - value of magnetic moment averaged over the Boltzmann distribution:

$$\mathbf{M} = N\langle\boldsymbol{\mu}\rangle = -Ng\mu_B \frac{\sum_{J_z=-J}^J J_z e^{\frac{g\mu_B B J_z}{k_B T}}}{\sum_{J_z=-J}^J e^{\frac{g\mu_B B J_z}{k_B T}}}$$

The summation can be done using the geometric progression formula.

Result for the magnetization:

$$M = NgJ\mu_B B_J(x)$$

$$x = \frac{g\mu_B JB}{kT}$$

and

$$B_J(x) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right)$$

- the Brillouin function

Magnetization for three different ions as a function of applied magnetic field

At relatively low fields and not too low temperatures we can expand the $\coth(x)$ assuming that $x \ll 1$:

$$\coth(x) \approx \frac{1}{x} + \frac{x}{3}$$

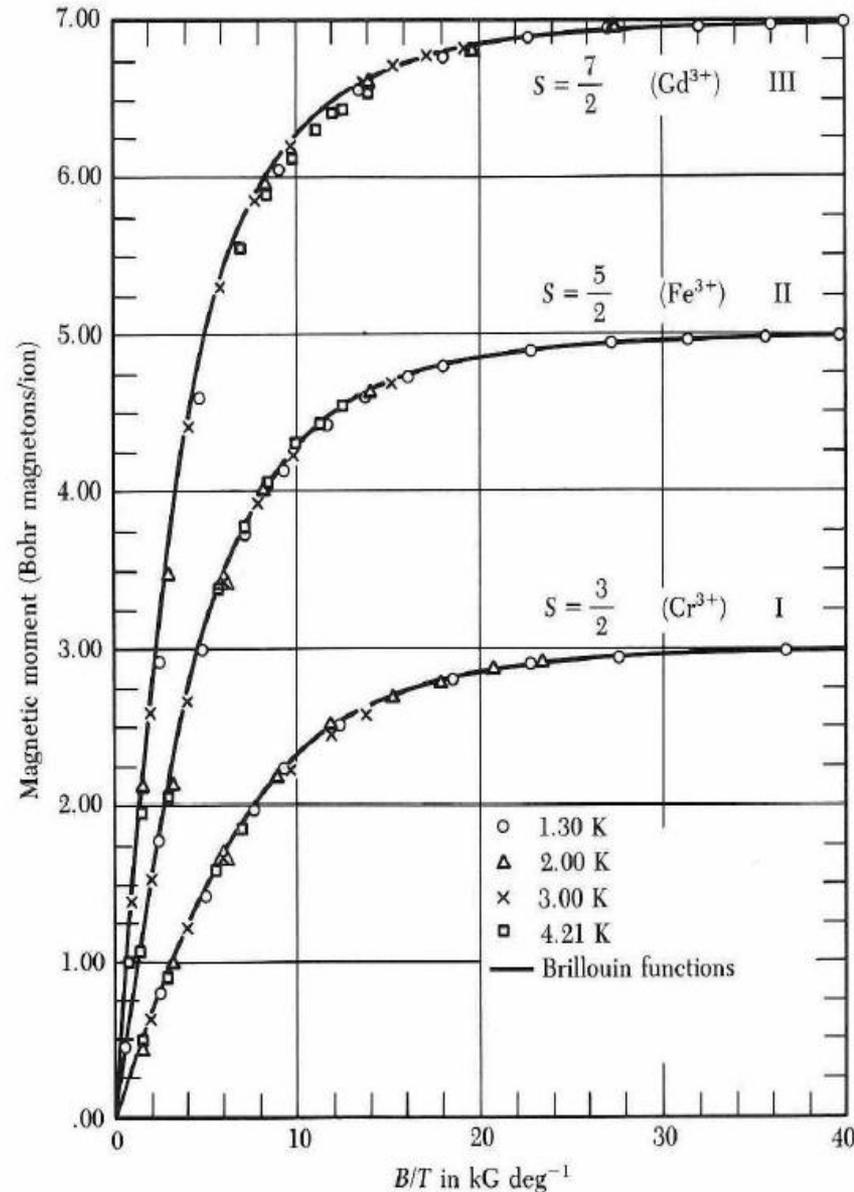
Get for the susceptibility:

$$\begin{aligned} \chi &= \mu_0 \frac{M}{B} = \frac{NJ(J+1)g^2\mu_B^2}{3k_B T} = \\ &= \frac{\mu_0 N p^2 \mu_B^2}{3k_B T} = \frac{C}{T} \quad \text{Curie Law} \end{aligned}$$

where p is the effective number of Bohr magnetons, defined as

$$p = g[J(J+1)]^{1/2}$$

C is the Curie constant



$$\chi = \frac{N}{V} \frac{(g\mu_B)^2 J(J+1)}{3 k_B T}, \quad (k_B T \gg g\mu_B H),$$

$$\chi^{\text{molar}} = N_A \frac{(g\mu_B)^2 J(J+1)}{3 k_B T}, \quad p = g [J(J+1)]^{1/2}$$

CALCULATED AND MEASURED EFFECTIVE MAGNETON NUMBERS p FOR THE IRON ($3d$) GROUP IONS^a

ELEMENT (AND IONIZATION)	BASIC ELECTRON CONFIGURATION	GROUND- STATE TERM	CALCULATED ^b p		MEASURED ^c p
			($J = S$)	($J = L \pm S $)	
Ti ³⁺	3d ¹	² D _{3/2}	1.73	1.55	—
V ⁴⁺	3d ¹	² D _{3/2}	1.73	1.55	1.8
V ³⁺	3d ²	³ F ₂	2.83	1.63	2.8
V ²⁺	3d ³	⁴ F _{3/2}	3.87	0.77	3.8
Cr ³⁺	3d ³	⁴ F _{3/2}	3.87	0.77	3.7
Mn ⁴⁺	3d ³	⁴ F _{3/2}	3.87	0.77	4.0
Cr ²⁺	3d ⁴	⁵ D ₀	4.90	0	4.8
Mn ³⁺	3d ⁴	⁵ D ₀	4.90	0	5.0
Mn ²⁺	3d ⁵	⁶ S _{5/2}	5.92	5.92	5.9
Fe ³⁺	3d ⁵	⁶ S _{5/2}	5.92	5.92	5.9
Fe ²⁺	3d ⁶	⁵ D ₄	4.90	6.70	5.4
Co ²⁺	3d ⁷	⁴ F _{9/2}	3.87	6.54	4.8
Ni ²⁺	3d ⁸	³ F ₄	2.83	5.59	3.2
Cu ²⁺	3d ⁹	² D _{5/2}	1.73	3.55	1.9

^a Because of quenching, much better theoretical values are obtained by taking J equal to S , the total spin, than by taking the value $J = |L \pm S|$ appropriate to the free ion.

^b Equation (31.50). In the case $J = S$, one takes $L = 0$.

^c Equation (31.49).

Source: J. H. Van Vleck, *The Theory of Electric and Magnetic Susceptibilities*, Oxford, 1952, p. 285; R. Kubo and T. Nagamiya, eds., *Solid State Physics*, McGraw-Hill, New York, 1969, p. 453.

Pauli Spin paramagnetism in metals

Arises from the fact that each conduction electron carries a spin magnetic moment which tends to align with the field.

An electron has spin $1/2$ - might expect a Curie-type paramagnetic contribution to the magnetization of the metal :

$$j = s = 1/2 \text{ and } g = 2. \quad g^2 j(j+1) = 2^2 \left(\frac{1}{2}\right) \left(\frac{3}{2}\right) = 3,$$

$$\chi = \frac{\mu_0 N \mu_B^2}{k_B T}$$

However, experiments show, that spin susceptibilities in metals are essentially **independent of temperature**, and the observed values are also considerably smaller.

Source of this discrepancy:

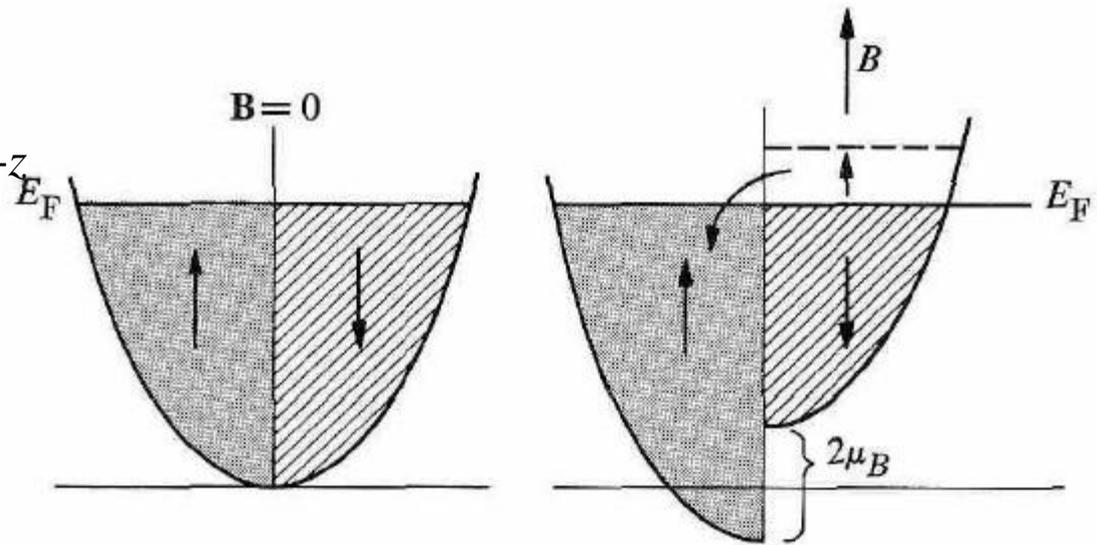
the above formula was derived on the basis of localized electrons obeying the Boltzmann distribution, whereas the conduction electrons are delocalized and satisfy the Fermi distribution.

The proper treatment must taking this into account.

No field: half the electrons have spins along $+z$, the other half – spins along $-z$ direction \Rightarrow

vanishing net magnetization.

Field is applied along the z -direction:



the energy of the spins $\parallel \mathbf{B}$ is lowered by the amount $\mu_B B$;
the energy of spins opposite to \mathbf{B} is raised by the same amount.

\Rightarrow some electrons near the Fermi level begin to transfer from the opposite-spin half to the parallel-spin one, leading to a net magnetization.

Note that only relatively few electrons near the Fermi level are able to flip their spins and align with the field. The other electrons, lying deep within the Fermi distribution, are prevented from doing so by the exclusion principle.

We can now estimate the magnetic susceptibility.

The electrons participating in the spin flip occupy an energy interval $\sim \mu_B B$

\Rightarrow their concentration is $N_{eff} = \frac{1}{2} D(E_F) \mu_B B$,

where $D(E_F)$ - the density of states at the Fermi energy level

Each spin flip increases the magnetization by $2\mu_B$ (from $-\mu_B$ to $+\mu_B$)

\Rightarrow the net magnetization

$$M = N_{eff} 2\mu_B = \mu_B^2 D(E_F) B$$

leading to a paramagnetic susceptibility

$$\chi = \mu_B^2 D(E_F) \mu_0$$

\Rightarrow susceptibility is determined by the density of states at the Fermi level.

$\Rightarrow \chi$ is essentially independent of temperature. (Temperature has only a small effect on the Fermi-Dirac distribution of the electrons)

\Rightarrow the derivation remains valid.

If we apply the results for free electrons: $D(E_F) = \frac{3N}{2E_F} = \frac{3N}{2k_B T_F}$,

then

$$\chi = \frac{3N \mu_B^2}{2k_B T_F} \mu_0$$

T_F - Fermi temperature ($E_F = k_B T_F$).

T_F is very large (often 30,000°K or higher), $\Rightarrow \chi$ is smaller by factor of $\sim 10^2$ - in agreement with experiment.

Landau diamagnetism

Conduction electrons also exhibit diamagnetism on account of the cyclotron motion they execute in the presence of the magnetic field.

Each electron loop is equivalent to a dipole moment whose direction is opposite to that of the applied field.

Classical treatment shows that the total diamagnetic contribution of all electrons is zero.

Quantum treatment however shows that for free electrons this causes a diamagnetic moment equal to $-1/3$ of the Pauli paramagnetic moment.

⇒ the total susceptibility of a free electrons gas is

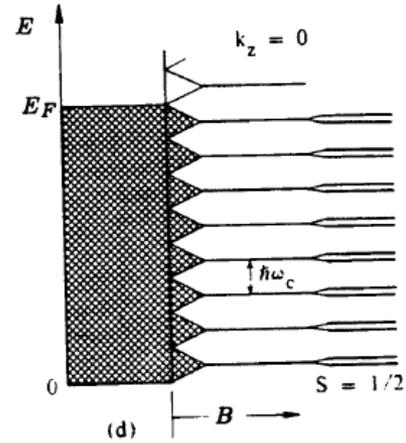
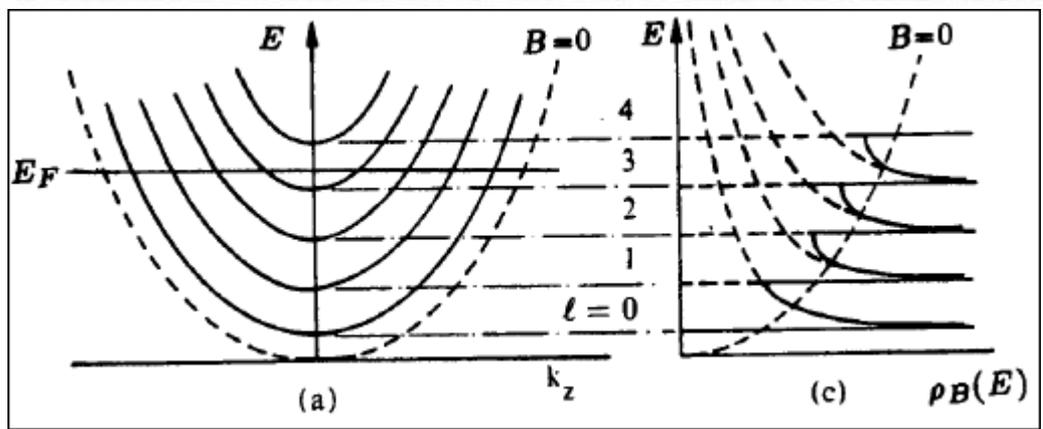
$$\chi = \frac{N\mu_B^2}{k_B T_F} \mu_0$$

The net response is therefore paramagnetic.

In comparing theoretical results with experiment, one must also include the diamagnetic effect of the ion cores (Langevin diamagnetism).

Susceptibilities of some monovalent and divalent metals $\times 10^6$ (Room T)

Element	χ_{total} (expt)	Experimental		Theoretical
		χ_{core}	$\chi_{\text{electron}} = \chi_{\text{total}} - \chi_{\text{core}}$	$\chi_{\text{electron}} = \chi_{\text{spin}} + \chi_{\text{orbit}}$
K	0.47	- 0.31	0.76	0.35
Rb	0.33	- 0.46	0.79	0.33
Cu	- 0.76	- 2.0	1.24	0.65
Ag	- 2.1	- 3.0	0.9	0.60
Au	- 2.9	- 4.3	1.4	0.60
Mg	0.95	- 0.22	1.2	0.65
Ca	1.7	- 0.43	2.1	0.5



Summary

- ❖ When a material medium is placed in a magnetic field, the medium is magnetized. Magnetisation is proportional to the magnetic field:

$$\mathbf{M} = \chi \mathbf{H} ; \quad \chi - \text{magnetic susceptibility of the medium}$$

$$\text{magnetic permittivity } \mu = \mu_0(1 + \chi); \quad \mu_r = 1 + \chi$$

- ❖ Langevin diamagnetism - ions or atoms with all electronic shells filled

negative magnetic susceptibility:

$$\chi = - \frac{\mu_0 e^2 N Z \langle r^2 \rangle}{6m}$$

- ❖ Langevin paramagnetism: if an atom has moment μ , then classical paramagnetic susceptibility

$$\chi = \frac{\mu_0 N \mu^2}{3k_B T}$$

quantum treatment \rightarrow same result for

- ❖ In metals, conduction electrons make a spin paramagnetic contribution: independent on T

$$\mu = g [J(J+1)]^{1/2} \mu_B$$

μ_0 -

- ❖ Conduction electrons also exhibit diamagnetism due to the cyclotron motion, which is equal to 1/3 of the spin $\chi = \mu_B^2 D(E_F)$ contribution.

- ❖ Ion core effect must also be taken into account

Magnetism / Magnetic Order

↑↑↑↑ ferromagnet

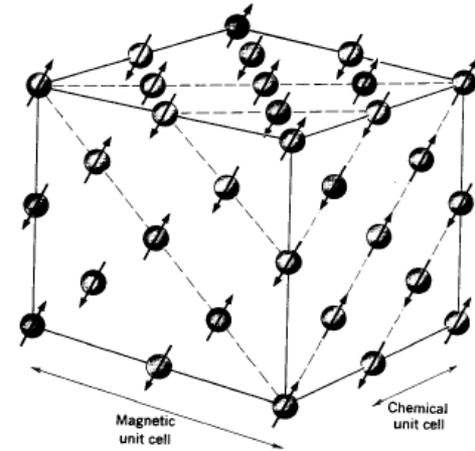
ferromagnet – the magnetic moments (spins) all line up parallel to one another, and

↑↓↑↓ antiferromagnet

antiferromagnet – equal magnetic moments (spins) on nearest neighbor sites which tend to line up antiparallel.

More complicated arrangements are possible like a

↑↓↑↓↑ ferrimagnet



Ordered arrangements of spins of the Mn²⁺ ions in MnO, as determined by neutron diffraction. The lattice for MnO is a NaCl structure, though the O²⁻ ions are not shown in the figure. The chemical and magnetic unit cells are indicated in the figure.

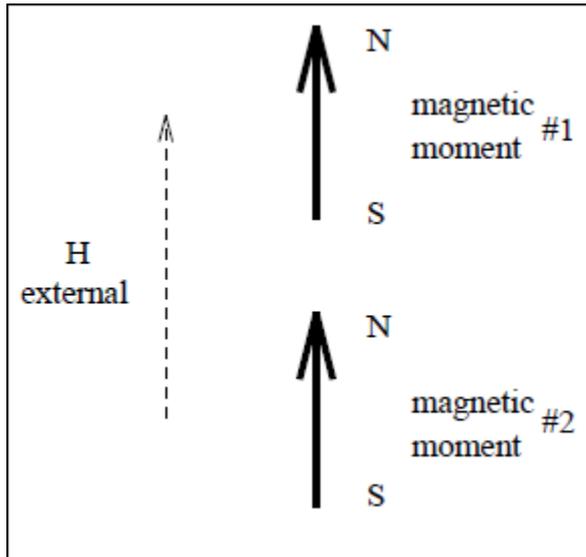
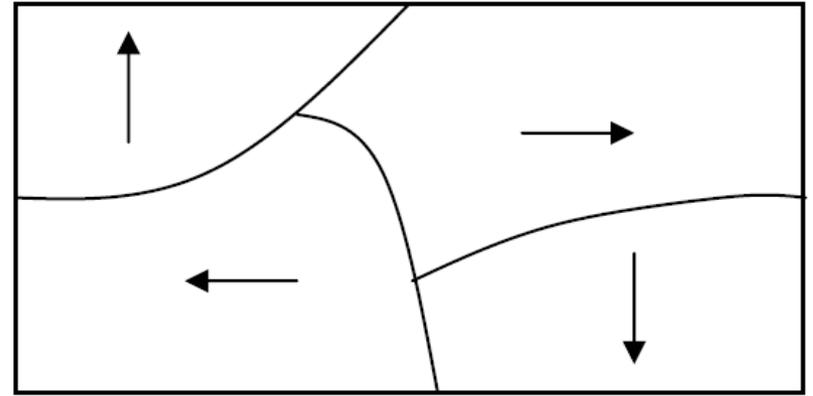


Table 7.1: Magnetic parameters for a number of ferromagnetic materials. Here n_B is the number of Bohr magnetons per atom and M_S is the spontaneous magnetization in units of gauss.

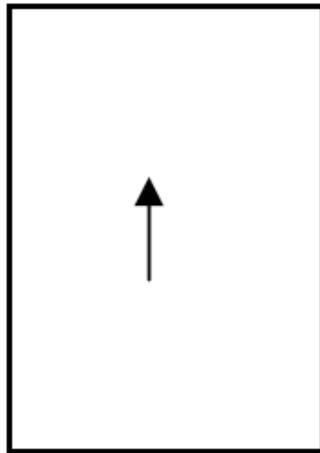
material	$M_S(300K)$	$M_S(0K)$	$n_B(0K)$	T_c (K)
Fe	1707	1740	2.22	1043
Co	1400	1446	1.72	1400
Ni	485	510	0.606	631
Gd	–	2010	7.10	292
Dy	–	2920	10.0	85
Cu ₂ MnAl	500	(550)	(4.0)	710
MnAs	670	870	3.4	318
MnBi	620	680	3.52	630
Mn ₄ N	183	–	1.0	743
MnSb	710	–	3.5	587
MnB	152	163	1.92	578

Domains

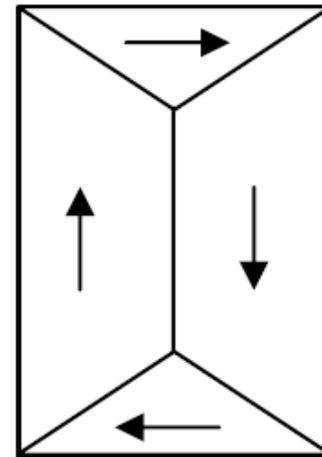
Spin in a material with long range magnetic ordering (ferromagnetic, antiferromagnetic etc.) form domains.



Reason for domain formation:



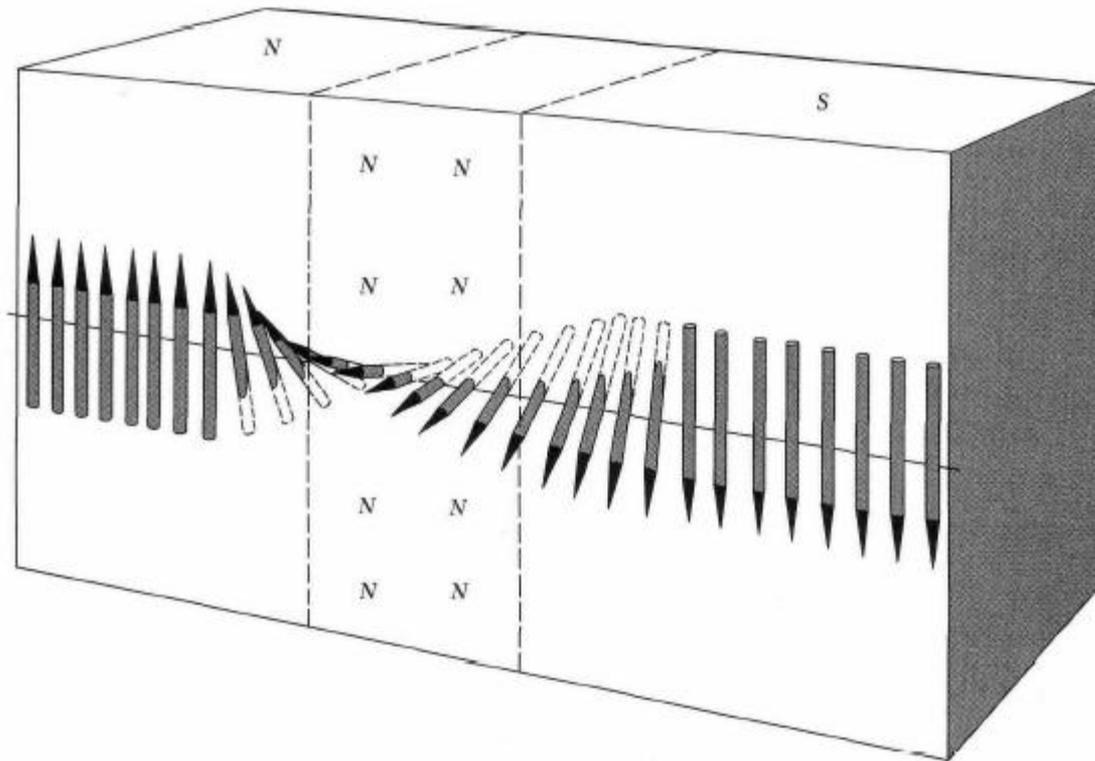
Higher energy



Lower energy

Domain walls

The boundary regions between neighboring domains



The typical size of the domains is 1-100 μm ;
the width of the domain walls is much smaller: ~ 100 nm.

→ the domain structure consists of uniformly magnetized domains separated by narrow boundaries.

Ferromagnetism

- *Ferromagnetism* is the phenomenon of spontaneous magnetization – the magnetization exists in a material in the absence of applied magnetic field.
- The best-known examples - transition metals Fe, Co, and Ni.
Also, other elements and alloys involving transition or rare-earth elements, such as the rare-earth metals Gd, Dy, and the insulating transition metal oxides (CrO_2).
- The phenomenon is restricted to transition and rare-earth elements \Rightarrow it is related to the unfilled $3d$ and $4f$ shells in these substances.
- Ferromagnetism involves the alignment of a significant fraction of the molecular magnetic moments in some favorable direction in the crystal.
- Ferromagnetism appears only below a certain temperature, which is known as the *ferromagnetic transition temperature* or simply as the *Curie temperature* (depends on the substance).

Curie-Weiss law

Above the Curie temperature, the moments are oriented randomly, resulting in a zero net magnetization.

In this region the substance is paramagnetic, and its susceptibility is given by

$$\chi = \frac{C}{T - T_C}$$

- *Curie-Weiss law.*
C - Curie constant
T_C - Curie temperature

The Curie-Weiss law can be derived using arguments (Weiss):

In the ferromagnetics the moments are magnetized spontaneously, which implies the presence of an internal field to produce this magnetization.

The field is assumed that this field is proportional to the magnetization:

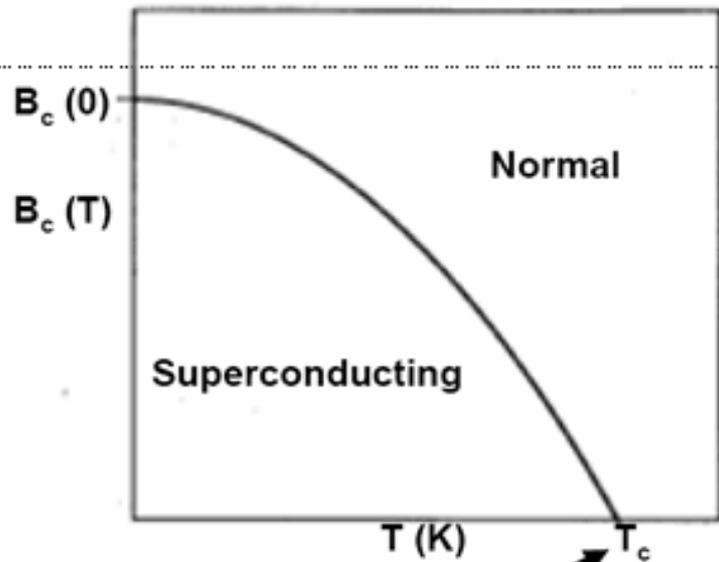
$$H_E = \lambda M \quad \lambda \text{ is the Weiss constant}$$

Weiss called this field the molecular field.

In reality, the origin of this field is the *exchange interaction*.

SUPERCONDUCTORS

B-T phase diagram



FERROMAGNETS

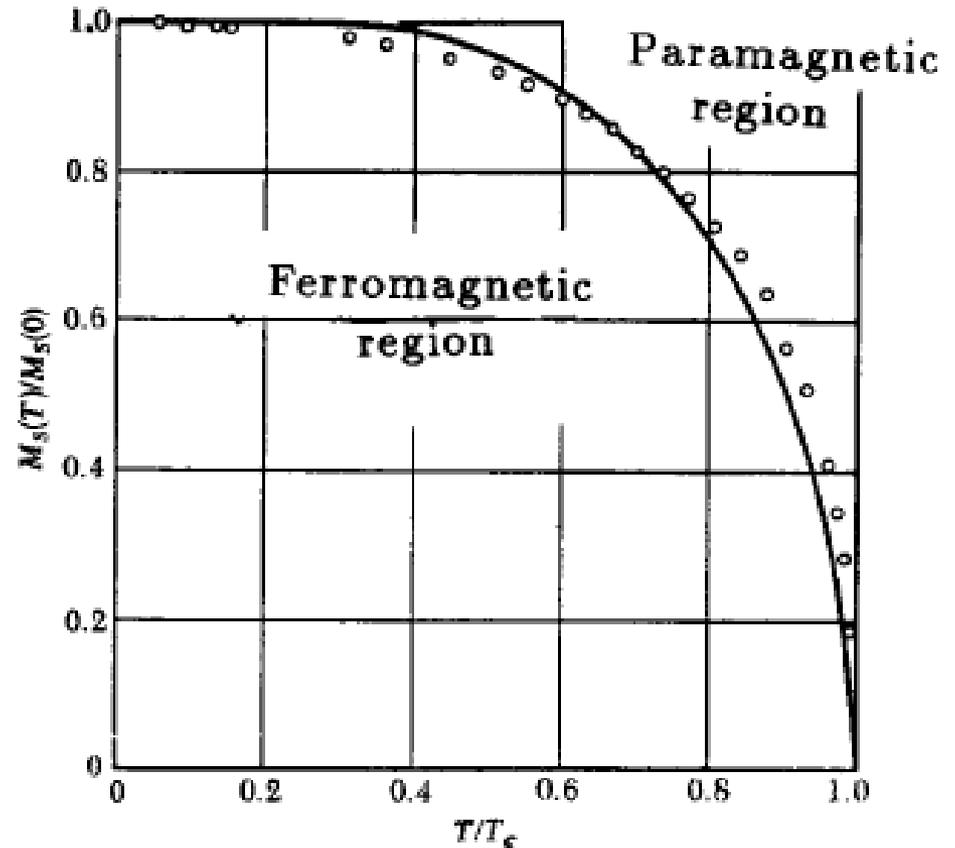


Figure 7.3: Phase diagram for a ferromagnetic system showing magnetization vs. T . The points are for Ni and the line is from mean field theory for $S = 1/2$.