Chapter 38

1. Let R be the rate of photon emission (number of photons emitted per unit time) of the Sun and let E be the energy of a single photon. Then the power output of the Sun is given by $P = RE$. Now

$$E = hf = hc/\lambda,$$

where $h = 6.626 \times 10^{-34}$ J·s is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and

$$R = \frac{\lambda P}{hc} = \frac{(550\,\text{nm})(3.9 \times 10^{26}\,\text{W})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^{8}\,\text{m/s})} = 1.0 \times 10^{45}\,\text{photons/s}.$$

2. We denote the diameter of the laser beam as d. The cross-sectional area of the beam is $A = \pi d^2/4$. From the formula obtained in Problem 38-1, the rate is given by

$$\frac{R}{A} = \frac{\lambda P}{hc\left(\pi d^2/4\right)} = \frac{4(633\,\text{nm})\left(5.0\times10^{-3}\,\text{W}\right)}{\pi\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(2.998\times10^{8}\,\text{m/s}\right)\left(3.5\times10^{-3}\,\text{m}\right)^2}$$
$$= 1.7\times10^{21}\,\text{photons/m}^2\cdot\text{s}.$$

3. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^8$ m/s,

$$hc = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.998 \times 10^{8}\,\text{m/s})}{(1.602 \times 10^{-19}\,\text{J/eV})(10^{-9}\,\text{m/nm})} = 1240\,\text{eV}\cdot\text{nm}.$$

Thus,

$$E = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda}.$$

With

$$\lambda = (1,650,763.73)^{-1}\,\text{m} = 6.0578021 \times 10^{-7}\,\text{m} = 605.78021\,\text{nm},$$

we find the energy to be

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{605.78021\,\text{nm}} = 2.047\,\text{eV}.$$

4. The energy of a photon is given by $E = hf$, where h is the Planck constant and f is the frequency. The wavelength λ is related to the frequency by $\lambda f = c$, so $E = hc/\lambda$. Since $h = 6.626 \times 10^{-34}$ J·s and $c = 2.998 \times 10^8$ m/s,

$$hc = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(2.998\times10^{8}\ \mathrm{m/s})}{(1.602\times10^{-19}\ \mathrm{J/eV})(10^{-9}\ \mathrm{m/nm})} = 1240\,\mathrm{eV\cdot nm}.$$

Thus,

$$E = \frac{1240\,\mathrm{eV\cdot nm}}{\lambda}.$$

With $\lambda = 589$ nm, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240\,\mathrm{eV\cdot nm}}{589\,\mathrm{nm}} = 2.11\,\mathrm{eV}.$$

5. (a) Let $E = 1240$ eV·nm/$\lambda_{\min} = 0.6$ eV to get $\lambda = 2.1 \times 10^3$ nm = 2.1 μm.

(b) It is in the infrared region.

6. Let

$$\frac{1}{2}m_e v^2 = E_{\text{photon}} = \frac{hc}{\lambda}$$

and solve for v:

$$v = \sqrt{\frac{2hc}{\lambda m_e}} = \sqrt{\frac{2hc}{\lambda m_e c^2}c^2} = c\sqrt{\frac{2hc}{\lambda (m_e c^2)}}$$
$$= (2.998\times10^{8}\ \mathrm{m/s})\sqrt{\frac{2(1240\,\mathrm{eV\cdot nm})}{(590\,\mathrm{nm})(511\times10^{3}\,\mathrm{eV})}} = 8.6\times10^{5}\ \mathrm{m/s}.$$

Since $v \ll c$, the non-relativistic formula $K = \frac{1}{2}mv^2$ may be used. The m_ec^2 value of Table 37-3 and $hc = 1240\,\mathrm{eV\cdot nm}$ are used in our calculation.

7. (a) Let R be the rate of photon emission (number of photons emitted per unit time) and let E be the energy of a single photon. Then, the power output of a lamp is given by $P = RE$ if all the power goes into photon production. Now, $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus

$$P = \frac{Rhc}{\lambda} \quad\Rightarrow\quad R = \frac{\lambda P}{hc}.$$

The lamp emitting light with the longer wavelength (the 700 nm infrared lamp) emits more photons per unit time. The energy of each photon is less, so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp. Then,

$$R=\frac{\lambda P}{hc}=\frac{(700\,\text{nm})(400\,\text{J/s})}{(1.60\times10^{-19}\,\text{J/eV})(1240\,\text{eV}\cdot\text{nm})}=1.41\times10^{21}\ \text{photon/s}.$$

8. (a) The rate at which solar energy strikes the panel is

$$P=(1.39\ \text{kW/m}^2)(2.60\ \text{m}^2)=3.61\ \text{kW}.$$

(b) The rate at which solar photons are absorbed by the panel is

$$R=\frac{P}{E_{\text{ph}}}=\frac{3.61\times10^3\ \text{W}}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})/(550\times10^{-9}\ \text{m})}$$
$$=1.00\times10^{22}\ \text{photons/s}.$$

(c) The time in question is given by

$$t=\frac{N_A}{R}=\frac{6.02\times10^{23}}{1.00\times10^{22}\ /\text{s}}=60.2\,\text{s}.$$

9. The total energy emitted by the bulb is $E = 0.93Pt$, where P = 60 W and

$$t = 730\ \text{h} = (730\ \text{h})(3600\ \text{s/h}) = 2.628\times10^6\ \text{s}.$$

The energy of each photon emitted is $E_{\text{ph}} = hc/\lambda$. Therefore, the number of photons emitted is

$$N=\frac{E}{E_{\text{ph}}}=\frac{0.93Pt}{hc/\lambda}=\frac{(0.93)(60\,\text{W})(2.628\times10^6\,\text{s})}{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^8\,\text{m/s})/(630\times10^{-9}\,\text{m})}=4.7\times10^{26}.$$

10. Following Sample Problem 38-1, we have

$$P=\frac{Rhc}{\lambda}=\frac{(100/\text{s})(6.63\times10^{-34}\,\text{J}\cdot\text{s})(2.998\times10^8\,\text{m/s})}{550\times10^{-9}\,\text{m}}=3.6\times10^{-17}\,\text{W}.$$

11. (a) We assume all the power results in photon production at the wavelength $\lambda=589$ nm. Let R be the rate of photon production and E be the energy of a single photon. Then,

$$P = RE = Rhc/\lambda,$$

where $E = hf$ and $f = c/\lambda$ are used. Here h is the Planck constant, f is the frequency of the emitted light, and λ is its wavelength. Thus,

$$R = \frac{\lambda P}{hc} = \frac{(589 \times 10^{-9}\ \text{m})(100\ \text{W})}{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})} = 2.96 \times 10^{20}\ \text{photon/s}.$$

(b) Let I be the photon flux a distance r from the source. Since photons are emitted uniformly in all directions, $R = 4\pi r^2 I$ and

$$r = \sqrt{\frac{R}{4\pi I}} = \sqrt{\frac{2.96\times 10^{20}\ \text{photon/s}}{4\pi\left(1.00\times 10^{4}\ \text{photon/m}^2\cdot\text{s}\right)}} = 4.86\times 10^{7}\ \text{m}.$$

(c) The photon flux is

$$I = \frac{R}{4\pi r^2} = \frac{2.96 \times 10^{20}\ \text{photon/s}}{4\pi(2.00\ \text{m})^2} = 5.89 \times 10^{18}\ \frac{\text{photon}}{\text{m}^2 \cdot \text{s}}.$$

12. The average power output of the source is

$$P_{\text{emit}} = \frac{\Delta E}{\Delta t} = \frac{7.2\ \text{nJ}}{2\ \text{s}} = 3.6\ \text{nJ/s} = 3.6\times 10^{-9}\ \text{J/s} = 2.25\times 10^{10}\ \text{eV/s}$$

Since the energy of each photon emitted is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{600\,\text{nm}} = 2.07\ \text{eV},$$

the rate at which photons are emitted by the source is

$$R_{\text{emit}} = \frac{P_{\text{emit}}}{E_{\text{ph}}} = \frac{2.25\times 10^{10}\ \text{eV/s}}{2.07\ \text{eV}} = 1.09\times 10^{10}\,\text{photons/s}.$$

Given that the source is isotropic, and the detector (located 12.0 m away) has an absorbing area of $A_{\text{abs}} = 2.00\times 10^{-6}\ \text{m}^2$ and absorbs 50% of the incident light, the rate of photon absorption is

$$R_{\text{abs}} = (0.50)\frac{A_{\text{abs}}}{4\pi r^2} R_{\text{emit}} = (0.50)\frac{2.00\times 10^{-6}\ \text{m}^2}{4\pi(12.0\ \text{m})^2}\left(1.09\times 10^{10}\,\text{photons/s}\right) = 6.0\ \text{photons/s}.$$

13. The rate at which photons are absorbed by the detector is related to the rate of photon emission by the light source via

$$R_{\text{abs}} = (0.80)\frac{A_{\text{abs}}}{4\pi r^2} R_{\text{emit}}.$$

Given that $A_{\text{abs}} = 2.00\times 10^{-6}\ \text{m}^2$ and $r = 3.00$ m, with $R_{\text{abs}} = 4.000$ photons/s, we find the rate at which photon is emitted to be

$$R_{\text{emit}} = \frac{4\pi r^2}{(0.80)A_{\text{abs}}} R_{\text{abs}} = \frac{4\pi(3.00\ \text{m})^2}{(0.80)(2.00\times 10^{-6}\ \text{m}^2)}\left(4.000\ \text{photons/s}\right) = 2.83\times 10^{8}\ \text{photons/s}.$$

Since the energy of each emitted photon is

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{500\,\text{nm}} = 2.48\ \text{eV},$$

the power output of source is

$$P_{\text{emit}} = R_{\text{emit}}E_{\text{ph}} = \left(2.83\times 10^{8}\text{photons/s}\right)(2.48\ \text{eV}) = 7.0\times 10^{8}\ \text{eV/s} = 1.1\times 10^{-10}\ \text{W}.$$

14. The rate at which photons are emitted from the argon laser source is given by $R = P/E_{\text{ph}}$, where $P = 1.5$ W is the power of the laser beam and $E_{\text{ph}} = hc/\lambda$ is the energy of each photon of wavelength λ. Since $\alpha = 84\%$ of the energy of the laser beam falls within the central disk, the rate of photon absorption of the central disk is

$$R' = \alpha R = \frac{\alpha P}{hc/\lambda} = \frac{(0.84)(1.5\ \text{W})}{\left[(6.63\times 10^{-34}\ \text{J}\cdot\text{s})(2.998\times 10^{8}\ \text{m/s})/(515\times 10^{-9}\ \text{m})\right]}$$
$$= 3.3\times 10^{18}\ \text{photons/s}.$$

15. The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}(m_e c^2)(v/c)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v = c\sqrt{\frac{2\left(E_{\text{photon}} - \Phi\right)}{m_e c^2}} = (2.998\times 10^{8}\ \text{m/s})\sqrt{\frac{2(5.80\,\text{eV} - 4.50\,\text{eV})}{511\times 10^{3}\ \text{eV}}} = 6.76\times 10^{5}\ \text{m/s}.$$

16. The energy of the most energetic photon in the visible light range (with wavelength of about 400 nm) is about $E = (1240\ \text{eV}\cdot\text{nm}/400\ \text{nm}) = 3.1$ eV (using the vlue $hc = 1240$ eV·nm). Consequently, barium and lithium can be used, since their work functions are both lower than 3.1 eV.

17. The energy of an incident photon is $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the electromagnetic radiation, and λ is its wavelength. The kinetic energy of the most energetic electron emitted is

$$K_m = E - \Phi = (hc/\lambda) - \Phi,$$

where Φ is the work function for sodium. The stopping potential V_0 is related to the maximum kinetic energy by $eV_0 = K_m$, so

$$eV_0 = (hc/\lambda) - \Phi$$

and

$$\lambda = \frac{hc}{eV_0 + \Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.0\,\text{eV} + 2.2\,\text{eV}} = 170\,\text{nm}.$$

Here $eV_0 = 5.0$ eV and $hc = 1240$ eV·nm are used.

18. We use Eq. 38-5 to find the maximum kinetic energy of the ejected electrons:

$$K_{\max} = hf - \Phi = \left(4.14\times10^{-15}\,\text{eV}\cdot\text{s}\right)\left(3.0\times10^{15}\,\text{Hz}\right) - 2.3\,\text{eV} = 10\,\text{eV}.$$

19. (a) We use the photoelectric effect equation (Eq. 38-5) in the form $hc/\lambda = \Phi + K_m$. The work function depends only on the material and the condition of the surface, and not on the wavelength of the incident light. Let λ_1 be the first wavelength described and λ_2 be the second. Let $K_{m1} = 0.710$ eV be the maximum kinetic energy of electrons ejected by light with the first wavelength, and $K_{m2} = 1.43$ eV be the maximum kinetic energy of electrons ejected by light with the second wavelength. Then,

$$\frac{hc}{\lambda_1} = \Phi + K_{m1}, \qquad \frac{hc}{\lambda_2} = \Phi + K_{m2}.$$

The first equation yields $\Phi = (hc/\lambda_1) - K_{m1}$. When this is used to substitute for Φ in the second equation, the result is

$$(hc/\lambda_2) = (hc/\lambda_1) - K_{m1} + K_{m2}.$$

The solution for λ_2 is

$$\lambda_2 = \frac{hc\lambda_1}{hc + \lambda_1(K_{m2} - K_{m1})} = \frac{(1240\,\text{V}\cdot\text{nm})(491\,\text{nm})}{1240\,\text{eV}\cdot\text{nm} + (491\,\text{nm})(1.43\,\text{eV} - 0.710\,\text{eV})}$$
$$= 382\,\text{nm}.$$

Here $hc = 1240$ eV·nm has been used.

(b) The first equation displayed above yields

$$\Phi = \frac{hc}{\lambda_1} - K_{m1} = \frac{1240 \text{ eV}\cdot\text{nm}}{491 \text{ nm}} - 0.710 \text{ eV} = 1.82 \text{ eV}.$$

20. We use Eq. 38-6 and the value hc = 1240 eV·nm:

$$K_{\max} = E_{\text{photon}} - \Phi = \frac{hc}{\lambda} - \frac{hc}{\lambda_{\max}} = \frac{1240\,\text{eV}\cdot\text{nm}}{254\,\text{nm}} - \frac{1240\,\text{eV}\cdot\text{nm}}{325\,\text{nm}} = 1.07\,\text{eV}.$$

21. (a) The kinetic energy K_m of the fastest electron emitted is given by

$$K_m = hf - \Phi = (hc/\lambda) - \Phi,$$

where Φ is the work function of aluminum, f is the frequency of the incident radiation, and λ is its wavelength. The relationship $f = c/\lambda$ was used to obtain the second form. Thus,

$$K_m = \frac{1240\,\text{eV}\cdot\text{nm}}{200\,\text{nm}} - 4.20\,\text{eV} = 2.00 \text{ eV},$$

where we have used hc = 1240 eV·nm.

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_0 is given by $K_m = eV_0$, so

$$V_0 = K_m/e = (2.00 \text{ eV})/e = 2.00 \text{ V}.$$

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus, $hc/\lambda = \Phi$, or

$$\lambda = hc/\Phi = (1240 \text{ eV·nm})/(4.2 \text{ eV}) = 295 \text{ nm}.$$

If the wavelength is longer, the photon energy is less and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

22. To find the longest possible wavelength $\lambda_{\max}$ (corresponding to the lowest possible energy) of a photon which can produce a photoelectric effect in platinum, we set $K_{\max} = 0$ in Eq. 38-5 and use $hf = hc/\lambda$. Thus $hc/\lambda_{\max} = \Phi$. We solve for $\lambda_{\max}$:

$$\lambda_{\max} = \frac{hc}{\Phi} = \frac{1240\,\text{eV}\cdot\text{nm}}{5.32\,\text{nm}} = 233\,\text{nm}.$$

23. (a) We use Eq. 38-6:

$$V_{\text{stop}} = \frac{hf - \Phi}{e} = \frac{hc/\lambda - \Phi}{e} = \frac{\left(1240\,\text{eV}\cdot\text{nm}/400\,\text{nm}\right) - 1.8\,\text{eV}}{e} = 1.3\,\text{V}.$$

(b) The speed v of the electron satisfies

$$K_{\max} = \tfrac{1}{2}m_e v^2 = \tfrac{1}{2}\left(m_e c^2\right)\left(v/c\right)^2 = E_{\text{photon}} - \Phi.$$

Using Table 37-3, we find

$$v = \sqrt{\frac{2\left(E_{\text{photon}} - \Phi\right)}{m_e}} = \sqrt{\frac{2eV_{\text{stop}}}{m_e}} = c\sqrt{\frac{2eV_{\text{stop}}}{m_e c^2}} = \left(2.998\times10^8\,\text{m/s}\right)\sqrt{\frac{2e\left(1.3\text{V}\right)}{511\times10^3\text{eV}}}$$
$$= 6.8\times10^5\,\text{m/s}.$$

24. Using the value hc = 1240 eV·nm, the number of photons emitted from the laser per unit time is

$$R = \frac{P}{E_{\text{ph}}} = \frac{2.00\times10^{-3}\ \text{W}}{(1240\ \text{eV}\cdot\text{nm}/600\ \text{nm})(1.60\times10^{-19}\ \text{J/eV})} = 6.05\times10^{15}\,/\,\text{s},$$

of which $(1.0 \times 10^{-16})(6.05 \times 10^{15}/\text{s}) = 0.605/\text{s}$ actually cause photoelectric emissions. Thus the current is

$$i = (0.605/\text{s})(1.60 \times 10^{-19}\ \text{C}) = 9.68 \times 10^{-20}\ \text{A}.$$

25. (a) From $r = m_e v/eB$, the speed of the electron is $v = rBe/m_e$. Thus,

$$K_{\max} = \frac{1}{2}m_e v^2 = \frac{1}{2}m_e\left(\frac{rBe}{m_e}\right)^2 = \frac{(rB)^2 e^2}{2m_e} = \frac{(1.88\times10^{-4}\ \text{T}\cdot\text{m})^2(1.60\times10^{-19}\ \text{C})^2}{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{J/eV})}$$
$$= 3.1\ \text{keV}.$$

(b) Using the value hc = 1240 eV·nm, the work done is

$$W = E_{\text{photon}} - K_{\max} = \frac{1240\ \text{eV}\cdot\text{nm}}{71\times10^{-3}\ \text{nm}} - 3.10\ \text{keV} = 14\ \text{keV}.$$

26. (a) For the first and second case (labeled 1 and 2) we have

$$eV_{01} = hc/\lambda_1 - \Phi\,, \qquad eV_{02} = hc/\lambda_2 - \Phi,$$

from which h and Φ can be determined. Thus,

$$h=\frac{e\left(V_1-V_2\right)}{c\left(\lambda_1^{-1}-\lambda_2^{-1}\right)}=\frac{1.85\text{eV}-0.820\text{eV}}{\left(3.00\times10^{17}\text{ nm/s}\right)\left[\left(300\text{nm}\right)^{-1}-\left(400\text{nm}\right)^{-1}\right]}=4.12\times10^{-15}\text{ eV}\cdot\text{s}.$$

(b) The work function is

$$\Phi=\frac{3(V_2\lambda_2-V_1\lambda_1)}{\lambda_1-\lambda_2}=\frac{(0.820\text{ eV})(400\text{ nm})-(1.85\text{ eV})(300\text{ nm})}{300\text{ nm}-400\text{ nm}}=2.27\text{ eV}.$$

(c) Let $\Phi = hc/\lambda_{\max}$ to obtain

$$\lambda_{\max}=\frac{hc}{\Phi}=\frac{1240\text{ eV}\cdot\text{nm}}{2.27\text{ eV}}=545\text{ nm}.$$

27. (a) The x-ray frequency is

$$f=\frac{c}{\lambda}=\frac{2.998\times10^{8}\text{ m/s}}{35.0\times10^{-12}\text{ m}}=8.57\times10^{18}\text{ Hz}.$$

(b) The x-ray photon energy is

$$E=hf=(4.14\times10^{-15}\text{ eV}\cdot\text{s})(8.57\times10^{18}\text{ Hz})=3.55\times10^{4}\text{ eV}.$$

(c) From Eq. 38-7,

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{35.0\times10^{-12}\text{ m}}=1.89\times10^{-23}\text{ kg}\cdot\text{m/s}=35.4\text{ keV}/c.$$

28. (a) The rest energy of an electron is given by $E = m_e c^2$. Thus the momentum of the photon in question is given by

$$p=\frac{E}{c}=\frac{m_e c^2}{c}=m_e c=(9.11\times10^{-31}\text{ kg})(2.998\times10^{8}\text{ m/s})=2.73\times10^{-22}\text{ kg}\cdot\text{m/s}$$
$$=0.511\text{ MeV}/c.$$

(b) From Eq. 38-7,

$$\lambda=\frac{h}{p}=\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2.73\times10^{-22}\text{ kg}\cdot\text{m/s}}=2.43\times10^{-12}\text{ m}=2.43\text{ pm}.$$

(c) Using Eq. 38-1,

$$f=\frac{c}{\lambda}=\frac{2.998\times10^{8}\text{ m/s}}{2.43\times10^{-12}\text{ m}}=1.24\times10^{20}\text{ Hz}.$$

29. (a) When a photon scatters from an electron initially at rest, the change in wavelength is given by

$$\Delta\lambda = (h/mc)(1 - \cos\phi),$$

where m is the mass of an electron and ϕ is the scattering angle. Now, $h/mc = 2.43 \times 10^{-12}$ m = 2.43 pm, so

$$\Delta\lambda = (h/mc)(1 - \cos\phi) = (2.43\text{ pm})(1 - \cos 30°) = 0.326\text{ pm}.$$

The final wavelength is

$$\lambda' = \lambda + \Delta\lambda = 2.4\text{ pm} + 0.326\text{ pm} = 2.73\text{ pm}.$$

(b) Now, $\Delta\lambda = (2.43\text{ pm})(1 - \cos 120°) = 3.645$ pm and

$$\lambda' = 2.4\text{ pm} + 3.645\text{ pm} = 6.05\text{ pm}.$$

30. (a) Eq. 38-11 yields

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\text{ pm})(1-\cos 180°) = +4.86\text{ pm}.$$

(b) Using the value hc = 1240 eV·nm, the change in photon energy is

$$\Delta E = \frac{hc}{\lambda'} - \frac{hc}{\lambda} = (1240\text{ eV}\cdot\text{nm})\left(\frac{1}{0.01\text{ nm} + 4.86\text{ pm}} - \frac{1}{0.01\text{ nm}}\right) = -40.6\text{ keV}.$$

(c) From conservation of energy, $\Delta K = -\Delta E = 40.6$ keV.

(d) The electron will move straight ahead after the collision, since it has acquired some of the forward linear momentum from the photon. Thus, the angle between $+x$ and the direction of the electron's motion is zero.

31. (a) The fractional change is

$$\frac{\Delta E}{E} = \frac{\Delta(hc/\lambda)}{hc/\lambda} = \lambda\Delta\left(\frac{1}{\lambda}\right) = \lambda\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) = \frac{\lambda}{\lambda'} - 1 = \frac{\lambda}{\lambda + \Delta\lambda} - 1$$
$$= -\frac{1}{\lambda/\Delta\lambda + 1} = -\frac{1}{(\lambda/\lambda_C)(1-\cos\phi)^{-1} + 1}.$$

If $\lambda = 3.0$ cm $= 3.0 \times 10^{10}$ pm and $\phi = 90°$, the result is

$$\frac{\Delta E}{E} = -\frac{1}{(3.0\times10^{10}\ \text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1} = -8.1\times10^{-11} = -8.1\times10^{-9}\ \%.$$

(b) Now λ = 500 nm = 5.00 × 10^5 pm and ϕ = 90°, so

$$\frac{\Delta E}{E} = -\frac{1}{(5.00\times10^{5}\ \text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1} = -4.9\times10^{-6} = -4.9\times10^{-4}\ \%.$$

(c) With λ = 25 pm and ϕ = 90°, we find

$$\frac{\Delta E}{E} = -\frac{1}{(25\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1} = -8.9\times10^{-2} = -8.9\ \%.$$

(d) In this case,

$$\lambda = hc/E = 1240\ \text{nm·eV}/1.0\ \text{MeV} = 1.24 \times 10^{-3}\ \text{nm} = 1.24\ \text{pm},$$

so

$$\frac{\Delta E}{E} = -\frac{1}{(1.24\,\text{pm}/2.43\,\text{pm})(1-\cos 90°)^{-1}+1} = -0.66 = -66\ \%.$$

(e) From the calculation above, we see that the shorter the wavelength the greater the fractional energy change for the photon as a result of the Compton scattering. Since $\Delta E/E$ is virtually zero for microwave and visible light, the Compton effect is significant only in the x-ray to gamma ray range of the electromagnetic spectrum.

32. (a) Using the value hc = 1240 eV·nm, we find

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{nm}\cdot\text{eV}}{0.511\ \text{MeV}} = 2.43\times10^{-3}\ \text{nm} = 2.43\ \text{pm}.$$

(b) Now, Eq. 38-11 leads to

$$\lambda' = \lambda + \Delta\lambda = \lambda + \frac{h}{m_e c}(1-\cos\phi) = 2.43\,\text{pm} + (2.43\,\text{pm})(1-\cos 90.0°)$$
$$= 4.86\,\text{pm}.$$

(c) The scattered photons have energy equal to

$$E' = E\left(\frac{\lambda}{\lambda'}\right) = (0.511\ \text{MeV})\left(\frac{2.43\ \text{pm}}{4.86\ \text{pm}}\right) = 0.255\ \text{MeV}.$$

33. (a) Since the mass of an electron is m = 9.109 × 10^{-31} kg, its Compton wavelength is

$$\lambda_C = \frac{h}{mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.109\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 2.426\times10^{-12}\ \text{m} = 2.43\ \text{pm}.$$

(b) Since the mass of a proton is $m = 1.673 \times 10^{-27}$ kg, its Compton wavelength is

$$\lambda_C = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(1.673\times10^{-27}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 1.321\times10^{-15}\ \text{m} = 1.32\ \text{fm}.$$

(c) We note that hc = 1240 eV·nm, which gives E = (1240 eV·nm)/λ, where E is the energy and λ is the wavelength. Thus for the electron,

$$E = (1240\ \text{eV}\cdot\text{nm})/(2.426\times10^{-3}\ \text{nm}) = 5.11\times10^{5}\ \text{eV} = 0.511\ \text{MeV}.$$

(d) For the proton,

$$E = (1240\ \text{eV}\cdot\text{nm})/(1.321\times10^{-6}\ \text{nm}) = 9.39\times10^{8}\ \text{eV} = 939\ \text{MeV}.$$

34. Referring to Sample Problem 38-4, we see that the fractional change in photon energy is

$$\frac{E-E_n}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{(h/mc)(1-\cos\phi)}{(hc/E)+(h/mc)(1-\cos\phi)}.$$

Energy conservation demands that $E - E' = K$, the kinetic energy of the electron. In the maximal case, $\phi = 180°$, and we find

$$\frac{K}{E} = \frac{(h/mc)(1-\cos180°)}{(hc/E)+(h/mc)(1-\cos180°)} = \frac{2h/mc}{(hc/E)+(2h/mc)}.$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\left(\frac{2/mc}{c/E+2/mc}\right) = \frac{E^2}{mc^2/2+E}.$$

35. (a) From Eq. 38-11

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = (2.43\,\text{pm})(1-\cos90°) = 2.43\,\text{pm}.$$

(b) The fractional shift should be interpreted as $\Delta\lambda$ divided by the original wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{2.425\,\text{pm}}{590\,\text{nm}} = 4.11\times10^{-6}.$$

(c) The change in energy for a photon with λ = 590 nm is given by

$$\begin{aligned}\Delta E_{\text{ph}} &= \Delta\left(\frac{hc}{\lambda}\right) \approx -\frac{hc\Delta\lambda}{\lambda^2} \\ &= -\frac{(4.14\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})(2.43\,\text{pm})}{(590\,\text{nm})^2} \\ &= -8.67\times10^{-6}\ \text{eV}\ .\end{aligned}$$

(d) For an x-ray photon of energy E_{ph} = 50 keV, Δλ remains the same (2.43 pm), since it is independent of E_{ph}.

(e) The fractional change in wavelength is now

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\lambda}{hc/E_{\text{ph}}} = \frac{(50\times10^{3}\ \text{eV})(2.43\,\text{pm})}{(4.14\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^{8}\,\text{m/s})} = 9.78\times10^{-2}\ .$$

(f) The change in photon energy is now

$$\Delta E_{\text{ph}} = hc\left(\frac{1}{\lambda+\Delta\lambda} - \frac{1}{\lambda}\right) = -\left(\frac{hc}{\lambda}\right)\frac{\Delta\lambda}{\lambda+\Delta\lambda} = -E_{\text{ph}}\left(\frac{\alpha}{1+\alpha}\right)$$

where $\alpha = \Delta\lambda/\lambda$. With E_{ph} = 50 keV and $\alpha = 9.78\times10^{-2}$, we obtain ΔE_{ph} = –4.45 keV. (Note that in this case $\alpha \approx 0.1$ is not close enough to zero so the approximation $\Delta E_{\text{ph}} \approx hc\Delta\lambda/\lambda^2$ is not as accurate as in the first case, in which $\alpha = 4.12\times10^{-6}$. In fact if one were to use this approximation here, one would get $\Delta E_{\text{ph}} \approx$ –4.89 keV, which does not amount to a satisfactory approximation.)

36. The initial wavelength of the photon is (using hc = 1240 eV·nm)

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{17500\,\text{eV}} = 0.07086\ \text{nm}$$

or 70.86 pm. The maximum Compton shift occurs for ϕ = 180°, in which case Eq. 38-11 (applied to an electron) yields

$$\Delta\lambda = \left(\frac{hc}{m_e c^2}\right)(1-\cos 180°) = \left(\frac{1240\,\text{eV}\cdot\text{nm}}{511\times10^{3}\ \text{eV}}\right)(1-(-1)) = 0.00485\ \text{nm}$$

where Table 37-3 is used. Therefore, the new photon wavelength is

$$\lambda' = 0.07086\ \text{nm} + 0.00485\ \text{nm} = 0.0757\ \text{nm}.$$

Consequently, the new photon energy is

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.0757\,\text{nm}} = 1.64\times10^4\ \text{eV} = 16.4\ \text{keV}\ .$$

By energy conservation, then, the kinetic energy of the electron must equal

$$E' - E = 17.5\ \text{keV} - 16.4\ \text{keV} = 1.1\ \text{keV}.$$

37. If E is the original energy of the photon and E' is the energy after scattering, then the fractional energy loss is

$$\frac{\Delta E}{E} = \frac{E-E'}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda}$$

using the result from Sample Problem 38-4. Thus

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta E/E}{1-\Delta E/E} = \frac{0.75}{1-0.75} = 3 = 300\ \%.$$

A 300% increase in the wavelength leads to a 75% decrease in the energy of the photon.

38. The initial energy of the photon is (using hc = 1240 eV·nm)

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.00300\ \text{nm}} = 4.13\times10^5\ \text{eV}\,.$$

Using Eq. 38-11 (applied to an electron), the Compton shift is given by

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) = \frac{h}{m_e c}(1-\cos 90.0^\circ) = \frac{hc}{m_e c^2} = \frac{1240\,\text{eV}\cdot\text{nm}}{511\times10^3\ \text{eV}} = 2.43\ \text{pm}$$

Therefore, the new photon wavelength is

$$\lambda' = 3.00\ \text{pm} + 2.43\ \text{pm} = 5.43\ \text{pm}.$$

Consequently, the new photon energy is

$$E' = \frac{hc}{\lambda'} = \frac{1240\,\text{eV}\cdot\text{nm}}{0.00543\,\text{nm}} = 2.28\times10^5\ \text{eV}$$

By energy conservation, then, the kinetic energy of the electron must be equal to

$$K_e = \Delta E = E - E' = 4.13\times10^5 - 2.28\times10^5\ \text{eV} = 1.85\times10^5\ \text{eV} \approx 3.0\times10^{-14}\ \text{J}\,.$$

39. (a) From Eq. 38-11,

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) .$$

In this case ϕ = 180° (so cos ϕ = –1), and the change in wavelength for the photon is given by $\Delta\lambda = 2h/m_e c$. The energy E' of the scattered photon (with initial energy $E = hc/\lambda$) is then

$$E' = \frac{hc}{\lambda+\Delta\lambda} = \frac{E}{1+\Delta\lambda/\lambda} = \frac{E}{1+(2h/m_e c)(E/hc)} = \frac{E}{1+2E/m_e c^2}$$
$$= \frac{50.0\,\text{keV}}{1+2(50.0\,\text{keV})/0.511\,\text{MeV}} = 41.8\,\text{keV} .$$

(b) From conservation of energy the kinetic energy K of the electron is given by

$$K = E - E' = 50.0\text{ keV} - 41.8\text{ keV} = 8.2\text{ keV}.$$

40. The (1 – cos ϕ) factor in Eq. 38-11 is largest when ϕ = 180°. Thus, using Table 37-3, we obtain

$$\Delta\lambda_{\max} = \frac{hc}{m_p c^2}(1-\cos 180°) = \frac{1240\,\text{MeV}\cdot\text{fm}}{938\,\text{MeV}}(1-(-1)) = 2.64\ \text{fm}$$

where we have used the value hc = 1240 eV·nm =1240 MeV·fm.

41. The magnitude of the fractional energy change for the photon is given by

$$\left|\frac{\Delta E_{\text{ph}}}{E_{\text{ph}}}\right| = \left|\frac{\Delta(hc/\lambda)}{hc/\lambda}\right| = \left|\lambda\Delta\left(\frac{1}{\lambda}\right)\right| = \lambda\left(\frac{1}{\lambda} - \frac{1}{\lambda+\Delta\lambda}\right) = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \beta$$

where β = 0.10. Thus $\Delta\lambda = \lambda\beta/(1-\beta)$. We substitute this expression for $\Delta\lambda$ in Eq. 38-11 and solve for cos ϕ:

$$\cos\phi = 1 - \frac{mc}{h}\Delta\lambda = 1 - \frac{mc\lambda\beta}{h(1-\beta)} = 1 - \frac{\beta(mc^2)}{(1-\beta)E_{\text{ph}}}$$
$$= 1 - \frac{(0.10)(511\text{ keV})}{(1-0.10)(200\,\text{keV})} = 0.716 .$$

This leads to an angle of ϕ = 44°.

42. (a) Using Table 37-3 and the value hc = 1240 eV·nm, we obtain

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{hc}{\sqrt{2m_ec^2K}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2(511000\text{eV})(1000\text{eV})}} = 0.0388\,\text{nm}.$$

(b) A photon's de Broglie wavelength is equal to its familiar wave-relationship value. Using the value hc = 1240 eV·nm,

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{keV}} = 1.24\,\text{nm}\ .$$

(c) The neutron mass may be found in Appendix B. Using the conversion from electron-volts to Joules, we obtain

$$\lambda = \frac{h}{\sqrt{2m_nK}} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(1.675\times10^{-27}\,\text{kg})(1.6\times10^{-16}\,\text{J})}} = 9.06\times10^{-13}\,\text{m}.$$

43. The de Broglie wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} = \frac{h}{\sqrt{2m_eeV}}\ ,$$

where V is the accelerating potential and e is the fundamental charge. This gives

$$\lambda = \frac{h}{\sqrt{2m_eeV}} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{C})(25.0\times10^{3}\,\text{V})}}$$
$$= 7.75\times10^{-12}\ \text{m} = 7.75\,\text{pm}.$$

44. Using Eq. 37-8, we find the Lorentz factor to be

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.9900)^2}} = 7.0888\ .$$

With $p = \gamma mv$ (Eq. 37-41), the de Broglie wavelength of the protons is

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(7.0888)(1.67\times10^{-27}\ \text{kg})(0.99\times3.00\times10^{8}\ \text{m/s})} = 1.89\times10^{-16}\ \text{m}\ .$$

The vertical distance between the second interference minimum and the center point is

$$y_2 = \left(1+\frac{1}{2}\right)\frac{\lambda L}{d} = \frac{3}{2}\frac{\lambda L}{d}$$

where L is the perpendicular distance between the slits and the screen. Therefore, the angle between the center of the pattern and the second minimum is given by

$$\tan\theta = \frac{y_2}{L} = \frac{3\lambda}{2d}.$$

Since $\lambda = \ d$, $\tan\theta \approx \theta$, and we obtain

$$\theta \approx \frac{3\lambda}{2d} = \frac{3(1.89\times10^{-16}\ \text{m})}{2(4.00\times10^{-9}\ \text{m})} = 7.07\times10^{-8}\,\text{rad} = (4.0\times10^{-6})^\circ.$$

45. (a) The momentum of the photon is given by $p = E/c$, where E is its energy. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{eV}} = 1240\,\text{nm}.$$

(b) The momentum of the electron is given by $p = \sqrt{2mK}$, where K is its kinetic energy and m is its mass. Its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}.$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}} = \frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}} = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

For $K = 1.00\ \text{eV}$, we have

$$\lambda = \frac{1.226\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{1.00\,\text{eV}}} = 1.23\ \text{nm}.$$

(c) For the photon,

$$\lambda = \frac{hc}{E} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\times10^{9}\,\text{eV}} = 1.24\times10^{-6}\,\text{nm} = 1.24\ \text{fm}.$$

(d) Relativity theory must be used to calculate the wavelength for the electron. According to Eq. 38-51, the momentum p and kinetic energy K are related by

$$(pc)^2 = K^2 + 2Kmc^2.$$

Thus,

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{\left(1.00\times10^{9}\,\text{eV}\right)^2 + 2\left(1.00\times10^{9}\text{eV}\right)\left(0.511\times10^{6}\,\text{eV}\right)}$$
$$= 1.00\times10^{9}\,\text{eV}.$$

The wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240\,\text{eV}\cdot\text{nm}}{1.00\times10^{9}\,\text{eV}} = 1.24\times10^{-6}\,\text{nm} = 1.24\ \text{fm}.$$

46. (a) The momentum of the electron is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}} = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$$

(b) The momentum of the photon is the same as that of the electron: $p = 3.3\times10^{-24}\,\text{kg}\cdot\text{m/s}$.

(c) The kinetic energy of the electron is

$$K_e = \frac{p^2}{2m_e} = \frac{\left(3.3\times10^{-24}\,\text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)} = 6.0\times10^{-18}\,\text{J}=38\text{eV}.$$

(d) The kinetic energy of the photon is

$$K_{\text{ph}} = pc = \left(3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)\left(2.998\times10^{8}\ \text{m/s}\right) = 9.9\times10^{-16}\ \text{J} = 6.2\,\text{keV}.$$

47. (a) The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus

$$K = (1.60\times10^{-19}\ \text{C})(300\ \text{V}) = 4.80\times10^{-17}\ \text{J}.$$

The mass of a single sodium atom is, from Appendix F,

$$m = (22.9898\ \text{g/mol})/(6.02\times10^{23}\ \text{atom/mol}) = 3.819\times10^{-23}\ \text{g} = 3.819\times10^{-26}\ \text{kg}.$$

Thus, the momentum of an ion is

$$p = \sqrt{2mK} = \sqrt{2\left(3.819\times10^{-26}\ \text{kg}\right)\left(4.80\times10^{-17}\ \text{J}\right)} = 1.91\times10^{-21}\ \text{kg}\cdot\text{m/s}.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{1.91\times10^{-21}\text{kg}\cdot\text{m/s}} = 3.46\times10^{-13}\,\text{m}.$$

48. (a) Since $K = 7.5\,\text{MeV} \ll m_\alpha c^2 = 4(932\,\text{MeV})$, we may use the non-relativistic formula $p = \sqrt{2m_\alpha K}$. Using Eq. 38-43 (and noting that 1240 eV·nm = 1240 MeV·fm), we obtain

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2m_\alpha c^2 K}} = \frac{1240\,\text{MeV}\cdot\text{fm}}{\sqrt{2(4\,\text{u})(931.5\,\text{MeV}/\text{u})(7.5\,\text{MeV})}} = 5.2\,\text{fm}.$$

(b) Since $\lambda = 5.2\,\text{fm} \ll 30\,\text{fm}$, to a fairly good approximation, the wave nature of the α particle does not need to be taken into consideration.

49. If K is given in electron volts, then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\,\text{kg})(1.602\times10^{-19}\,\text{J/eV})K}} = \frac{1.226\times10^{-9}\,\text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$

$$= \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}},$$

where K is the kinetic energy. Thus,

$$K = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\lambda}\right)^2 = \left(\frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{590\,\text{nm}}\right)^2 = 4.32\times10^{-6}\,\text{eV}.$$

50. (a) We need to use the relativistic formula

$$p = \sqrt{(E/c)^2 - m_e^2c^2} \approx E/c \approx K/c$$

$\left(\text{since } E \gg m_e c^2\right)$. So

$$\lambda = \frac{h}{p} \approx \frac{hc}{K} = \frac{1240\,\text{eV}\cdot\text{nm}}{50\times10^9\,\text{eV}} = 2.5\times10^{-8}\,\text{nm} = 0.025\ \text{fm}.$$

(b) With $R = 5.0\ \text{fm}$, we obtain $R/\lambda = 2.0\times10^2$.

51. The wavelength associated with the unknown particle is

$$\lambda_p = \frac{h}{p_p} = \frac{h}{m_p v_p},$$

where p_p is its momentum, m_p is its mass, and v_p is its speed. The classical relationship $p_p = m_p v_p$ was used. Similarly, the wavelength associated with the electron is $\lambda_e = h/(m_e v_e)$, where m_e is its mass and v_e is its speed. The ratio of the wavelengths is

$$\lambda_p/\lambda_e = (m_e v_e)/(m_p v_p),$$

so

$$m_p = \frac{v_e \lambda_e}{v_p \lambda_p} m_e = \frac{9.109 \times 10^{-31}\text{ kg}}{(3.1813 \times 10^{-4})} = 1.675 \times 10^{-27}\text{ kg}.$$

According to Appendix B, this is the mass of a neutron.

52. (a) We use the value $hc = 1240\,\text{nm}\cdot\text{eV}$:

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\,\text{nm}} = 1.24\,\text{keV}.$$

(b) For the electron, we have

$$K = \frac{p^2}{2m_e} = \frac{(h/\lambda)^2}{2m_e} = \frac{(hc/\lambda)^2}{2m_e c^2} = \frac{1}{2(0.511\text{ MeV})}\left(\frac{1240\,\text{eV}\cdot\text{nm}}{1.00\,\text{nm}}\right)^2 = 1.50\,\text{eV}.$$

(c) In this case, we find

$$E_{\text{photon}} = \frac{1240\,\text{nm}\cdot\text{eV}}{1.00\times10^{-6}\,\text{nm}} = 1.24\times10^{9}\text{ eV} = 1.24\,\text{GeV}.$$

(d) For the electron (recognizing that 1240 eV·nm = 1240 MeV·fm)

$$\begin{aligned} K &= \sqrt{p^2c^2 + (m_e c^2)^2} - m_e c^2 = \sqrt{(hc/\lambda)^2 + (m_e c^2)^2} - m_e c^2 \\ &= \sqrt{\left(\frac{1240\,\text{MeV}\cdot\text{fm}}{1.00\,\text{fm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 1.24 \times 10^{3}\text{ MeV} = 1.24\,\text{GeV}. \end{aligned}$$

We note that at short λ (large K) the kinetic energy of the electron, calculated with the relativistic formula, is about the same as that of the photon. This is expected since now $K \approx E \approx pc$ for the electron, which is the same as $E = pc$ for the photon.

53. (a) Setting $\lambda = h/p = h/\sqrt{(E/c)^2 - m_e^2c^2}$, we solve for $K = E - m_e c^2$:

$$\begin{aligned} K &= \sqrt{\left(\frac{hc}{\lambda}\right)^2 + m_e^2c^4} - m_e c^2 = \sqrt{\left(\frac{1240\,\text{eV}\cdot\text{nm}}{10\times10^{-3}\,\text{nm}}\right)^2 + (0.511\,\text{MeV})^2} - 0.511\,\text{MeV} \\ &= 0.015\,\text{MeV} = 15\,\text{keV}. \end{aligned}$$

(b) Using the value $hc = 1240\,\text{eV}\cdot\text{nm}$

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{10\times10^{-3}\,\text{nm}} = 1.2\times10^{5}\,\text{eV} = 120\,\text{keV}.$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.

54. The same resolution requires the same wavelength, and since the wavelength and particle momentum are related by $p = h/\lambda$, we see that the same particle momentum is required. The momentum of a 100 keV photon is

$$p = E/c = (100\times10^{3}\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})/(3.00\times10^{8}\,\text{m/s}) = 5.33\times10^{-23}\,\text{kg}\cdot\text{m/s}.$$

This is also the magnitude of the momentum of the electron. The kinetic energy of the electron is

$$K = \frac{p^2}{2m} = \frac{\left(5.33\times10^{-23}\,\text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\,\text{kg}\right)} = 1.56\times10^{-15}\,\text{J}.$$

The accelerating potential is

$$V = \frac{K}{e} = \frac{1.56\times10^{-15}\,\text{J}}{1.60\times10^{-19}\,\text{C}} = 9.76\times10^{3}\,\text{V}.$$

55. (a) We solve v from $\lambda = h/p = h/(m_p v)$:

$$v = \frac{h}{m_p\lambda} = \frac{6.626\times10^{-34}\,\text{J}\cdot\text{s}}{\left(1.6705\times10^{-27}\,\text{kg}\right)\left(0.100\times10^{-12}\,\text{m}\right)} = 3.96\times10^{6}\,\text{m/s}.$$

(b) We set $eV = K = \frac{1}{2}m_p v^2$ and solve for the voltage:

$$V = \frac{m_p v^2}{2e} = \frac{\left(1.6705\times10^{-27}\,\text{kg}\right)\left(3.96\times10^{6}\,\text{m/s}\right)^2}{2\left(1.60\times10^{-19}\,\text{C}\right)} = 8.18\times10^{4}\,\text{V} = 81.8\ \text{kV}.$$

56. (a) Using Euler's formula $e^{i\phi} = \cos\phi + i\sin\phi$, we rewrite $\psi(x)$ as

$$\psi(x) = \psi_0 e^{ikx} = \psi_0\left(\cos kx + i\sin kx\right) = \left(\psi_0\cos kx\right) + i\left(\psi_0\sin kx\right) = a + ib,$$

where $a = \psi_0\cos kx$ and $b = \psi_0\sin kx$ are both real quantities.

(b) The time-dependent wave function is

$$\psi(x,t) = \psi(x)e^{-i\omega t} = \psi_0 e^{ikx} e^{-i\omega t} = \psi_0 e^{i(kx-\omega t)}$$
$$= [\psi_0 \cos(kx - \omega t)] + i[\psi_0 \sin(kx - \omega t)].$$

57. We plug Eq. 38-17 into Eq. 38-16, and note that

$$\frac{d\psi}{dx} = \frac{d}{dx}\left(Ae^{ikx} + Be^{-ikx}\right) = ikAe^{ikx} - ikBe^{-ikx}.$$

Also,

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left(ikAe^{ikx} - ikBe^{-ikx}\right) = -k^2 Ae^{ikx} - k^2 Be^{ikx}.$$

Thus,

$$\frac{d^2\psi}{dx^2} + k^2\psi = -k^2 Ae^{ikx} - k^2 Be^{ikx} + k^2\left(Ae^{ikx} + Be^{-ikx}\right) = 0.$$

58. (a) The product nn^* can be rewritten as

$$nn^* = (a+ib)(a+ib)^* = (a+ib)(a^* + i^* b^*) = (a+ib)(a-ib)$$
$$= a^2 + iba - iab + (ib)(-ib) = a^2 + b^2,$$

which is always real since both a and b are real.

(b) Straightforward manipulation gives

$$|nm| = |(a+ib)(c+id)| = |ac + iad + ibc + (-i)^2 bd| = |(ac-bd) + i(ad+bc)|$$
$$= \sqrt{(ac-bd)^2 + (ad+bc)^2} = \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}.$$

However, since

$$|n||m| = |a+ib||c+id| = \sqrt{a^2+b^2}\sqrt{c^2+d^2}$$
$$= \sqrt{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2},$$

we conclude that $|nm| = |n|\,|m|$.

59. The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$ and the wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now, the kinetic energy K and the momentum are related by $K = p^2/2m$, where m is the mass of the particle. Thus $p = \sqrt{2mK}$ and

$$k = \frac{2\pi\sqrt{2mK}}{h}.$$

60. The wave function is now given by

$$\Psi(x,t) = \psi_0 e^{-i(kx+\omega t)}.$$

This function describes a plane matter wave traveling in the negative x direction. An example of the actual particles that fit this description is a free electron with linear momentum $\vec{p} = -(hk/2\pi)\hat{i}$ and kinetic energy

$$K = \frac{p^2}{2m_e} = \frac{h^2k^2}{8\pi^2 m_e} \; .$$

61. For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

We substitute $\psi = \psi_0 e^{ikx}$. The second derivative is

$$\frac{d^2\psi}{dx^2} = -k^2\psi_0 e^{ikx} = -k^2\psi.$$

The result is

$$-k^2\psi + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0.$$

Solving for k, we obtain

$$k = \sqrt{\frac{8\pi^2 m}{h^2}[E - U_0]} = \frac{2\pi}{h}\sqrt{2m[E - U_0]}.$$

62. (a) The wave function is now given by

$$\Psi(x,t) = \psi_0\left[e^{i(kx-\omega t)} + e^{-i(kx+\omega t)}\right] = \psi_0 e^{-i\omega t}(e^{ikx} + e^{-ikx}).$$

Thus,

$$\begin{aligned}|\Psi(x,t)|^2 &= \left|\psi_0 e^{-i\omega t}(e^{ikx} + e^{-ikx})\right|^2 = \left|\psi_0 e^{-i\omega t}\right|^2 \left|e^{ikx} + e^{-ikx}\right|^2 = \psi_0^2\left|e^{ikx} + e^{-ikx}\right|^2 \\ &= \psi_0^2\,|(\cos kx + i\sin kx) + (\cos kx - i\sin kx)|^2 = 4\psi_0^2(\cos kx)^2 \\ &= 2\psi_0^2(1 + \cos 2kx).\end{aligned}$$

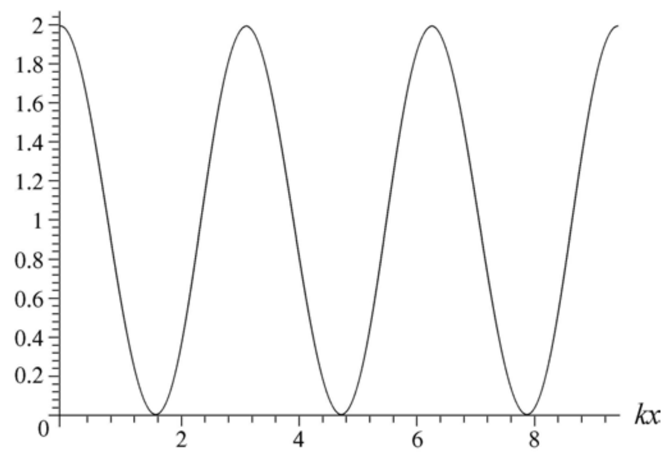
(b) Consider two plane matter waves, each with the same amplitude $\psi_0/\sqrt{2}$ and traveling in opposite directions along the x axis. The combined wave Ψ is a standing wave:

$$\Psi(x,t) = \psi_0 e^{i(kx-\omega t)} + \psi_0 e^{-i(kx+\omega t)} = \psi_0(e^{ikx} + e^{-ikx})e^{-i\omega t} = (2\psi_0 \cos kx)e^{-i\omega t}.$$

Thus, the squared amplitude of the matter wave is

$$|\Psi(x,t)|^2 = (2\psi_0 \cos kx)^2 \left|e^{-i\omega t}\right|^2 = 2\psi_0^2(1+\cos 2kx),$$

which is shown below.



(c) We set $|\Psi(x,t)|^2 = 2\psi_0^2(1+\cos 2kx) = 0$ to obtain $\cos(2kx) = -1$. This gives

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = (2n+1)\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x:

$$x = \frac{1}{4}(2n+1)\lambda.$$

(d) The most probable positions for finding the particle are where $|\Psi(x,t)| \propto (1+\cos 2kx)$ reaches its maximum. Thus $\cos 2kx = 1$, or

$$2kx = 2\left(\frac{2\pi}{\lambda}\right) = 2n\pi, \quad (n = 0, 1, 2, 3, \ldots)$$

We solve for x and find $x = \frac{1}{2}n\lambda$.

63. If the momentum is measured at the same time as the position, then

$$\Delta p \approx \frac{\hbar}{\Delta x} = \frac{6.63\times10^{-34}\ \mathrm{J\cdot s}}{2\pi(50\ \mathrm{pm})} = 2.1\times10^{-24}\ \mathrm{kg\cdot m/s}\ .$$

64. (a) Using the value $hc = 1240\,\mathrm{nm\cdot eV}$, we have

$$E = \frac{hc}{\lambda} = \frac{1240\,\mathrm{nm\cdot eV}}{10.0\times10^{-3}\ \mathrm{nm}} = 124\,\mathrm{keV}\ .$$

(b) The kinetic energy gained by the electron is equal to the energy decrease of the photon:

$$\begin{aligned}\Delta E &= \Delta\left(\frac{hc}{\lambda}\right) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda+\Delta\lambda}\right) = \left(\frac{hc}{\lambda}\right)\left(\frac{\Delta\lambda}{\lambda+\Delta\lambda}\right) = \frac{E}{1+\lambda/\Delta\lambda}\\ &= \frac{E}{1+\frac{\lambda}{\lambda_C(1-\cos\phi)}} = \frac{124\,\mathrm{keV}}{1+\frac{10.0\,\mathrm{pm}}{(2.43\,\mathrm{pm})(1-\cos180^\circ)}}\\ &= 40.5\,\mathrm{keV}.\end{aligned}$$

(c) It is impossible to "view" an atomic electron with such a high-energy photon, because with the energy imparted to the electron the photon would have knocked the electron out of its orbit.

65. We use the uncertainty relationship $\Delta x\Delta p \ge \hbar$. Letting $\Delta x = \lambda$, the de Broglie wavelength, we solve for the minimum uncertainty in p:

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\lambda} = \frac{p}{2\pi}$$

where the de Broglie relationship $p = h/\lambda$ is used. We use $1/2\pi = 0.080$ to obtain $\Delta p = 0.080p$. We would expect the measured value of the momentum to lie between $0.92p$ and $1.08p$. Measured values of zero, $0.5p$, and $2p$ would all be surprising.

66. (a) The rate at which incident protons arrive at the barrier is

$$n = 1.0\,\mathrm{kA}/1.60\times10^{-19}\ \mathrm{C} = 6.25\times10^{21}/\mathrm{s}\ .$$

Letting $nTt = 1$, we find the waiting time t:

$$t = (nT)^{-1} = \frac{1}{n}\exp\left(2L\sqrt{\frac{8\pi^2 m_p (U_b - E)}{h^2}}\right)$$
$$= \left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left(\frac{2\pi(0.70\text{nm})}{1240\text{eV}\cdot\text{nm}}\sqrt{8(938\text{MeV})(6.0\text{eV}-5.0\text{eV})}\right)$$
$$= 3.37\times10^{111}\,\text{s} \approx 10^{104}\,\text{y},$$

which is much longer than the age of the universe.

(b) Replacing the mass of the proton with that of the electron, we obtain the corresponding waiting time for an electron:

$$t = (nT)^{-1} = \frac{1}{n}\exp\left[2L\sqrt{\frac{8\pi^2 m_e (U_b - E)}{h^2}}\right]$$
$$= \left(\frac{1}{6.25\times10^{21}/\text{s}}\right)\exp\left[\frac{2\pi(0.70\text{nm})}{1240\text{eV}\cdot\text{nm}}\sqrt{8(0.511\text{MeV})(6.0\text{eV}-5.0\text{eV})}\right]$$
$$= 2.1\times10^{-19}\,\text{s}.$$

The enormous difference between the two waiting times is the result of the difference between the masses of the two kinds of particles.

67. (a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

If the change ΔU_b in U_b is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU_b}\Delta U_b = -2LT\frac{db}{dU_b}\Delta U_b.$$

Now,

$$\frac{db}{dU_b} = \frac{1}{2\sqrt{U_b - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U_b - E)}\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}} = \frac{b}{2(U_b - E)}.$$

Thus,

$$\Delta T = -LTb\frac{\Delta U_b}{U_b - E} .$$

For the data of Sample Problem 38-7, $2bL = 10.0$, so $bL = 5.0$ and

$$\frac{\Delta T}{T} = -bL\frac{\Delta U_b}{U_b - E} = -(5.0)\frac{(0.010)(6.8\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = -0.20 .$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\Delta L = -2be^{-2bL}\Delta L = -2bT\Delta L$$

and

$$\frac{\Delta T}{T} = -2b\Delta L = -2\left(6.67\times10^{9}\ \text{m}^{-1}\right)(0.010)\left(750\times10^{-12}\ \text{m}\right) = -0.10 .$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\Delta E = -2Le^{-2bL}\frac{db}{dE}\Delta E = -2LT\frac{db}{dE}\Delta E.$$

Now, $db/dE = -db/dU_b = -b/2(U_b - E)$, so

$$\frac{\Delta T}{T} = bL\frac{\Delta E}{U_b - E} = (5.0)\frac{(0.010)(5.1\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = 0.15 .$$

There is a 15% increase in the transmission coefficient.

68. With

$$T \approx e^{-2bL} = \exp\left(-2L\sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}\right),$$

we have

$$E = U_b - \frac{1}{2m}\left(\frac{h\ln T}{4\pi L}\right)^2 = 6.0\,\text{eV} - \frac{1}{2(0.511\,\text{MeV})}\left[\frac{(1240\,\text{eV}\cdot\text{nm})(\ln 0.001)}{4\pi(0.70\,\text{nm})}\right]^2$$
$$= 5.1\,\text{eV}.$$

69. (a) The transmission coefficient T for a particle of mass m and energy E that is incident on a barrier of height U_b and width L is given by

$$T = e^{-2bL},$$

where

$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{h^2}}.$$

For the proton, we have

$$b = \sqrt{\frac{8\pi^2 (1.6726\times10^{-27}\,\text{kg})(10\,\text{MeV} - 3.0\,\text{MeV})(1.6022\times10^{-13}\ \text{J/MeV})}{(6.6261\times10^{-34}\,\text{J}\cdot\text{s})^2}}$$
$$= 5.8082\times10^{14}\,\text{m}^{-1}.$$

This gives $bL = (5.8082\times10^{14}\ \text{m}^{-1})(10\times10^{-15}\ \text{m}) = 5.8082,$ and

$$T = e^{-2(5.8082)} = 9.02\times10^{-6}.$$

The value of b was computed to a greater number of significant digits than usual because an exponential is quite sensitive to the value of the exponent.

(b) Mechanical energy is conserved. Before the proton reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero. After passing through the barrier, the proton again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(c) Energy is also conserved for the reflection process. After reflection, the proton has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

(d) The mass of a deuteron is 2.0141 u = 3.3454×10^{-27} kg, so

$$b = \sqrt{\frac{8\pi^2 (3.3454\times10^{-27}\,\text{kg})(10\,\text{MeV} - 3.0\,\text{MeV})(1.6022\times10^{-13}\ \text{J/MeV})}{(6.6261\times10^{-34}\,\text{J}\cdot\text{s})^2}}$$
$$= 8.2143\times10^{14}\,\text{m}^{-1}.$$

This gives $bL = (8.2143\times10^{14}\,\text{m}^{-1})(10\times10^{-15}\,\text{m}) = 8.2143,$ and

$$T = e^{-2(8.2143)} = 7.33\times10^{-8}.$$

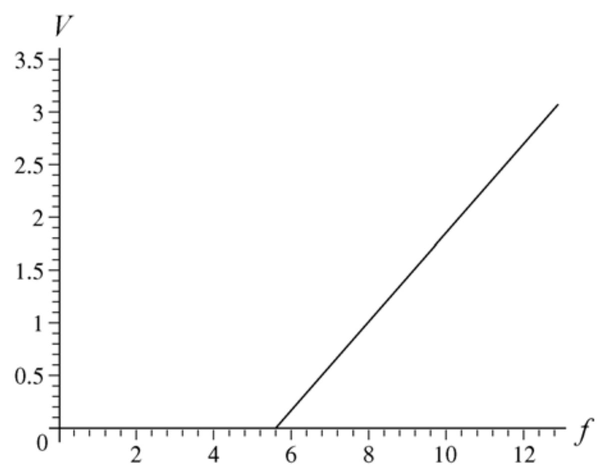
(e) As in the case of a proton, mechanical energy is conserved. Before the deuteron reaches the barrier, it has a kinetic energy of 3.0 MeV and a potential energy of zero.

After passing through the barrier, the deuteron again has a potential energy of zero, thus a kinetic energy of 3.0 MeV.

(f) Energy is also conserved for the reflection process. After reflection, the deuteron has a potential energy of zero, and thus a kinetic energy of 3.0 MeV.

70. (a) We calculate frequencies from the wavelengths (expressed in SI units) using Eq. 38-1. Our plot of the points and the line which gives the least squares fit to the data is shown below. The vertical axis is in volts and the horizontal axis, when multiplied by 10^{14}, gives the frequencies in Hertz.

From our least squares fit procedure, we determine the slope to be 4.14×10^{-15} V·s, which, upon multiplying by e, gives 4.14×10^{-15} eV·s. The result is in very good agreement with the value given in Eq. 38-3.



(b) Our least squares fit procedure can also determine the y-intercept for that line. The y-intercept is the negative of the photoelectric work function. In this way, we find Φ = 2.31 eV.

71. We rewrite Eq. 38-9 as

$$\frac{h}{m\lambda} - \frac{h}{m\lambda'}\cos\phi = \frac{v}{\sqrt{1-(v/c)^2}}\cos\theta \; ,$$

and Eq. 38-10 as

$$\frac{h}{m\lambda'}\sin\phi = \frac{v}{\sqrt{1-(v/c)^2}}\sin\theta \; .$$

We square both equations and add up the two sides:

$$\left(\frac{h}{m}\right)^2\left[\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\cos\phi\right)^2 + \left(\frac{1}{\lambda'}\sin\phi\right)^2\right] = \frac{v^2}{1-(v/c)^2} \; ,$$

where we use $\sin^2\theta + \cos^2\theta = 1$ to eliminate θ. Now the right-hand side can be written as

$$\frac{v^2}{1-(v/c)^2} = -c^2\left[1-\frac{1}{1-(v/c)^2}\right],$$

so

$$\frac{1}{1-(v/c)^2} = \left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1 .$$

Now we rewrite Eq. 38-8 as

$$\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1 = \frac{1}{\sqrt{1-(v/c)^2}} .$$

If we square this, then it can be directly compared with the previous equation we obtained for $[1-(v/c)^2]^{-1}$. This yields

$$\left[\frac{h}{mc}\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\right)+1\right]^2 = \left(\frac{h}{mc}\right)^2\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda'}\cos\phi\right)^2+\left(\frac{1}{\lambda'}\sin\phi\right)^2\right]+1 .$$

We have so far eliminated θ and v. Working out the squares on both sides and noting that $\sin^2\phi + \cos^2\phi = 1$, we get

$$\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1-\cos\phi) .$$

72. (a) The average kinetic energy is

$$K = \frac{3}{2}kT = \frac{3}{2}\left(1.38\times10^{-23}\ \text{J/K}\right)\left(300\ \text{K}\right) = 6.21\times10^{-21}\ \text{J} = 3.88\times10^{-2}\ \text{eV}.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{\sqrt{2m_nK}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(1.675\times10^{-27}\ \text{kg}\right)\left(6.21\times10^{-21}\ \text{J}\right)}} = 1.46\times10^{-10}\ \text{m}.$$

73. (a) The average de Broglie wavelength is

$$\lambda_{\text{avg}} = \frac{h}{p_{\text{avg}}} = \frac{h}{\sqrt{2mK_{\text{avg}}}} = \frac{h}{\sqrt{2m(3kT/2)}} = \frac{hc}{\sqrt{2(mc^2)kT}}$$

$$= \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{3(4)(938\,\text{MeV})(8.62\times10^{-5}\,\text{eV/K})(300\,\text{K})}}$$

$$= 7.3\times10^{-11}\,\text{m} = 73\,\text{pm}.$$

(b) The average separation is

$$d_{\text{avg}} = \frac{1}{\sqrt[3]{n}} = \frac{1}{\sqrt[3]{p/kT}} = \sqrt[3]{\frac{(1.38\times10^{-23}\,\text{J/K})(300\,\text{K})}{1.01\times10^5\,\text{Pa}}} = 3.4\,\text{nm}.$$

(c) Yes, since $\lambda_{\text{avg}} \ll d_{\text{avg}}$.

74. (a) Since

$$E_{\text{ph}} = h/\lambda = 1240\ \text{eV}\cdot\text{nm}/680\ \text{nm} = 1.82\ \text{eV} < \Phi = 2.28\ \text{eV},$$

there is no photoelectric emission.

(b) The cutoff wavelength is the longest wavelength of photons which will cause photoelectric emission. In sodium, this is given by

$$E_{\text{ph}} = hc/\lambda_{\max} = \Phi,$$

or

$$\lambda_{\max} = hc/\Phi = (1240\ \text{eV}\cdot\text{nm})/2.28\ \text{eV} = 544\ \text{nm}.$$

(c) This corresponds to the color green.

75. Using the value $hc = 1240\,\text{eV}\cdot\text{nm}$, we obtain

$$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{21\times10^7\ \text{nm}} = 5.9\times10^{-6}\ \text{eV} = 5.9\,\mu\text{eV}.$$

76. We substitute the classical relationship between momentum p and velocity v, $v = p/m$ into the classical definition of kinetic energy, $K = \frac{1}{2}mv^2$ to obtain $K = p^2/2m$. Here m is the mass of an electron. Thus $p = \sqrt{2mK}$. The relationship between the momentum and the de Broglie wavelength λ is $\lambda = h/p$, where h is the Planck constant. Thus,

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

If K is given in electron volts, then

$$\lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.109\times10^{-31}\ \text{kg})(1.602\times10^{-19}\ \text{J/eV})K}} = \frac{1.226\times10^{-9}\ \text{m}\cdot\text{eV}^{1/2}}{\sqrt{K}}$$
$$= \frac{1.226\,\text{nm}\cdot\text{eV}^{1/2}}{\sqrt{K}}.$$

77. The de Broglie wavelength for the bullet is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J.s}}{(40\times10^{-3}\ \text{kg})(1000\,\text{m/s})} = 1.7\times10^{-35}\ \text{m}\ .$$

78. (a) Since $p_x = p_y = 0,\ \Delta p_x = \Delta p_y = 0$. Thus from Eq. 38-20 both Δx and Δy are infinite. It is therefore impossible to assign a y or z coordinate to the position of an electron.

(b) Since it is independent of y and z the wave function $\Psi(x)$ should describe a plane wave that extends infinitely in both the y and z directions. Also from Fig. 38-12 we see that $|\Psi(x)|^2$ extends infinitely along the x axis. Thus the matter wave described by $\Psi(x)$ extends throughout the entire three-dimensional space.

79. The uncertainty in the momentum is

$$\Delta p = m\,\Delta v = (0.50\ \text{kg})(1.0\ \text{m/s}) = 0.50\ \text{kg·m/s},$$

where Δv is the uncertainty in the velocity. Solving the uncertainty relationship $\Delta x \Delta p \ge \eta$ for the minimum uncertainty in the coordinate x, we obtain

$$\Delta x = \frac{\eta}{\Delta p} = \frac{0.60\,\text{J}\cdot\text{s}}{2\pi(0.50\,\text{kg}\cdot\text{m/s})} = 0.19\,\text{m}\ .$$

80. The kinetic energy of the car of mass m moving at speed v is given by $E = \frac{1}{2}mv^2$, while the potential barrier it has to tunnel through is $U_b = mgh$, where h = 24 m. According to Eq. 38-21 and 38-22 the tunneling probability is given by $T \approx e^{-2bL}$, where

$$b = \sqrt{\frac{8\pi^2 m\left(U_b - E\right)}{h^2}} = \sqrt{\frac{8\pi^2 m\left(mgh - \frac{1}{2}mv^2\right)}{h^2}}$$
$$= \frac{2\pi\left(1500\text{kg}\right)}{6.63\times10^{-34}\,\text{J}\cdot\text{s}}\sqrt{2\left[\left(9.8\ \text{m/s}^2\right)\left(24\text{m}\right) - \frac{1}{2}\left(20\ \text{m/s}\right)^2\right]}$$
$$= 1.2\times10^{38}\,\text{m}^{-1}.$$

Thus,

$$2bL = 2\left(1.2\times10^{38}\,\text{m}^{-1}\right)\left(30\text{m}\right) = 7.2\times10^{39}.$$

One can see that $T \approx e^{-2bL}$ is very small (essentially zero).

81. We note that

$$|e^{ikx}|^2 = (e^{ikx})^*(e^{ikx}) = e^{-ikx}e^{ikx} = 1.$$

Referring to Eq. 38-14, we see therefore that $|\psi|^2 = |\Psi|^2$.

82. From Sample Problem 38-4, we have

$$\frac{\Delta E}{E} = \frac{\Delta\lambda}{\lambda+\Delta\lambda} = \frac{(h/mc)(1-\cos\phi)}{\lambda'} = \frac{hf'}{mc^2}(1-\cos\phi)$$

where we use the fact that $\lambda + \Delta\lambda = \lambda' = c/f'$.

83. With no loss of generality, we assume the electron is initially at rest (which simply means we are analyzing the collision from its initial rest frame). If the photon gave all its momentum and energy to the (free) electron, then the momentum and the kinetic energy of the electron would become

$$p = \frac{hf}{c}, \quad K = hf,$$

respectively. Plugging these expressions into Eq. 38-51 (with m referring to the mass of the electron) leads to

$$(pc)^2 = K^2 + 2Kmc^2$$
$$(hf)^2 = (hf)^2 + 2hfmc^2$$

which is clearly impossible, since the last term ($2hfmc^2$) is not zero. We have shown that considering total momentum and energy absorption of a photon by a free electron leads to an inconsistency in the mathematics, and thus cannot be expected to happen in nature.

84. The difference between the electron-photon scattering process in this problem and the one studied in the text (the Compton shift, see Eq. 38-11) is that the electron is in motion relative with speed v to the laboratory frame. To utilize the result in Eq. 38-11, shift to a new reference frame in which the electron is at rest before the scattering. Denote the quantities measured in this new frame with a prime ($'$), and apply Eq. 38-11 to yield

$$\Delta\lambda' = \lambda' - \lambda'_0 = \frac{h}{m_e c}(1-\cos\pi) = \frac{2h}{m_e c},$$

where we note that $\phi = \pi$ (since the photon is scattered back in the direction of incidence). Now, from the Doppler shift formula (Eq. 38-25) the frequency f'_0 of the photon prior to the scattering in the new reference frame satisfies

$$f'_0 = \frac{c}{\lambda'_0} = f_0\sqrt{\frac{1+\beta}{1-\beta}},$$

where $\beta = v/c$. Also, as we switch back from the new reference frame to the original one after the scattering

$$f = f'\sqrt{\frac{1-\beta}{1+\beta}} = \frac{c}{\lambda'}\sqrt{\frac{1-\beta}{1+\beta}}\ .$$

We solve the two Doppler-shift equations above for λ' and λ'_0 and substitute the results into the Compton shift formula for $\Delta\lambda'$:

$$\Delta\lambda' = \frac{1}{f}\sqrt{\frac{1-\beta}{1+\beta}} - \frac{1}{f_0}\sqrt{\frac{1-\beta}{1+\beta}} = \frac{2h}{m_e c^2}\ .$$

Some simple algebra then leads to

$$E = hf = hf_0\left[1 + \frac{2h}{m_e c^2}\sqrt{\frac{1+\beta}{1-\beta}}\right]^{-1}.$$