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Gebrauchsanweisung
Instruction Sheet

332 101

Gravitations-Drehwaage
Gravitation Torsion balance

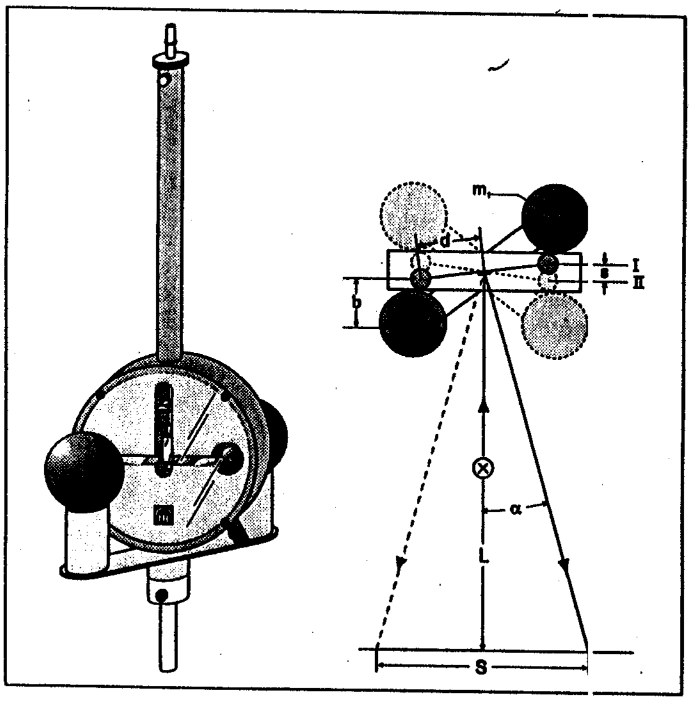


Fig. 1
Gravitations-Drehwaage (332 101) sowie schematische Darstellung zum Meßprinzip
Gravitation torsion balance (332 101) and schematic diagram of measuring principle

Das Gerät dient zur Demonstration der durch Gravitationskräfte verursachten Massenanziehung und zur Bestimmung der Gravitationskonstanten *f*.

Meßprinzip (Fig. 1)

Ein Torsionspendel mit einer Schwingungsdauer von etwa 10 min wird durch eine Positionsänderung der äußeren Massen m_1, die auf den hantelförmigen Pendelkörper wirken, in seinem statischen Gleichgewicht (Stellung I) gestört; es führt gedämpfte Schwingungen aus und schwingt in eine neue Gleichgewichtslage (Stellung II) ein. Der Winkel zwischen beiden Gleichgewichtslagen ist ein Maß für die wirksame Gravitationskraft.

Die Schwingung des Pendels, an dem ein Hohlspiegel angebracht ist, wird durch eine Lichtmarke angezeigt, wahlweise

- mit sichtbarem Licht direkt auf einer mm-Skala oder
- mit Infrarot-Licht unter Verwendung des IR-Position-Detectors (332 11), der eine Schreiberaufzeichnung oder eine computerunterstützte Meßwerterfassung ermöglicht.

Aus dem zeitlichen Verlauf der Schwingung, der Masse m_1 und der Geometrie der Anordnung ermittelt man die Gravitationskonstante *f* entweder nach der Endausschlagmethode oder (bei verkürztem Meßverfahren) nach der Beschleunigungsmethode.

Bei der *Endausschlagmethode* werden die Schwingungsdauer *T* des Torsionspendels und der Abstand *S* zwischen den Lichtzeigerpositionen für die beiden Gleichgewichtslagen ausgewertet:

$$f = \frac{\pi^2 \cdot b^2 \cdot d \cdot S}{m_1 \cdot T^2 \cdot L} \qquad \text{(I)}$$

Bei der *Beschleunigungsmethode* wird die Beschleunigung $a = 2s/t^2$ des Torsionspendels nach der Störung seiner Gleichgewichtslage durch die Massen m_1 ausgewertet:

$$f = \frac{S \cdot d \cdot b^2}{2\,m_1 \cdot t^2 \cdot L} \qquad \text{(II)}$$

The device can be used to demonstrate the gravitational attractive force between masses and to determine the gravitational constant *f*.

Measurement principle (Fig. 1)

The static equilibrium (position I) of a torsion pendulum with a period of approx. 10 min. is disturbed by a change in position of the outer masses m_1, which affect the dumbbell-shaped pendulum body. The oscillations become damped and the pendulum takes up a new equilibrium position (position 2). The angle between the two positions is a measure of the active gravitational force.

The oscillation of the pendulum, which is equipped with a concave mirror, is indicated by a light pointer. This is possible using

- visible light directly on a mm-scale or
- infrared light along with the IR position detector (332 11). The latter enables the measurement values to be plotted or evaluated by computer.

The gravitational constant *f* can be obtained from the oscillation curve with respect to time, the mass m_1 and the geometry of the arrangement using either the end deflection method or (in a quicker process) the acceleration method.

In the *end deflection method* the torsion pendulum period *T* and the distance *S* separating the light pointer positions are evaluated for the two equilibrium positions:

$$f = \frac{\pi^2 \cdot b^2 \cdot d \cdot S}{m_1 \cdot T^2 \cdot L} \qquad \text{(I)}$$

In the *acceleration method*, the acceleration of the torsion pendulum $a = 2s/t^2$ is evaluated using the masses m_1 after the equilibrium position has been disturbed:

$$f = \frac{S \cdot d \cdot b^2}{2\,m_1 \cdot t^2 \cdot L} \qquad \text{(II)}$$

1 Sicherheitshinweise

Das empfindliche Bronzeband des Torsionspendels vor unkontrollierter mechanischer Belastung schützen.

- Arretierschrauben ⑦ für das Schwingungssystem - siehe Fig. 2 - erst lösen, wenn das Gerät ordnungsgemäß montiert und in Versuchsposition gebracht ist.
- Schwingungssystem stets arretieren, wenn das Gerät nicht benutzt wird; Arretierung insbesondere beim Transport und bei der Montage sicherstellen.

Rändelschraube (3.1) - siehe Fig. 2 - zur Fixierung des bei Lieferung vorjustierten Torsionskopf nur dann geringfügig lockern, wenn nach ordnungsgemäßer Montage der Drehwaage bei deren Inbetriebnahme eine Feinjustierung des Nullpunktes erforderlich sein sollte.

Madenschraube (3.2) - siehe Fig. 2 - zur Fixierung des Pendelhalters nur lösen beim Austausch des Torsionsbandes gemäß Abschnitt 4.

2 Beschreibung, technische Daten, Lieferumfang (siehe Fig. 2)

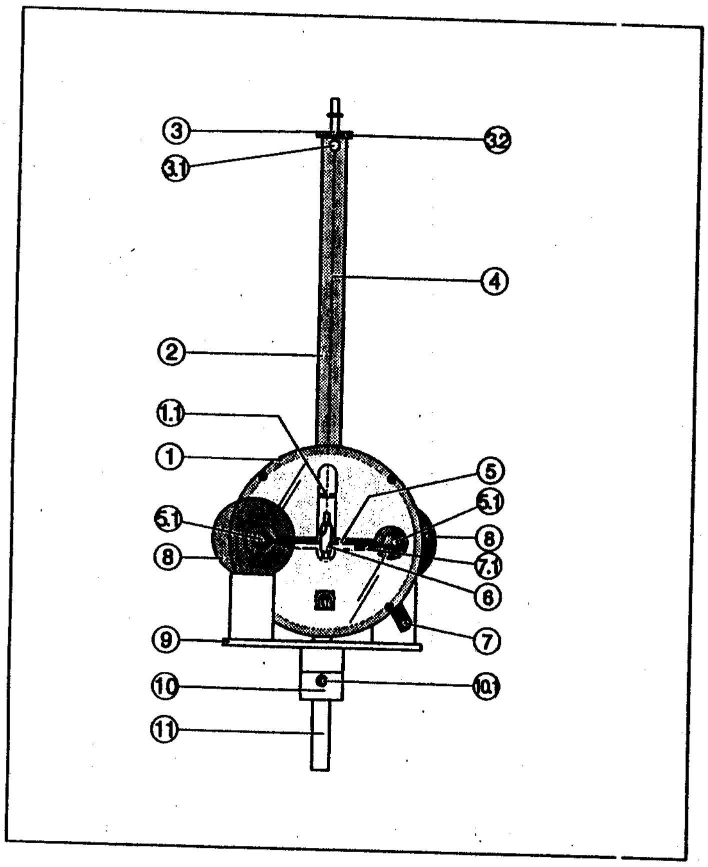


Fig.2
Gravitations-Drehwaage.
Gravitation torsion balance

① Metallgehäuse (Ø15 cm), mit Metallring und Glasabdeckungen; zweiteilige Blende (1.1) zur Verminderung störender Konvektionen zwischen Gehäuse und Schutzrohr ②
② Schutzrohr (25 cm lang) für Torsionsband ④
③ Torsionskopf mit Pendelhalter, bei gelockerter Schraube (3.1) drehbar zur Justierung der Gleichgewichtslage des Pendels; Pendelhalter mit Madenschraube (3.2) fixiert
Wichtig!
Rändelschraube (3.1) nur lösen, wenn eine Nullpunktjustierung gemäß Abschnitt 3.5 erforderlich ist.
Madenschraube (3.2) nur lösen beim Austausch des Torsionsbandes gemäß Abschnitt 4.
④ Torsionsband aus Bronze, 26 cm lang
Ersatzteil-Nr: 683 21
⑤ hantelförmiger Pendelkörper, bestehend aus 2 Bleikugeln (5.1) auf Metallstab
⑥ Hohlspiegel für Lichtmarken-Anzeige der Pendelbewegung

1 Safety notes

Protect the sensitive bronze band of the torsion pendulum from uncontrolled mechanical loading:

- Do not unscrew the oscillating system's locking screws ⑦ - see Fig. 2 - before the device has been correctly assembled and brought into position.
- Always lock the oscillating system when the device is not in use; in particular, make sure it is locked during transport and assembly.

The knurled screw (3.1) - see Fig. 2 - for fixing the torsion head (preadjusted on delivery) is only to be slightly loosened if a fine adjustment of the zero point proves necessary on putting the correctly-assembled torsion balance into operation. The grub screw (3.2) - see Fig. 2 - for fixing the pendulum holder is only to be loosened when replacing the torsion band as described in section 4.

2 Description, technical data, scope of supply (see Fig. 2)

① Metal housing (15 cm dia.), with metal ring and glass covers; two-part slide (1.1) to prevent interference from convection currents between the housing and protective tube (2)
② Protective tube (25 cm long) for torsion band ④
③ Torsion head with pendulum holder,
can be rotated for adjustment of the pendulum equilibrium position when screw has been loosened (3.1);
pendulum holder fixed with grub screw (3.2)
Important!
Do not loosen the knurled screw (3.1) unless it is essential to carry out a zero-point adjustment in accordance with section 3.5. Only loosen the grub screw (3.2) when the torsion band is being replaced as described in section 4.
④ Torsion band made from bronze, 26 cm long
Spare part no.: 683 21
⑤ Dumbbell-shaped pendulum body, consisting of 2 lead balls (5.1) on a metal rod

Brennweite *f* ca. 30 cm

⑦ Schrauben zur Arretierung des Pendelkörpers ⑤ durch Federpaar (7.1)

⑧ Paar große Bleikugeln
Ersatzteil-Nr. für 1 Kugel: 683 22

⑨ Kugelträger, um Stativstange ⑪ schwenkbar, zur versuchsgerechten Anordnung der großen Bleikugeln ⑧

⑩ Auflagering mit Fixierschraube (10.1) für Kugelträger

⑪ Stativstange (9 cm x 1,2 cm Ø) zu Aufbau des Gerätes in Stativmaterial

Im Lieferumfang enthalten:
1 m selbstklebendes Skalenband mit cm- und mm-Teilung

Versuchswichtige Daten (siehe Fig. 1):

Gehäusetiefe: 30 mm

Torsionspendel

Schwingungsdauer: etwa 10 min

Durchmesser / Masse m_2 einer Bleikugel (5.1): 15 mm / 20 g

Abstand *d* eines Kugelmittelpunktes zur Drehachse: 50 mm

Durchmesser / Masse m_1 einer großen Bleikugel ⑧: 64 mm / 1.5 kg ±5 g

Abstand *b* zwischen den Mittelpunkten der goßen Kugel (bei Gehäuseberührung) und der kleinen Kugel (in Nullage): 47 mm

3 Bedienung

Wichtig !
Zufriedenstellende Versuchsergebnisse werden nur dann erzielt, wenn das Torsionspendel einwandfrei justiert ist und wenn die durch die Massenanziehung bewirkten Torsionsschwingungen durch keine unerwünschten Pendelbewegungen beeinträchtigt werden.
Das Pendel reagiert sehr empfindlich auf Erschütterungen, die auf die Versuchsanordnung übertragen werden. Ein stabiler Aufbau an einer festen Wand oder auf einem schweren Tisch ist daher unerläßlich.

Temperaturschwankungen bewirken im Gehäuse der Drehwaage Konvektionen, die zu unerwünschten Bewegungen (s. Fig. 6) des Torsionspendels führen.
Deshalb ist der Experimentierplatz so zu wählen, daß die Drehwaage keiner Sonneneinstrahlung und keinen Luftbewegungen ausgesetzt ist.

3.1 Zusätzlich erforderliche Geräte

3.1.1. Schwingungsanzeige durch (sichtbare) Lichtmarke auf einer mm-Skala

1 Lampengehäuse	450 60
1 Lampe, 6 V, 30 W	450 51
1 asphärischer Kondensor (mit 1-mm-Spaltblende)	460 20
1 Wechselspannungsquelle, 6 V, 30 W z.B	562 73
1 Rollbandmaß	311 77
1 Stoppuhr z.B.	313 05
1 Schwebemagnet	510 44

Wand-Aufbau (siehe Fig. 3)

1 Großer Stativfuß	300 01
1 Paar Stellschrauben	301 06
1 Leybold-Muffe	301 01
1 Drehmuffe	301 03
1 Stativstange, 47 cm	300 42

Montagehilfen und -zubehör:
Bohrmaschine, Steinbohrer, Dübel (6 mm)

Tisch-Aufbau

1 Optische Bank mit Normalprofil, 1 m	460 32
2 Optikreiter z. B.	460 351

⑥ Concave mirror for light pointer indication of pendulum motion
Focal length *f* approx.30 cm

⑦ Screws for locking the pendulum body ⑤ using pair of springs (7.1)

⑧ Pair of large lead balls
Spare part no. for 1 ball: 683 22

⑨ Ball carrier, can be rotated around stand rod ⑪, for correct experimental arrangement of the large lead balls ⑧

⑩ Supporting ring with fixing screw (10.1) for ball carrier

⑪ Stand rod (9 cm x 1.2 cm dia.) for assembly of the device in stand material

Included in scope of supply:
1 m self-adhesive scale tape with cm and mm divisions

Important experimental data (see Fig. 1):

Housing depth: 30 mm

Torsion pendulum

Period: approx. 10 min

Diameter / mass m_2 of a lead ball (5.1): 15 mm / 20 g

Distance *d* between the center of a ball and the axis of rotation: 50 mm

Diameter / mass m_1 of a large lead ball ⑧: 64 mm / 1.5 kg ±5 g

Distance *b* between the center of the large ball (when contact is made with housing) and the small ball (in the equilibrium position): 47 mm

3 Operation

Important !
Satisfactory experiment results are only possible when the torsion pendulum has been properly adjusted and the torsion oscillations produced by attraction between the masses are not affected by unwanted pendulum movements. The pendulum is very sensitive to any disturbance of the experiment setup: make sure that the experiment setup is absolutely stable, e.g. by attaching it to a solid wall or placing it on a sturdy bench or table.

Temperature variations cause convection in the housing of the torsion balance, which in turn cause undesired motions of the torsion pendulum (see Fig. 6).
For this reason, select an experiment site which does not stand in direct sunlight or drafts.

3.1 Additional equipment required

3.1.1. Oscillation indication using (visible) light marks on a mm-scale

1 Lamp housing	450 60
1 Lamp, 6 V, 30 W	450 51
1 Aspherical condenser (with 1-mm slit screen)	460 20
1 A.C. voltage source, 6 V, 30 W e.g.	562 73
1 Steel tape measure	311 77
1 Stop-clock e.g.	313 05
1 Suspended magnet	510 44

Wall assembly (see Fig. 3)

1 Large stand base	300 31
1 Pair of levelling screws	300 06
1 Leybold multiclamp	301 01
1 Rotatable clamp	301 03
1 Stand rod, 47 cm	300 42

Assembly aids and accessories:
Electric drill, masonry drill bit, wall plugs (6 mm)

Benchtop assembly

1 Optical bench with normal profile, 1 m	460 32
2 Optical riders e.g.	460 351

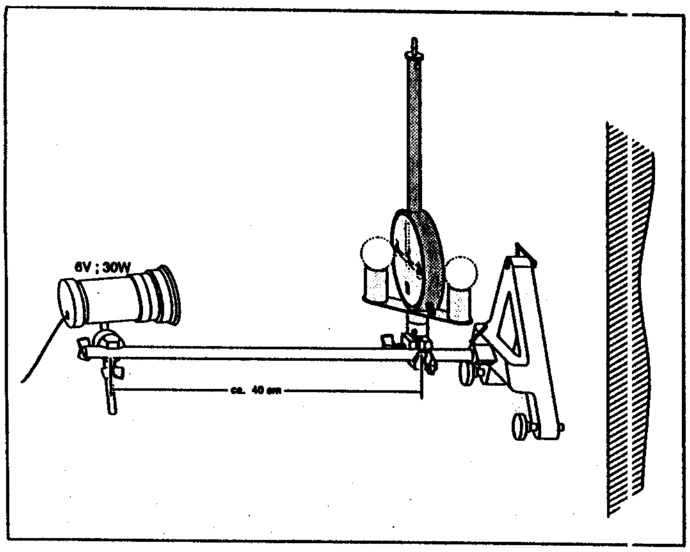


Fig. 3
Aufbau der Gravitations-Drehwaage; Anzeige des Schwingungsverlaufs durch (sichtbaren) Lichtzeiger auf einer mm-Skala (für IR-Positions-Detector, 332 11, ungeeignet!)

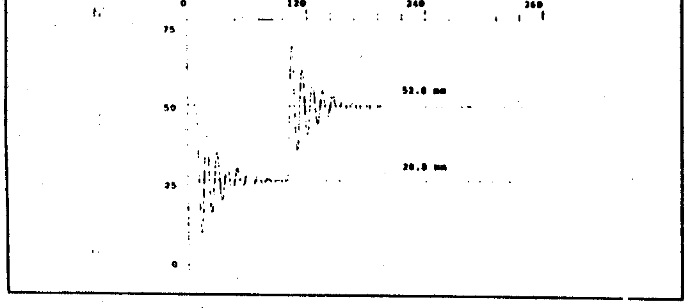
Setup for the gravitation torsion balance; display of the oscillation curve by means of (visible) light pointer on mm-scale (not suitable for IR position detector 332 11!)

3.1.2 Schreiberaufzeichnung oder computerunterstützte Erfassung der Schwingung mit Infrarot-Licht; Schienenaufbau auf einem Tisch (siehe Fig. 4)

1 Infrarot-Position-Detector	332 11
1 Wechselspannungsquelle, 12 V z.B.	562 73
1 TY-Schreiber z. B.	575 701
oder	
MS-DOS-Rechner über serielle Schnittstelle	
RS 232-Kabel	z.B. 530 008
Programm mit versuchsspezifischen Voreinstellungen z.B. auf der Demo-Diskette zur "Universellen Meßwerterfassung", enthalten in Lieferumfang des IR-Position-Detector (332 11); Meßbeispiel s. Fig. 4.1	
oder	
MS-DOS-Rechner mit Interface	
CASSYpack-E	524 007
Programm "Messen und Auswerten"	524 111
1 Rollbandmaß	311 77
1 Schwebemagnet	510 44
1 Optische Bank mit Normalprofil, 1 m	460 32
2 Optikreiter z. B.	460 351
1 Stativstange, 25 cm z. B.	300 41

Hinweis:

Weitere Informationen zum Einsatz des Infrarot-Position-Detector finden Sie in der zugehörigen Gebrauchsanweisung.



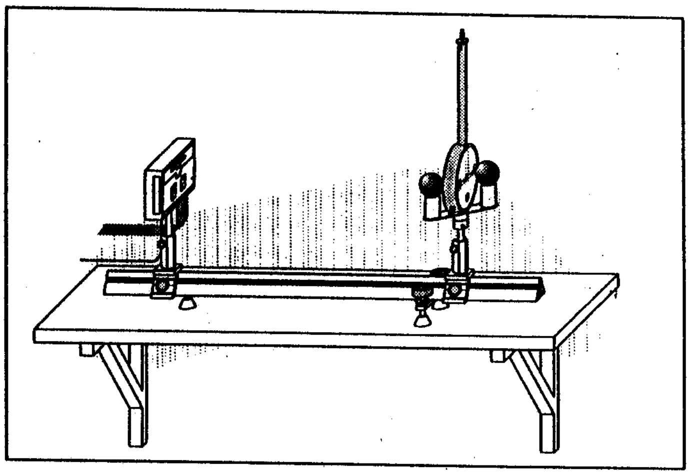


Fig. 4
Aufbau der Gravitations-Drehwaage; elektronische Erfassung des Schwingungsverlaufs durch IR-Position-Detector (332 11); Aufzeichnung mit TY-Schreiber oder Weiterverarbeitung mit Computerunterstützung (s. Fig. 4.1)

Benchtop assembly of the gravitation torsion balance; electronic determination of the oscillation curve using an IR position detector (332 11); recording with TY-recorder or further processing with computer assistance (see Fig. 4.1)

3.1.2 Recorder plotting or computer-assisted recording of the oscillation using infrared light; rail assembly on a bench (see Fig. 4)

1 Infrared position detector	332 11
1 A.C. voltage source, 12 V e.g.	562 73
1 TY-recorder e.g.	575 701
or	
MS DOS computer via serial interface	
RS 232 cable	e.g. 530 008
Program with experiment-specific default settings e.g. contained on the demo disk for "Universal Data Acquisition", included in the scope of supply of the IR position detector (332 11); see measuring example Fig. 4.1.	
or	
MS DOS computer with interface	
CASSYpack-E	524 007
Program "Measuring and Evaluating"	524 112
1 Steel tape measure	311 77
1 Suspended magnet	510 44
1 Optical bench with normal profile, 1 m	460 32
2 Optical riders e.g.	460 351
1 Stand rod, 25 cm e.g.	300 41

Note:

Further information on the use of the infrared position detector can be found in the corresponding Instruction Sheet.

Fig. 4.1
Computerunterstützte Aufzeichnung der Schwingungen der Gravitationsdrehwaage

Computer-assisted recording of the oscillations of the gravitation torsion balance

3.2 Wandmontage des Stativfußes (vor Erstinbetriebnahme erforderlich für einen Aufbau an der Wand gemäß Fig. 3)

Wichtig:
Diese Wandmontage des Stativmaterials ist ungeeignet für den Aufbau mit IR-Position-Detector (s. Abschnitt 3.1.2; Fig. 4)

Stativfuß an seiner Scheitelbohrung gemäß Fig. 5.1 - 5.3 fachgerecht eindübeln und mit Hilfe der Stellschrauben parallel zur Wand ausrichten; falls sich die Stellschrauben in die Wand eindrücken, feste Unterlage benutzen.

3.2 Mounting the stand base on the wall (required for wall mounting as shown in Fig. 3 before using the setup for the first time.)

Important:
Wall mounting of the stand material is not suitable for the experiement setup with the IR position detector 332 11 (see section 3.1.2; Fig. 4).

Correctly mount the stand base on its apex hole as shown in Fig. 5.1 - 3.3 and align parallel to the wall using levelling screws; use solid base material if the levelling screws press into the wall.

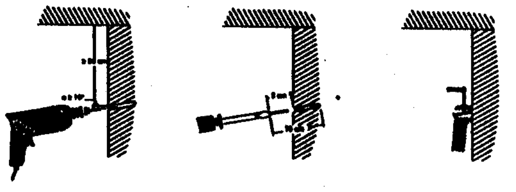


Fig. 5.1-5.3

Wandmontage des Stativfußes
Wall-mounting of the stand base

3.3 Versuchsvorbereitung

Anordnung - zunächst ohne die großen Bleikugeln - gemäß Fig. 3 an der Wand (oder sinngemäß auf einem stabilen Tisch auf der Optischen Bank) aufbauen.

Lampengehäuse so ausrichten, daß der reflektierte Strahl ungehindert auf die mindestens 5 m entfernte Skala projiziert wird.

Leuchtwendel lotrecht ausrichten, bevor der Kondensor mit Blendenhalter und Spalt (lotrecht!) auf das Lampengehäuse aufgesteckt wird.

Durch Verschieben des Einsatzes im Lampengehäuse Wendel scharf auf den Spiegel der Drehwaage abbilden. Dazu ein weißes Blatt Papier direkt vor die Waage halten, auf welchem die Leuchtwendel sichtbar wird.

Lampengehäuse so verschieben, daß der Spalt scharf auf der Skala abgebildet wird.

Erforderlichenfalls den Nullpunkt gemäß Abschnitt 3.5 justieren.

Bleikugeln auflegen und in eine Extremstellung bringen. Dabei Berühren des Gehäuses durch Finger oder Bleikugeln unbedingt vermeiden.

3.4 Versuchsdurchführung; Meßbeispiel

Wichtig!
Anordnung nach dem Aufbau gemäß Abschnitt 3.3 mindestens zwei Stunden erschütterungsfrei stehen lassen, sodaß das Pendel in die Gleichgewichtslage einschwingen kann.
Kugelträger ⑨ ohne Gehäuseberührung umschwenken.
Zur Verkürzung der Beruhigungszeit für das Pendel den Diamagnetismus des Bleis ausnutzen: Wenn sich eine der Bleikugeln des Pendelkörpers der Glasabdeckung nähert, einen starken Magneten bis zur Umkehr des Systems dagegenhalten, ohne das Glas zu berühren.
Versuchsanordung während der Meßwerterfassung keiner mechanischen Erschütterung und keinen Temperaturschwankungen, die zu Konvektionen in Gehäuse der Drehwaage führen können, aussetzen (s. Fig. 6)

Vor Beginn der Messungen Stabilität des Nullpunktes kontrollieren. Nullpunktschwankungen erforderlichenfalls über mindestens 10 Minuten beobachten und dokumentieren; daraus x_0 mitteln.

Zum Zeitpunkt $t = 0$ den Träger mit den Bleikugeln zügig, aber so vorsichtig von der einen in die andere Extremstellung schwenken, daß das Gehäuse weder von den Fingern noch von den Bleikugeln berührt wird. Unmittelbar nach dem Umschwenken Stoppuhr starten.

Für die Endausschlagmethode über mindestens 3 Schwingungsperioden, für die Beschleunigungsmethode über 1 Periode die Stellung des Lichtzeigers auf der Skala alle 30 s ablesen und notieren (Fig. 6)

3.3 Preparing for the experiment

Assemble the arrangement - initially without the large lead balls - on the wall as shown in Fig. 3 (or similarly on the optical bench on a sturdy lab bench).

Align the lamp housing so that the reflected beam is projected unobstructed onto the scale, which should be positioned at least 5 m distant.

Align the filament so that it is vertical before attaching the condenser with screen holder and slit (vertically!) to the lamp housing.

Move the insert in the lamp housing so that a focused image of the filament is projected onto the torsion balance mirror. To do this, hold a white sheet of paper, on which the filament forms an image, directly in front of the balance.

Move the lamp housing so that the slit is focused onto the scale.

If necessary, adjust the zero-point as described in section 3.5.

Attach the lead balls and bring the arrangement to an extreme position. In carrying out this step make absolutely sure that the housing does not come into contact with either your finger or a lead ball.

3.4 Experiment procedure; measuring example

Important!
After the experiment has been set up as described in section 3.3, it must be left undisturbed for at least two hours so that the pendulum can settle in its equilibrium state.
Turn the ball carrier ⑨ without touching the housing.
To shorten this settling time, make use of the diamagnetism of the lead: each time one of the pendulum lead balls approaches the glass cover, hold up a strong magnet - without touching the glass - until the motion of the system is reversed.
The experiment setup must not be subjected to any shocks or temperature variations which may cause convention in the torsion balance housing while recording measured values (see Fig. 6)

Check the zero-point stability before starting the measurement. If necessary, observe and note the zero-point fluctuations for at least 10 minutes so that you can determine an average value for x_0.

At time $t = 0$ rotate the carrier with the lead balls quickly from one extreme position to the other. However, in carrying out this movement, make sure that fingers or lead balls do not come into contact with the housing. Start the stop-clock immediately after shifting the carrier.

Read off and note the position of the light pointer on the scale every 30 s (Fig. 6) for at least 3 periods when using the end displacement method and for 1 period when using the acceleration method.

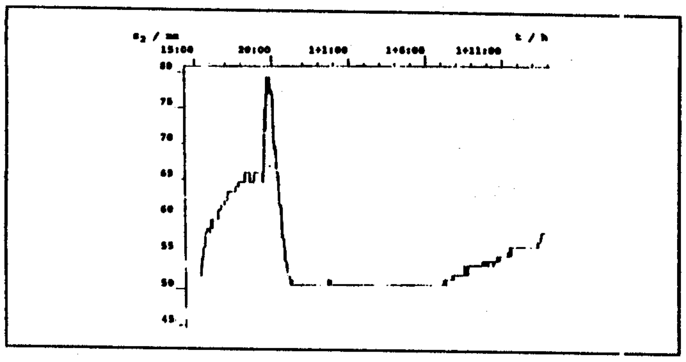


Fig. 6

24-Stunden-Protokoll über die Position des Torsionspendels: Positionsänderung im wesentlichen bedingt durch tageszeitabhängige Temperaturschwankungen (besonders ausgeprägt bei direkter Sonneneinstrahlung)

24-hour log of the position of the torsion pendulum: position changes are essentially due to (day)time dependent variations in temperature (particularly noticeable for exposure to direct sunlight)

Meßbeispiel:

Gerätekonstanten:

Masse der großen Bleikugel: m_1=1,5 kg
Abstand des Kugelmittelpunktes zur Drehachse: d = 0,05 m
Abstand zwischen den Mittelpunkten der goßen Kugel (bei Gehäuseberührung) und der kleinen Kugel (in Gleichgewichtslage): b = 0,047 mm

Abstand Spiegel - Drehwaage: L = 4,425 m

Schwingungsdauer T (aus Fig. 7):

$$T = \frac{(2790 - 315)\ \mathrm{s}}{4} = 618{,}8\ \mathrm{s}$$

Anfangsgleichgewichtslage: x_0 = 47 cm

Endgleichgewichtslage x_{00} (aus drei aufeinanderfolgenden Extrema ermitteln) z.B.

$$x_{00} = \frac{(x_1+x_3)/2 + x_2}{2} = \frac{x_1}{4}+\frac{x_2}{2}+\frac{x_3}{4} = 62{,}3\ \mathrm{cm}$$

Differenz S der Lichtzeigerpositionen für die Anfangs- und Endgleichgewichtslage des Pendels:

$$S = x_{00} - x_0 = 62{,}3\ \mathrm{cm} - 47\ \mathrm{cm} = 15{,}3\ \mathrm{cm}$$

Durch Einsetzen in Gleichung (1) für die Endausschlagmethode ergibt sich die Gravitationskonstante f

$$f = \frac{\pi^2 \cdot (0{,}047\ \mathrm{m})^2 \cdot 0{,}05\ \mathrm{m} \cdot 0{,}153\ \mathrm{m}}{1{,}5\ \mathrm{kg} \cdot (618{,}8\ \mathrm{s})^2 \cdot 4{,}425\ \mathrm{m}}$$

$$= 6{,}56\ 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^2}.$$

3.5 Nullpunkt-Justierung

Ziel der Nullpunkt-Justierung ist es, das Torsionspendel so zu positionieren, daß der hantelförmige Pendelkörper ⑤ (siehe Fig. 2) parallel zu den Glasplatten des Gehäuses ausgerichtet ist.

Eine Justierung ist erforderlich

eventuell vor der Erstinbetriebnahme, falls die vom Hersteller vorgenommene Justierung z. B. durch den Transport beeinträchtigt wurde,

nach unsachgemäßer Behandlung (durch unkontrolliertes Drehen von Torsionskopf ③),

nach dem Einsetzen eines neuen Torsionsbandes gemäß Abschnitt 4.

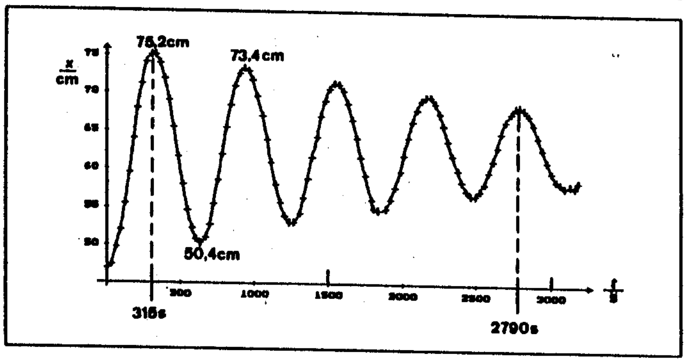


Fig. 7

Schwingungen der Gravitationsdrehwaage um die Endgleichgewichtslage x_{00}

Oscillations of the gravitation torsion balance around the final equilibrium position x_{00}

Measuring example:

Equipment constants:

Mass of the large lead ball: m_1=1.5 kg
Distance between ball center and axis of rotation: d = 0.05 m
Distance between center point of the large ball (when touching housing) and the small ball (in equilibrium position): b = 0.047 mm

Distance between mirror and torsion balance: L = 4.425 m

Oscillation period T (from Fig. 7):

T = (2790 - 315)/4 s= 618.8 s.

Equilibrium position at start: x_0 = 47 cm

Equilibrium position at end x_{00} (determined from three sequential extremes) e.g.

$$x_{00} = \frac{(x_1+x_3)/2 + x_2}{2} = \frac{x_1}{4}+\frac{x_2}{2}+\frac{x_3}{4} = 62.3\ \mathrm{cm}$$

Difference S between light pointer positions for the initial and final pendulum equilibrium states:

$$S = x_{00} - x_0 = 62.3\ \mathrm{cm} - 47\ \mathrm{cm} = 15.3\ \mathrm{cm}$$

Substitution in equation (1) for the end displacement method gives the gravitation constant f as

$$f = \frac{\pi^2 \cdot (0.047\ \mathrm{m})^2 \cdot 0.05\ \mathrm{m} \cdot 0.153\ \mathrm{m}}{1.5\ \mathrm{kg} \cdot (618.8\ \mathrm{s})^2 \cdot 4.425\ \mathrm{m}}$$

$$= 6.56\ 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^2}.$$

3.5 Zero-point adjustment

The objective of zero-point adjustment is to position the torsion pendulum so that the dumbbell-shaped pendulum body ⑤ (see Fig. 2) is aligned parallel to the glass plates of the housing in the equilibrium position.

An adjustment is necessary:

possibly before first using the system if the manufacturer's adjustment has been modified, e.g. as a result of transportation,

after improper handling (through uncontrolled rotation of torsion head) ③),

after a new torsion band has been inserted as described in section 4.

Vorbereitung zur Nullpunktjustierung:
Drehwaage mit Lichtzeigeranordnung z. B. entsprechend Abschnitt 3.3 (bzw. nach den entsprechenden Angaben in der Gebrauchsanweisung 332 11), jedoch ohne große Bleikugeln, auf einem Tisch aufbauen.

Torsionspendel durch Lösen von Schraube ⑦ (siehe Fig. 2) entarretieren;

Drehwaage über die Stellschrauben am Stativfuß (bzw. an der Optischen Bank) so ausrichten, daß das Torsionspendel lotrecht hängt (Kontrolle: Stift am Pendelende hängt in der Achse der Bohrung)

Anordnung nach diesen Vorbereitungen etwa 1 Tag erschütterungsfrei "aushängen" lassen, und dann die Nullpunktjustierung vornehmen.

Nullpunktjustierung:
Torsionspendel mit Schraube ⑦ arretieren und Drehwaage am Experimentierort aufbauen (z.B. Wandmontage gemäß Fig. 3).
Durch Entarretieren in Schwingungen versetzen.
Die beidseitigen Endausschläge des Lichtzeigers auf der Skala notieren.
Beobachten, zu welcher Ruhelage das System tendiert
zur Verkürzung der Beobachtungszeit Schwingungen diamagnetisch dämpfen: Wenn sich eine der Bleikugeln des Pendelkörpers der Glasabdeckung nähert, einen starken Magneten bis zur Umkehr des Systems dagegenhalten, ohne das Glas zu berühren.
Falls der Lichtzeiger nicht in eine Ruhelage tendiert, die etwa in der Mitte zwischen den beiden notierten Umkehrpunkten liegt, Torsionskopf ③ nach dem Lösen von Rändelschrauben (3.1) um einen kleinen Winkel in Richtung des "Soll-Nullpunktes" drehen.
Erneut die beiden Endausschläge sowie die Nullpunkttendenz des Lichtzeigers beobachten.
Die Nullpunktjustierung fortsetzen, bis der "Soll-Nullpunkt" in der Mitte zwischen den beiden Umkehrpunkten des Lichtzeigers erreicht ist.

4 Austausch des Torsionsbandes

Ein defektes Torsionsband kann gegen ein neues Band ausgetauscht werden, das als Ersatzteil (683 21) montagegerecht geliefert wird.
Der Austausch wird folgendermaßen durchgeführt:

Vorbereitung

- Drehwaage (mit arretiertem Pendelkörper) so in einem nivellierbaren Stativfuß (z. B. 300 01/06) aufbauen, daß sich die Rückseite vorne befindet; Schutzrohr mit Ersatz-Torsionsband (683 21) zur sicheren Handhabung in einer Klemme haltern (Fig. 8.1).
- Schraube (a_1) entfernen; danach die anderen Schrauben, welche die Glasplatte fixieren, lockern, bis die Platte entfernt werden kann (Fig. 8.2).
- Glasscheibe (a_2) und Abdeckung (a_3) entfernen sowie den vorderen Teil (a_4) der Blende herausziehen.

Ausbau des defekten Torsionsbandes

- Schraube (c_1) (Fig. 8.4) und - bei festgehaltenem Bandhalter (b) - Madenschraube (b_1) lösen und Bandhalter entnehmen (Fig. 8.3).
- Bei gerissenem Torsionsband Endstück (c) entfernen (Fig. 8.4).

Einbau des neuen Torsionsbandes

- Schraube x_1 lösen und bei festgehaltenem Bandhalter (B) Schraube x_2 entfernen (Fig. 8.5).
- Anordnung mit großer Vorsicht und ohne Wandberührung des sehr empfindlichen Torsionsbandes aus der Schutzhülle nehmen und in die Drehwaage fädeln, bis das Endstück (C) 1 mm bis 2 mm über der Bohrung (c_2) hängt; sodann Bandhalter (B) durch Anziehen von Schraube (b_1) fixieren (Fig. 8.6).

Preparation for zero-point adjustment:
Assemble the torsion balance with light pointer arrangement on the lab bench e.g. as described in section 3.3 (or in accordance with the corresponding information in the Instruction Sheet 332 11), but without the large lead balls.

Unlock the torsion pendulum by loosening the screws ⑦ (see Fig. 2).

Using the levelling screws on the stand base (or on the optical bench), align the torsion balance so that the torsion pendulum hangs vertically (check: pin at the end of the pendulum hangs in the axis of the hole).

After these preparatory steps have been carried out, allow the arrangement to hang disturbance-free for around 1 day and then carry out the zero-point adjustment.

Zero-point adjustment:
Lock the torsion pendulum with screw ⑦ and set up the torsion balance at the experiment location (e.g. wall mounting as shown in Fig. 3); then unlock the pendulum to set it in motion.
Note the end deflections of the light pointer on both sides of the scale.
Observe the equilibrium position to which the system tends;
to shorten the observation time damp the oscillations diamagnetically: each time one of the pendulum lead balls approaches the glass cover, use a strong magnet to reverse the motion of the system - without touching the glass.
Should the light pointer not show a tendency towards an equilibrium position roughly in the center between the two noted direction reversal points, unscrew the knurled screws (3.1) and turn the torsion head ③ through a small angle in the direction of the "desired zero-point".
Once again observe the two end deflections and the zero-point tendency of the light pointer.
Continue this zero-point adjustment until the "desired zero-point" in the middle between the two direction reversal points of the light pointer has been reached.

4 Replacing the torsion band

A defective torsion band can be replaced by a new band which is supplied in a ready-for-assembly state as spare part (683 21).
The replacement is carried out as follows:

Preparation

- Assemble the torsion balance (with locked pendulum body) on a stand base which can be levelled (e.g. 300 01/06) in such a way that the back side is at the front; to ensure safe handling, support the protective tube with replacement torsion band (683 21) in a clamp (Fig. 7).
- Remove screw (a_1); subsequently loosen the other screws fixing the glass plate until it can be removed (Fig. 8.2).
- Remove glass plate (a_2) and cardboard seal (a_3) and pull out the front section of the cardboard slide (a_4).

Removing the defective torsion band

- Loosen grub screw (b_1) while holding the band holder (b) tightly. Remove the band holder (Fig. 8.3).
- If the torsion band is broken, remove end piece (c) (Fig. 8.4).

Inserting the new torsion band

- Loosen screw x_1 and remove screw x_2 while holding band holder (B) tightly.
- Very carefully remove the arrangement from the protective cover and "thread" it into the torsion balance until the end piece (C) hangs 1 mm to 2 mm above the hole (c_2). Make sure the very sensitive torsion band does not come into contact with the walls. Subsequently fix pendulum holder (B) by tightening screw ((b_1) (Fig. 8.6).
- Wait until the torsion band has settled and, if necessary, use the levelling screws on the stand base to align the torsion band so that the end piece (C) hangs vertically above the hole.

- Beruhigung des Torsionsbandes abwarten und erforderlichenfalls Torsionsband über die Stellschrauben des Stativfußes so ausrichten, daß das Endstück (C) lotrecht über der Bohrung (c_2) hängt.
- Bandhalter (B) erneut festhalten und bei gelöster Schraube (b_1) um einige mm absenken, ohne ihn zu verdrehen, bis der abgedrehte Teil von Endstück (C) vollständig in die Bohrung (c_1) gleitet; Bandhalter erst dann loslassen, wenn er durch festes Anziehen der Madenschraube (b_1) (Fig. 8.7) sorgfältig fixiert ist; danach Endstück (C) mit Schraube (c_1) befestigen.
- Vorderen Blendenteil (a_4) einführen, Gehäuse nach dem ordnungsgemäßen Einsetzen von Abdeckung (a_3) und Glasplatte (a_2) wieder mit Schraube (a_1) verschließen.

Nach der Montage entarretiert man das Torsionspendel und überprüft, ob der Pendelkörper frei schwingen kann; liegt er trotz Entarretierung auf den Arretierfedern auf, so hebt man den Bandhalter (B) bei gelöster Schraube (b_1) geringfügig an und fixiert ihn dann wieder sorgfältig.

Bevor die erforderliche Nullpunkt-Justierung gemäß Abschnitt 3.5 durchgeführt werden kann, muß das Torsionspendel mindestens 12 Stunden "aushängen".

- Loosen screw (b_1). Hold the band holder (B) tightly and lower it a few mm - without twisting it - until the machined part of the end piece (C) has been completely guided into the hole (c_1); do not release the band holder until it has been carefully fixed in place by tightening the grub screw (b_1) (Fig. 8.7); finally fix the end piece (C) with screw (c_1).
- Insert the front section of the slide (a_4). After correctly reinserting the cardboard disc (a_3) and the glass plate (a_2), close the housing with screw (a_1).
- Insert cardboard slide (a_4), and then reclose the housing with screw (a_1) after correctly inserting sealing disc (a_3) and glass plate (a_2).

After assembly has been carried out, unlock the torsion pendulum and check if the pendulum body can oscillate freely; should it lie on the locking springs despite unlocking, slightly raise the band holder (B), with screw (b_1) loosened, and again carefully fix in place.

The torsion pendulum must hang undisturbed for at least 12 hours before the required zero-point adjustment as described in section 3.5 can be carried out.

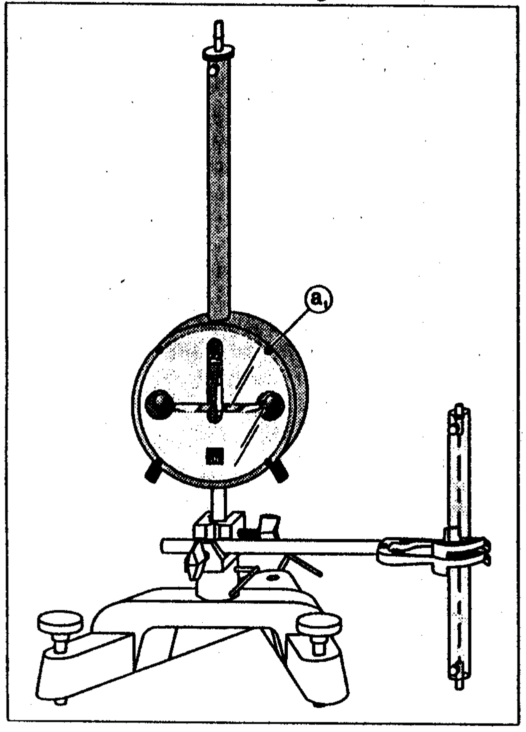


Fig. 8.1

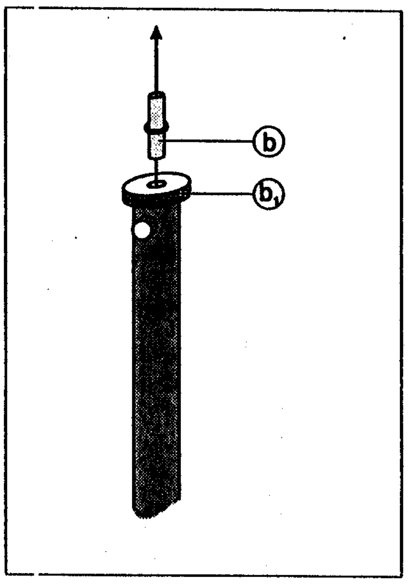


Fig. 8.3

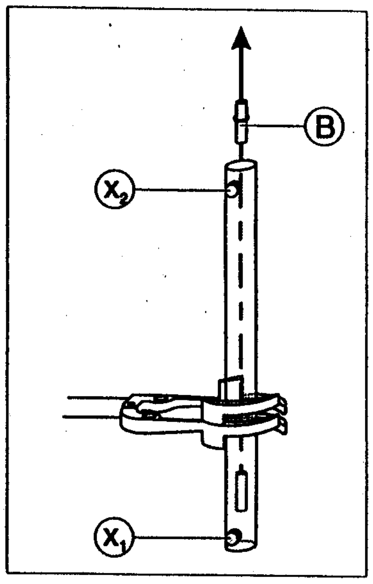


Fig. 8.5

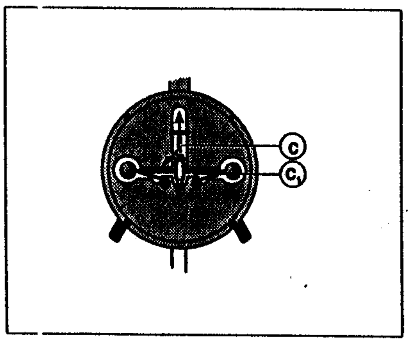


Fig. 8.4

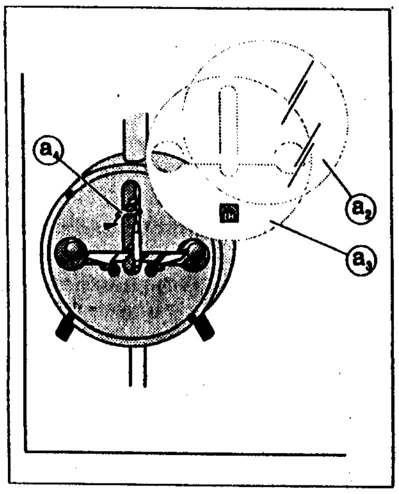


Fig. 8.2

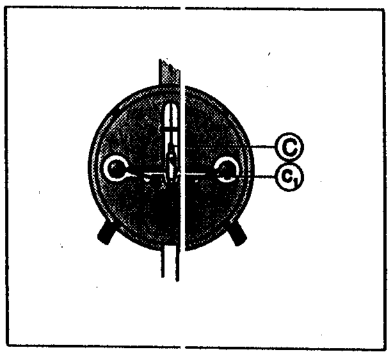


Fig. 8.7

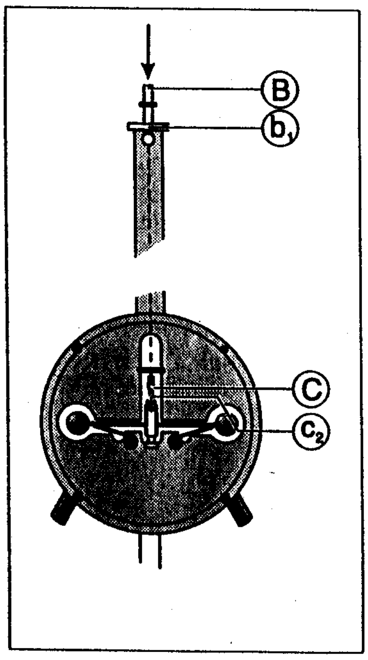


Fig. 8.6

The enormous strength of the Earth's attraction for the small masses, in comparison with their attraction for the large masses, is what originally made the measurement of the gravitational constant such a difficult task. The torsion balance (invented by Charles Coulomb) provides a means of negating the otherwise overwhelming effects of the Earth's attraction in this experiment. It also provides a force delicate enough to counterbalance the tiny gravitational force that exists between the large and small masses. This force is provided by twisting a very thin beryllium copper ribbon.

The large masses are first arranged in Position I, as shown in Figure 12, and the balance is allowed to come to equilibrium. The swivel support that holds the large masses is then rotated, so the large masses are moved to Position II, forcing the system into disequilibrium. The resulting oscillatory rotation of the system is then observed by watching the movement of the light spot on the scale, as the light beam is deflected by the mirror.

Any of three methods can be used to determine the gravitational constant, *G*, from the motion of the small masses. In Method I, the final deflection method, the motion is allowed to come to resting equilibrium—a process that requires several hours—and the result is accurate to within approximately 5%. In method II, the equilibrium method, the experiment takes 90 minutes or more and produces an accuracy of approximately 5% when graphical analysis is used in the procedure. In Method III, the acceleration method, the motion is observed for only 5 minutes, and the result is accurate to within approximately 15%.

➤ **Note:** 5% accuracy is possible in Method I if the experiment is set up on a sturdy table in an isolated location where it will not be disturbed by vibration or air movement.

➤ **Note:** 5% accuracy is possible in Method II if the resting equilibrium points are determined using a graphical analysis program.

METHOD I: Measurement by Final Deflection

Setup Time: ~ 45 minutes; Experiment Time: several hours
Accuracy: ~ 5%

Theory

With the large masses in Position I (Figure 13), the gravitational attraction, *F*, between each small mass (m_2) and its neighboring large mass (m_1) is given by the law of universal gravitation:

$$F = Gm_1m_2/b^2 \qquad (1.1)$$

where b = the distance between the centers of the two masses.

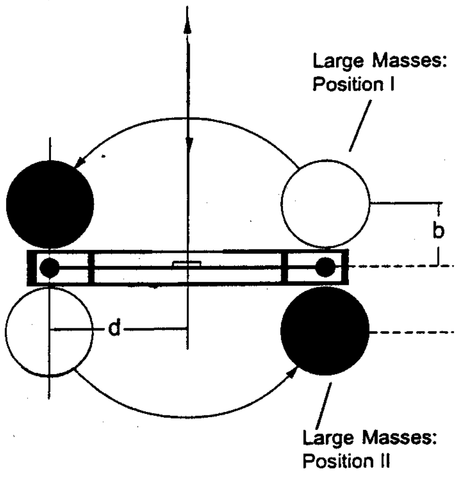


Figure 13
Origin of variables *b* and *d*

The gravitational attraction between the two small masses and their neighboring large masses produces a net torque (τ_{grav}) on the system:

$$\tau_{grav} = 2Fd \qquad (1.2)$$

where d is the length of the lever arm of the pendulum bob crosspiece.

Since the system is in equilibrium, the twisted torsion band must be supplying an equal and opposite torque. This torque (τ_{band}) is equal to the torsion constant for the band (κ) times the angle through which it is twisted (θ), or:

$$\tau_{band} = -\kappa\theta. \qquad (1.3)$$

Combining equations 1.1, 1.2, and 1.3, and taking into account that $\tau_{grav} = -\tau_{band}$, gives:

$$\kappa\theta = 2dGm_1m_2/b^2$$

Rearranging this equation gives an expression for G:

$$G = \frac{\kappa\theta b^2}{2dm_1m_2} \qquad (1.4)$$

To determine the values of θ and κ — the only unknowns in equation 1.4 — it is necessary to observe the oscillations of the small mass system when the equilibrium is disturbed. To disturb the equilibrium (from S_1), the swivel support is rotated so the large masses are moved to Position II. The system will then oscillate until it finally slows down and comes to rest at a new equilibrium position (S_2) (Figure 14).

At the new equilibrium position S_2, the torsion wire will still be twisted through an angle θ, but in the opposite direction of its twist in Position I, so the total change in angle is equal to 2θ. Taking into account that the angle is also doubled upon reflection from the mirror (Figure 15):

$$\Delta S = S_2 - S_1,$$

$$4\theta = \Delta S/L \qquad \text{or}$$

$$\theta = \Delta S/4L \qquad (1.5)$$

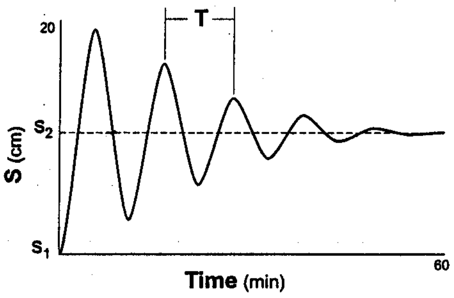


Figure 14
Graph of Small Mass Oscillations

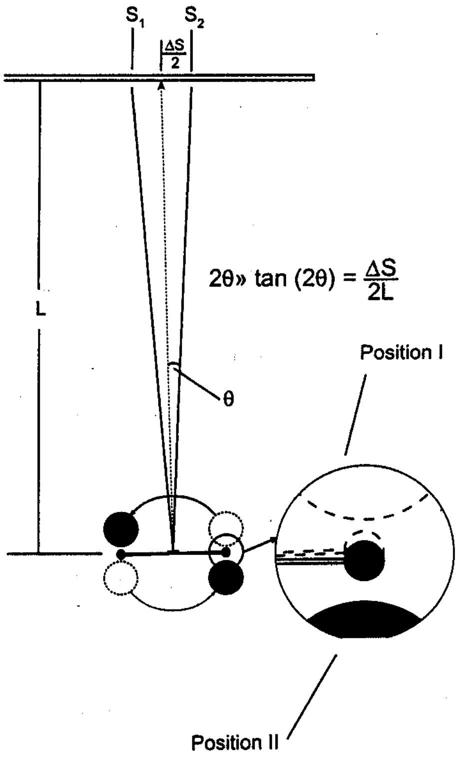


Figure 15
Diagram of the experiment showing the optical lever.

The torsion constant can be determined by observing the period (T) of the oscillations, and then using the equation:

$$T^2 = 4\pi^2 I/\kappa \qquad (1.6)$$

where I is the moment of inertia of the small mass system.

The moment of inertia for the mirror and support system for the small masses is negligibly small compared to that of the masses themselves, so the total inertia can be expressed as:

$$I = 2m_2(d^2 + \tfrac{2}{5}r^2) \qquad (1.7)$$

Therefore:

$$\kappa = 8\pi^2 m_2 \frac{d^2 + \tfrac{2}{5}r^2}{T^2} \qquad (1.8)$$

Substituting equations 1.5 and 1.8 into equation 1.4 gives:

$$G = \pi^2 \Delta S b^2 \frac{(d^2 + \tfrac{2}{5}r^2)}{T^2 m_1 L d} \qquad (1.9)$$

All the variables on the right side of equation 1.9 are known or measurable:

r = 9.55 mm
d = 50 mm
b = 46.5 mm
m_1 = 1.5 kg
L = (Measure as in step 1 of the setup.)

By measuring the total deflection of the light spot (ΔS) and the period of oscillation (T), the value of G can therefore be determined.

Procedure

1. Once the steps for leveling, aligning, and setup have been completed (with the large masses in Position I), allow the pendulum to stop oscillating.
2. Turn on the laser and observe the Position I end point of the balance for several minutes to be sure the system is at equilibrium. Record the Position I end point (S_1) as accurately as possible, and indicate any variation over time as part of your margin of error in the measurement.

3. Carefully rotate the swivel support so that the large masses are moved to Position II. The spheres should be just touching the case, but take care to avoid knocking the case and disturbing the system.

Note: You can reduce the amount of time the pendulum requires to move to equilibrium by moving the large masses in a two-step process: first move the large masses and support to an intermediate position that is in the midpoint of the total arc (Figure 16), and wait until the light beam has moved as far as it will go in the period; then move the sphere across the second half of the arc until the large mass support just touches the case. Use a slow, smooth motion, and avoid hitting the case when moving the mass support.

4. Immediately after rotating the swivel support, observe the light spot and record its position (S_1).

5. Use a stop watch to determine the time required for one period of oscillation (*T*). For greater accuracy, include several periods, and then find the average time required for one period of oscillation.

Note: The accuracy of this period value (*T*) is very important, since the *T* is squared in the calculation of G.

6. Wait until the oscillations stop, and record the resting equilibrium point (S_2).

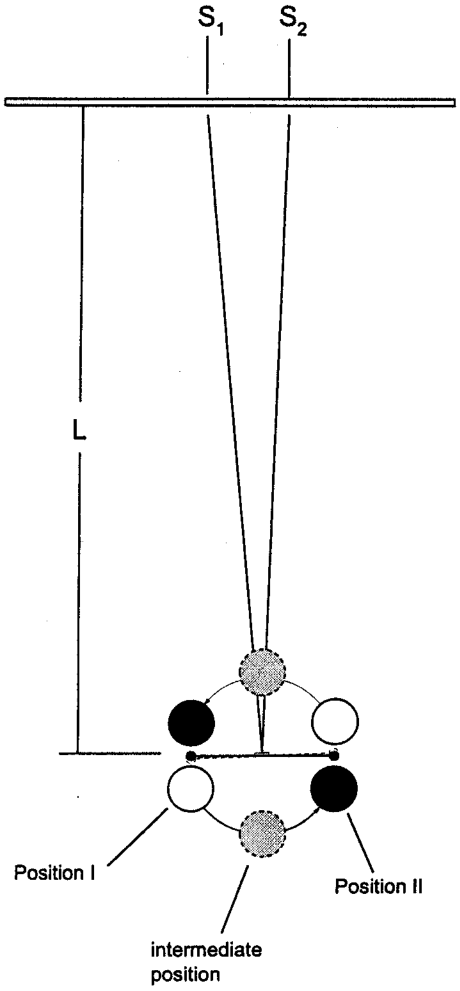


Figure 16
Two-step process of moving the large masses to reduce the time required to stop oscillating

Analysis

1. Use your results and equation 1.9 to determine the value of *G*.

2. The value calculated in step 2 is subject to the following systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, though with a much smaller force. The

geometry for this second force is shown in Figure 17 (the vector arrows shown are not proportional to the actual forces).

From Figure 17,

$$f = F_0 \sin\Phi$$

$$\sin\Phi = \frac{b}{(b^2 + 4d^2)^{1/2}}$$

The force, F_0 is given by the gravitational law, which translates, in this case, to:

$$F_0 = \frac{Gm_2m_1}{(b^2 + 4d^2)}$$

and has a component *f* that is opposite to the direction of the force *F*:

$$f = \frac{Gm_2m_1b}{(b^2 + 4d^2)(b^2 + 4d^2)^{\frac{1}{2}}} = \beta F$$

This equation defines a dimensionless parameter, *b*, that is equal to the ratio of the magnitude of *f* to that of *F*. Using the equation $F = Gm_1m_2/b^2$, it can be determined that:

$$b = b^3/(b^2 + 4d^2)^{3/2}$$

From Figure 17,

$$Fnet = F - f = F - bF = F(1 - b)$$

where F_{net} is the value of the force acting on each small sphere from *both* large masses, and *F* is the force of attraction to the nearest large mass only.

Similarly,

$$G = G_0(1 - b)$$

where *G* is your experimentally determined value for the gravitational constant, and G_0 is corrected to account for the systematic error.

Finally,

$$G_0 = G/(1 - b)$$

Use this equation with equation 1.9 to adjust your measured value.

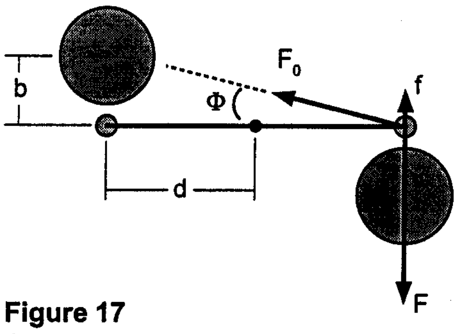


Figure 17
Correcting the measured value of *G*

METHOD II: Measurement by Equilibrium Positions

Observation Time: ~ 90+ minutes
Accuracy: ~ 5 %

Theory

When the large masses are placed on the swivel support and moved to either Position I or Position II, the torsion balance oscillates for a time before coming to rest at a new equilibrium position. This oscillation can be described by a damped sine wave with an offset, where the value of the offset represents the equilibrium point for the balance. By finding the equilibrium point for both Position I and Position II and taking the difference, the value of ΔS can be obtained. The remainder of the theory is identical to that described in Method I.

Note: To obtain an accuracy of 5% with this method, it is important to use graphical analysis of the position and time data to extrapolate the resting equilibrium positions, S_1 and S_2.

Procedure

1. Set up the experiment following steps 1–3 of Method I.
2. Immediately after rotating the swivel support to Position II, observe the light spot. Record the position of the light spot (*S*) and the time (*t*) every 15 seconds. Continue recording the position and time for about 45 minutes.
3. Rotate the swivel support to Position I. Repeat the procedure described in step 2.

Note: Although it is not imperative that step 3 be performed immediately after step 2, it is a good idea to proceed with it as soon as possible in order to minimize the risk that the system will be disturbed between the two measurements. Waiting more than a day to perform step 3 is not advised.

Analysis

1. Construct a graph of light spot position versus time for both Position I and Position II. You will now have a graph similar to Figure 18.
2. Find the equilibrium point for each configuration by analyzing the corresponding graphs using graphical analysis to extrapolate the resting equilibrium points S_1 and S_2 (the equilibrium point will be the center line about which the oscillation occurs). Find the difference between

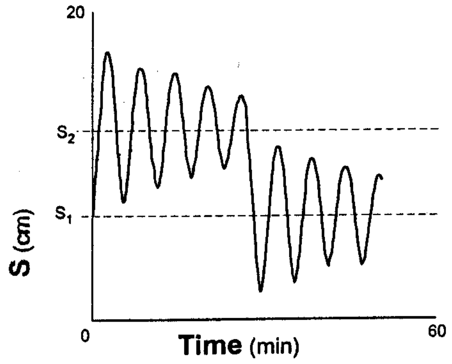


Figure 18
Typical pendulum oscillation pattern showing equilibrium positions

the two equilibrium positions and record the result as *ΔS.*

3. Determine the period of the oscillations of the small mass system by analyzing the two graphs. Each graph will produce a slightly different result. Average these results and record the answer as *T.*
4. Use your results and equation 1.9 to determine the value of *G.*
5. The value calculated in step 4 is subject to the same systematic error as described in Method I. Perform the correction procedure described in that section (*Analysis, step 3*) to find the value of G_0.

METHOD III: Measurement by Acceleration

Observation Time: ~ 5 minutes
Accuracy: ~ 15%

Theory

With the large masses in Position I, the gravitational attraction, *F*, between each small mass (m_2) and its neighboring large mass (m_1) is given by the law of universal gravitation:

$$F = Gm_1m_2/b^2 \qquad (3.1)$$

This force is balanced by a torque from the twisted torsion ribbon, so that the system is in equilibrium. The angle of twist, *θ*, is measured by noting the position of the light spot where the reflected beam strikes the scale. This position is carefully noted, and then the large masses are moved to Position II. The position change of the large masses disturbs the equilibrium of the system, which will now oscillate until friction slows it down to a new equilibrium position.

Since the period of oscillation of the small masses is long (approximately 10 minutes), they do not move significantly when the large masses are first moved from Position I to Position II. Because of the symmetry of the setup, the large masses exert the same gravitational force on the small masses as they did in Position I, but now in the opposite direction. Since the equilibrating force from the torsion band has not changed, the total force (F_{total}) that is now acting to accelerate the small masses is equal to twice the original gravitational force from the large masses, or:

$$F_{total} = 2F = 2Gm_1m_2/b^2 \qquad (3.2)$$

Each small mass is therefore accelerated toward its neighboring large mass, with an initial acceleration (a_0) that is expressed in the equation:

$$m_2a_0 = 2Gm_1\,m_2/b^2 \qquad (3.3)$$

Of course, as the small masses begin to move, the torsion ribbon becomes more and more relaxed so that the force decreases and their acceleration is reduced. If the system is observed over a relatively long period of time, as in Method I, it will be seen to oscillate. If, however, the acceleration of the small masses can be measured before the torque from the torsion ribbon changes appreciably, equation 3.3 can be used to determine G. Given the nature of the motion—damped harmonic—the initial acceleration is constant to within about 5% in the first one tenth of an oscillation. Reasonably good results can therefore be obtained if the acceleration is measured in the first minute after rearranging the large masses, and the following relationship is used:

$$G = b^2a_0/2m_1 \qquad (3.4)$$

The acceleration is measured by observing the displacement of the light spot on the screen. If, as is shown in Figure 19:

Δs = the linear displacement of the small masses,

d = the distance from the center of mass of the small masses to the axis of rotation of the torsion balance,

ΔS = the displacement of the light spot on the screen, and

L = the distance of the scale from the mirror of the balance,

then, taking into account the doubling of the angle on reflection,

$$\Delta S = \Delta s(2L/d) \qquad (3.5)$$

Using the equation of motion for an object with a constant

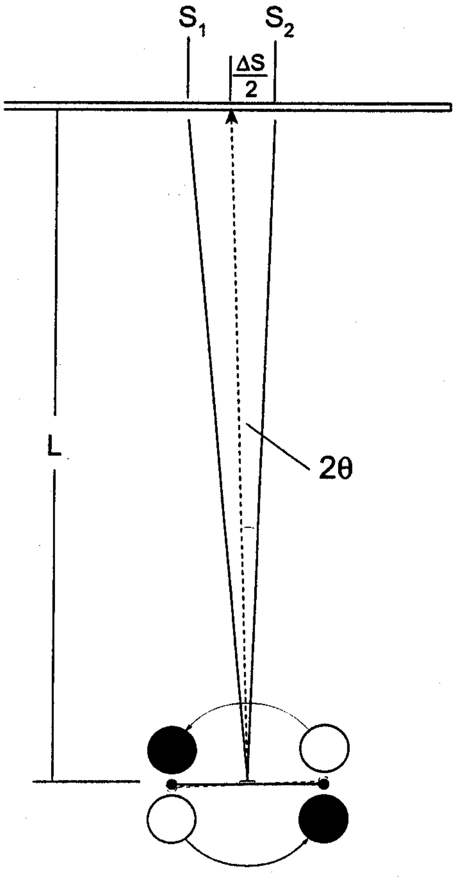


Figure 19
Source of data for calculations in Method III

acceleration ($x = 1/2\ at^2$), the acceleration can be calculated:

$$a_0 = 2\Delta s/t^2 = \Delta Sd/t^2L \qquad (3.6)$$

By monitoring the motion of the light spot over time, the acceleration can be determined using equation 3.6, and the gravitational constant can then be determined using equation 3.4.

Procedure

1. Begin the experiment by completing steps 1–3 of the procedure detailed in Method I.
2. Immediately after rotating the swivel support, observe the light spot. Record the position of the light spot (*S*) and the time (*t*) every 15 seconds for about two minutes.

Analysis

1. Construct a graph of light spot displacement ($\Delta S = S - S_1$) versus time squared (t^2), with t^2 on the horizontal axis (Figure 20). Draw a best-fit line through the observed data points over the first minute of observation.
2. Determine the slope of your best-fit line.
3. Use equations 3.4 and 3.6 to determine the gravitational constant.
4. The value calculated in step 3 is subject to a systematic error. The small sphere is attracted not only to its neighboring large sphere, but also to the more distant large sphere, although with a much smaller force. Use the procedure detailed in Method I (*Analysis, step 3*) to correct for this force.

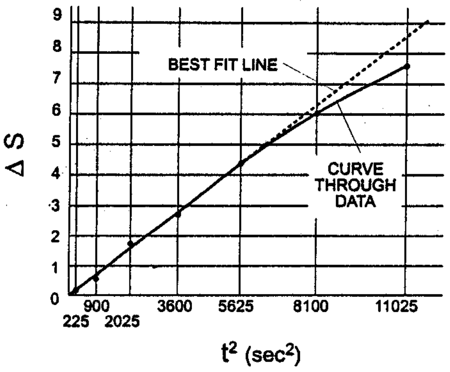


Figure 20
Sample data and best-fit line