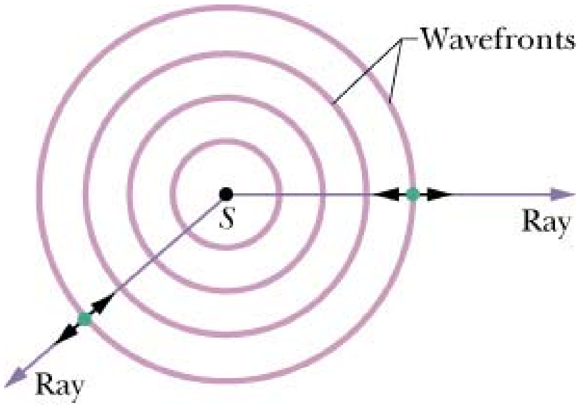
# Chapter 17 Waves II

**In this chapter we will study sound waves and concentrate on the following topics:**

Speed of sound waves
Relation between displacement and pressure amplitude
Interference of sound waves
Sound intensity and sound level
Beats
The Doppler effect

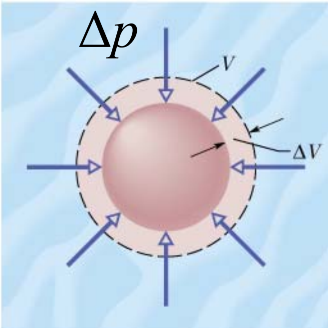
**(17 – 2)**




Sound waves are mechanical **longitudinal** waves that propagate in solids liquids and gases. Seismic waves used by oil explorers propagate in the earth's crust. Sound waves generated by a sonar system propagate in the sea. An orchestra creates sound waves that propagate in the air.

The locus of the points of a sound wave that has the same displacement is called a "**wavefront**". Lines perpendicular to the wavefronts are called "**rays**" and they point along the direction which the sound wave propagates. An example of a point source of sound waves is given in the figure. We assume that the surrounding medium is isotropic i.e. sound propagates with the same speed for all directions. In this case the sound wave spreads outwards uniformly and the wavefronts are spheres centered at the point source. The single arrows indicate the rays. The double arrows indicate the motion of the molecules of the medium in which sound propagates.

$$\boxed{v = \sqrt{\frac{B}{\rho}}}$$




**Bulk modulus** **(17 – 3)**

If we apply an overpressure $\Delta p$ on an object of volume $V$, this results in a change of volume $\Delta V$ as shown in the figure. The bulk modulus of the compressed material is defined as: $B = -\frac{\Delta p}{\Delta V / V}$ SI unit: the Pascal

**Note :** The negative sign denotes the **decrease** in volume when $\Delta p$ is positive.

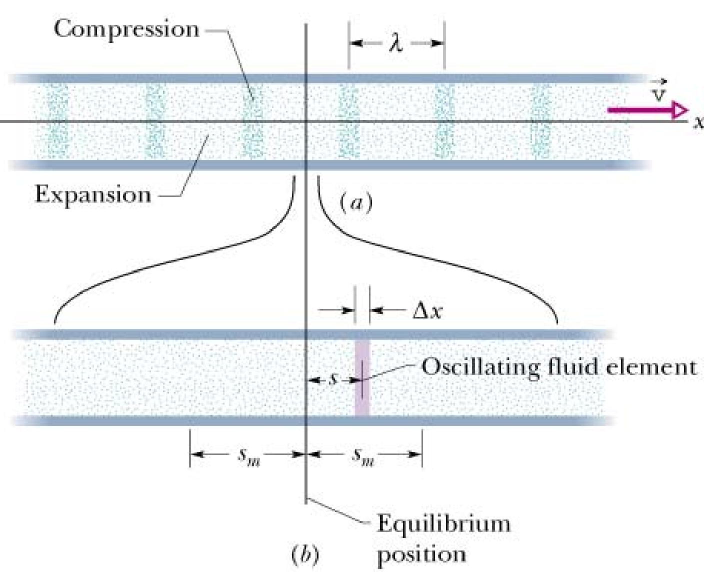
**The speed of sound**

Using the above definition of the bulk modulus and combining it with Newton's second law one can show that the speed of sound in a homogeneous isotropic medium with bulk modulus $B$ and density $\rho$ is given by the equation: $v = \sqrt{\frac{B}{\rho}}$

**Note 1 :** $|\Delta V| = \frac{pV}{B}$ Bulk modulus is smaller for more compressible media. Such media exhibit lower speed of sound.

**Note 2 :** Denser materials (higher $\rho$ ) have lower speed of sound

## Traveling sound waves.

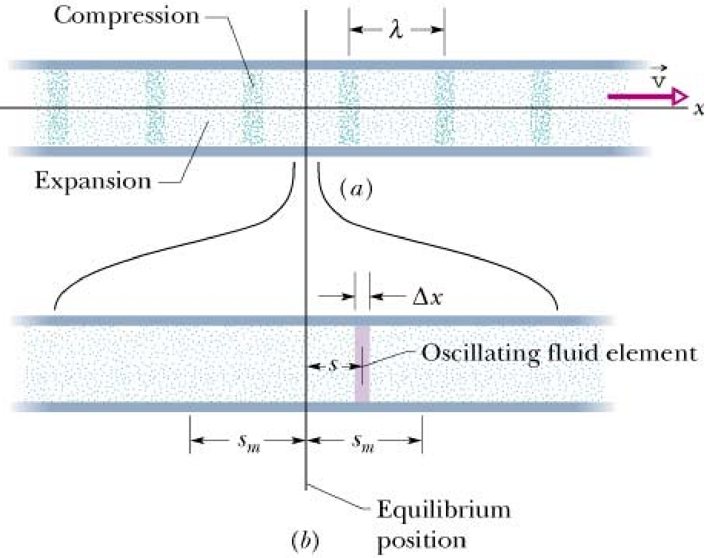



a) $$\underbrace{s(x,t)}_{\text{Displacement}} = \underbrace{s_m}_{\text{Displacement amplitude}} \underbrace{\cos(kx-\omega t)}_{\text{Oscillating term}}$$

b) $$\underbrace{\Delta p(x,t)}_{\text{Pressure variation}} = \underbrace{\Delta p_m}_{\text{Pressure amplitude}} \underbrace{\sin(kx-\omega t)}_{\text{Oscillating term}}$$

Consider the tube filled with air shown in the figure. We generate a harmonic sound wave traveling to the right along the axis of the tube. One simple method is to place a speaker at the left end of the tube and drive it at a particular frequency. Consider an air element of thickness $\Delta x$ which is located at position $x$ before the sound wave is generated. This is known as the "equlibrium position" of the element. Under these conditions the pressure inside the tube is constant In the presence of the sound wave the element oscillates about the equlibrium position. At the same time the pressure at the location of the element oscillates about its static value. The sound wave in the tube can be described using one of two parameters:

$$\boxed{\Delta p_m = (v\upsilon\omega)s_m}$$




a) $$\overbrace{s(x,t)}^{\text{Displacement}} = \underbrace{s_m}_{\text{Displacement amplitude}}\,\underbrace{\cos(kx-\omega t)}_{\text{Oscillating term}}$$

b) $$\underbrace{\Delta p(x,t)}_{\text{Pressure variation}} = \underbrace{\Delta p_m}_{\text{Pressure amplitude}}\,\overbrace{\sin(kx-\omega t)}^{\text{Oscillating term}}$$

**Traveling sound waves.**

One such parameter is the distance $s(x,t)$ of the element from its equilibrium position $s(x,t) = s_m\cos(kx-\omega t)$. The constant $s_m$ is the **displacement amplitude** of the wave. The angular wavenumber $k$ and the angular frequency $\omega$ hase the same meaning as in the case of the transverse waves studied in chapter 16.

The second possibility is to use the pressure variation $\Delta p$ from the static value. $\Delta p(x,t) = \Delta p_m \sin(kx-\omega t)$

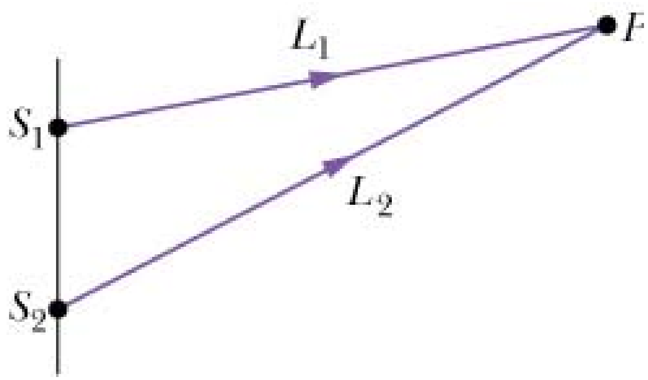
The constant $\Delta p_m$ is the wave's pressure **amplitude.**

The two amplitudes are connected by the equation:

$$\Delta p_m = (v\upsilon\omega)s_m$$

**Note :** The displacement and the pressure variation have a phase difference of 90°. As a result when one parameter has a maximum the other has a minimum and vice versa.

$$\boxed{\phi = \frac{2\pi}{\lambda}\Delta L}$$




## Interference

Consider two point sources of sound waves $S_1$ and $S_2$ shown in the figure. The two sources are in phase and emit sound waves of the same frequency.

Waves from both sources arrive at point P whose distance from $S_1$ and $S_2$ is $L_1$ and $L_2$ respectively.

The two waves interfere at point P.

At time $t$ the phase of sound wave 1 arriving from $S_1$ at point P is $\phi_1 = kL_1 - \omega t$

At time $t$ the phase of sound wave 2 arriving from $S_2$ at point P is $\phi_2 = kL_2 - \omega t$

In general the two waves at P have a phase difference

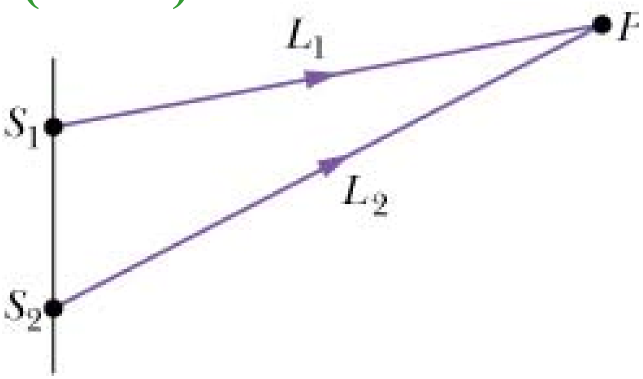
$$\phi = |\phi_2 - \phi_1| = |kL_2 - \omega t - (kL_1 - \omega t)| = k|L_2 - L_1| = \frac{2\pi}{\lambda}|L_2 - L_1|$$

The quantity $|L_2 - L_1|$ is known as the "***path length difference***" $\Delta L$

between the two waves. Thus $\phi = \frac{2\pi}{\lambda}\Delta L$

Here $\lambda$ is the wavelength of the two waves.

**(17 – 7)**




**Constructive intereference.**

The wave at P resulting from the interference of the two waves that arrive from $S_1$ and $S_2$ has a maximum amplitude when the phase difference $\phi = 2\pi m$

$$m = 0, 1, 2, \ldots \;.\quad \rightarrow \frac{2\pi}{\lambda}\Delta L = 2\pi m \quad \rightarrow \Delta L = m\lambda$$

$$\Delta L = 0,\ \lambda,\ 2\lambda, \ldots$$

**Destructive intereference.**

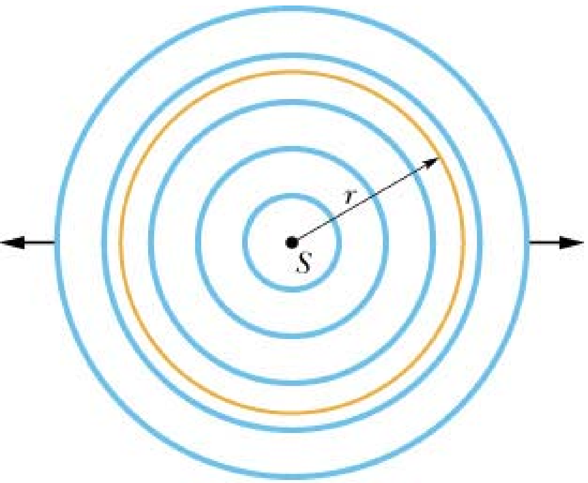
The wave at P resulting from the interference of the two waves that arrive from $S_1$ and $S_2$ has a miniimum amplitude when the phase difference

$$\phi = \pi(2m+1) \qquad m = 0, 1, 2, \ldots \;.\quad \rightarrow \frac{2\pi}{\lambda}\Delta L = \pi(2m+1) \quad \rightarrow$$

$$\Delta L = \left(m + \frac{1}{2}\right)\lambda \qquad \Delta L = \lambda/2,\ 3\lambda/2,\ 5\lambda/2, \ldots$$

$\Delta L$ equal to an integral multiple of $\lambda$ **constructive interference**

$\Delta L$ equal to a half-integral multiple of $\lambda$ $\rightarrow$ **destructive interference**




## Intensity of a sound wave

Consider a wave that is incident normally on a surface of area $A$. The wave transports energy. As a result power $P$ (energy per unit time) passes through $A$. We define at the wave intensity $I$ the ratio $P/A$

$$I = \frac{P}{A}$$ SI units: $\text{W/m}^2$

The intensity of a harmonic wave with displacement amplitude $s_m$ is given by:

$$I = \left(\frac{\rho v \omega^2}{2}\right) s_m^2.$$ In terms of the pressure amplitude $$I = \left(\frac{1}{2\rho v}\right) \Delta p_m^2$$

Consider a point source S emitting a power $P$ in the form of sound waves of a particular frequency. The surrounding medium is isotropic so the waves spread uniformly. The corresponding wavefronts are spheres that have S as their center. The sound intensity at a distance $r$ from S is: $I = \dfrac{P}{4\pi r^2}$

The intensity of a sound wave for a point sources is proportional to $\dfrac{1}{r^2}$

## The decibel

The auditory sensation in humans is proportional to the logarithm of the sound intensity $I$. This allows the ear to percieve a wide range of sound intensities. The threshold of hearing $I_o$ is defined as the lowest sound intensity that can be detected by the human ear. $I_0 = 10^{-12}\ \text{W/m}^2$

The sound level $\beta$ is defined in such a way as to mimic the response of the human ear. $\beta = 10\log\left(\frac{I}{I_o}\right)$ $\beta$ is expressed in decibels (dB)

We can invert the equation above and express $I$ in terms of $\beta$ as:

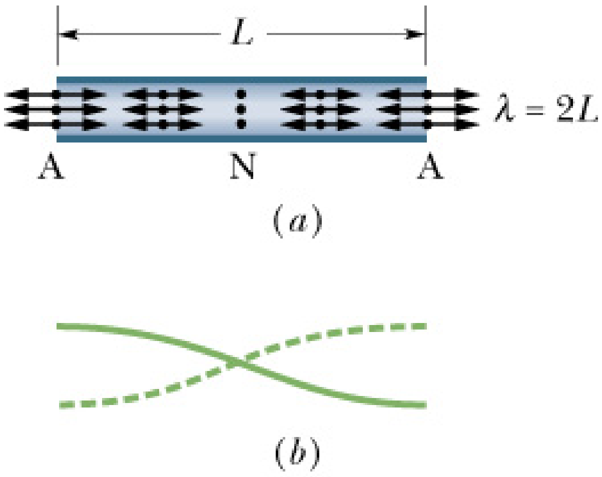
$I = I_o \times 10^{(\beta/10)}$

**Note 1:** For $I = I_o$ we have: $\beta = 10\log 1 = 0$

**Note2:** $\beta$ increases by 10 decibels every time $I$ increases by a factor of 10

For example $\beta = 40$ dB corresponds to $I = 10^4 I_o$

## Sound standing waves in pipes




Consider a pipe filled with air that is open at both ends. Sound waves that have walengths that satisfy a particular relation with the length L of the pipe setup standing waves that have sustained amplitudes.

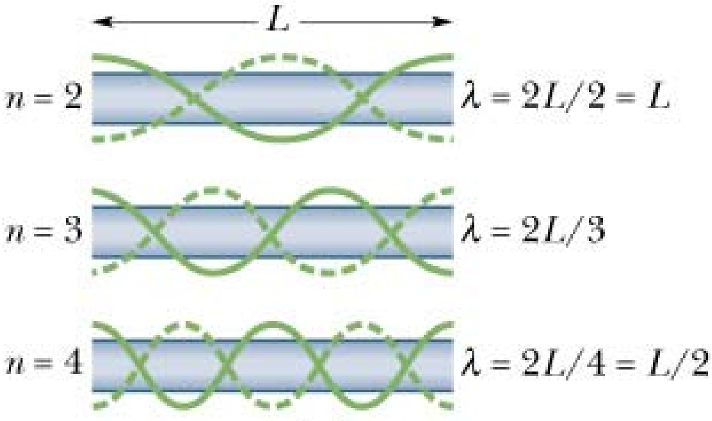
The simplest pattern can be set up in a pipe that is open at both ends as shown in fig.a.
In such a pipe standing waves have a antinode (maximum) in the dispacement amplitude
The amplitude of the standing wave is plotted as function of distance in fig.b.
The pattern has an node at the pipe center since two adjacent antinodes are separated by an anode (minimum). The distance between two adjacent antinodes is $\lambda/2$.

Thus $L = \lambda / 2 \rightarrow \lambda = 2L$ Its frequency $f = \dfrac{v}{\lambda} = \dfrac{v}{2L}$

The standing wave of fig.b is known as the "***fundamnetal mode***" or "***first harmonic***" of the tube.

**Note :** Antinodes in the displacement amplitude correspond to nodes in the pressure amplitude. This is because $s_m$ and $\Delta p_m$ are 90° out of phase.

$$\boxed{\lambda_n = \frac{2L}{n}}$$



(a)

**Standing waves in tubes open at both ends**

The next three standing wave patterns are shown in fig.a. The wavelength $\lambda_n = \dfrac{2L}{n}$ where $n = 1, 2, 3, \ldots$ The integer n is known as the ***harmonic number***

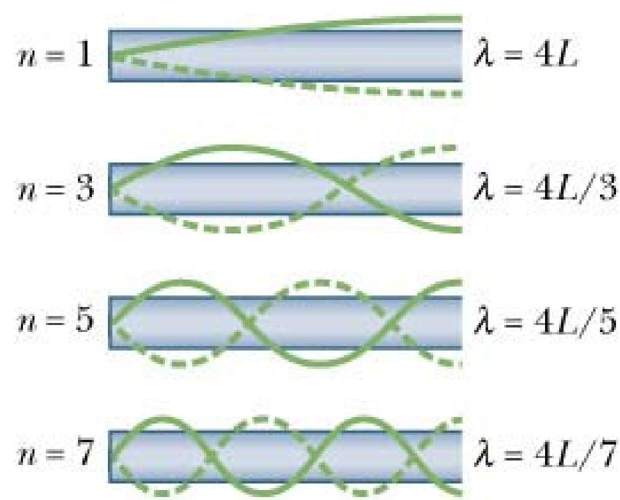
The corresponding frequencies $f_n = \dfrac{nv}{2L}$

**Standing waves in tubes open at one end and closed at the other**

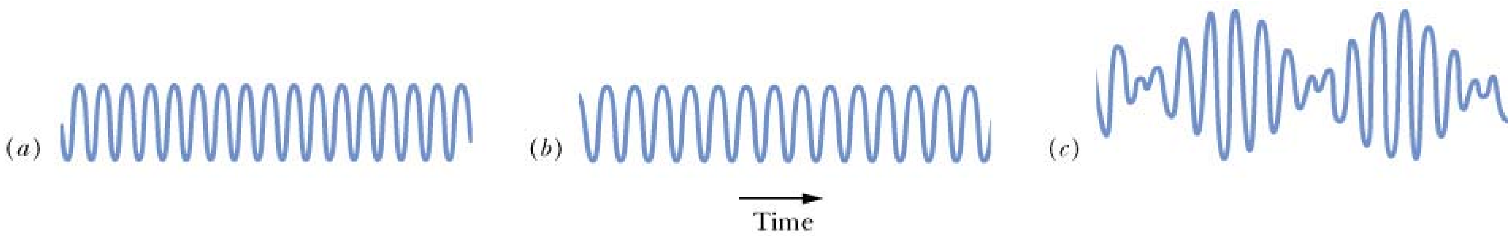
The first four standing wave patterns are shown in fig.a. They have an antinode at the open end and an node at the closed end.

The wavelength $\lambda_n = \dfrac{2L}{n+1/2}$

$$\boxed{\lambda_n = \frac{2L}{n+1/2}}$$



(b)




**Beats.**

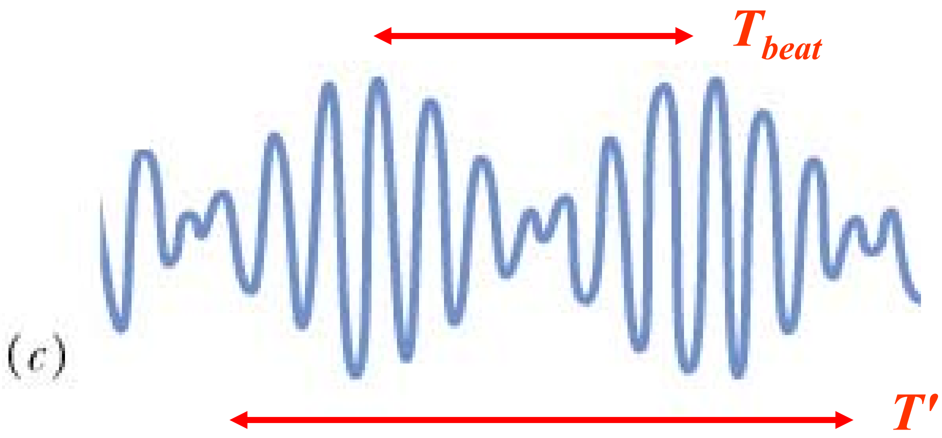
If we listen to two sound waves of equal amplitude and frequencies $f_1$ and $f_2$ $\left(f_1 > f_2 \text{ and } f_1 \approx f_2\right)$ we perceive them as a sound of frequency $f_{av} = \dfrac{f_1 + f_2}{2}$. in addition we also perceive "beats" which are variations in the intensity of the sound with frequency $f_{beat} = f_1 - f_2$. The displacements of the two sound waves are given by the equations: $s_1 = s_m \cos \omega_1 t$, and $s_2 = s_m \cos \omega_2 t$. These are plotted in fig.a and fig.b.

Using the principle of superposition we can determine the resultant displacement as:

$$s = s_1 + s_2 = s_m\left(\cos \omega_1 t + \cos \omega_2 t\right) = 2 s_m \cos\left[\left(\frac{\omega_1 - \omega_2}{2}\right)t\right]\cos\left[\left(\frac{\omega_1 + \omega_2}{2}\right)t\right]$$

$$s = \left[2 s_m \cos \omega' t\right]\cos \omega t \quad \text{where} \quad \omega' = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega = \frac{\omega_1 + \omega_2}{2}$$

Since $\omega_1 \approx \omega_2 \rightarrow \omega\ ?\ \ \omega'$




$$\boxed{f_{beat} = f_1 - f_2}$$

$$s = \left[2s_m \cos\omega' t\right]\cos\omega t \quad \text{where} \quad \omega' = \frac{\omega_1 - \omega_2}{2} \quad \text{and} \quad \omega = \frac{\omega_1 + \omega_2}{2}$$

The displacement s is plotted as function of time in the figure. We can regard it as a cosine function whose amplitude is equal to $\left|2s_m \cos\omega' t\right|$.

The amplitude is time dependent but varies slowly with time. The amplitude exhibits a maximum whenever $\cos\omega' t$ is equal to either +1 or -1 which happens twice within one period of the $\cos\omega' t$ function.

Thus the angual frequancy of the beats $\omega_{beat} = 2\omega' = 2\left(\frac{\omega_1 - \omega_2}{2}\right) = \omega_1 - \omega_2$

The frequancy of the beats $f_{beat} = 2\pi\omega_{beat} = 2\pi\omega_1 - 2\pi\omega_2 = f_1 - f_2$

# The Doppler effect

Consider the source and the detector of sound waves shown in the figure. We assume that the frequency of the source is equal to $f$.

We take as the reference frame that surrounding air through which the sound waves propagate. If there is relative motion between the source and the detector then the detector perceives the frequancy of the sound as $f' \neq f$. If the source or the detector move towards to each other $f' > f$. if on the other hand the source or the detector move away from each other $f' < f$. This is known as the "***Doppler***" effect. The frequecy $f'$ is given by the equation: $f' = f\dfrac{v \pm v_D}{v \pm v_S}$. Here $v_S$ and $v_D$ are the speeds of the source and detector with respect to air, respsctively.

When the motion of the detector or source is **towards** each other the sign of the speed must give an **upward** shift in frequency. If on the other hand the motoion is **away** from each other the sign of the speed must give a **downward** shift in frequency.

The four possible combinantions are illustrated in the next page.

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$$f' = f\,\frac{v + v_D}{v - v_S} \qquad f' > f$$


$$f' = f\,\frac{v - v_D}{v + v_S} \qquad f' < f$$


$$f' = f\,\frac{v - v_D}{v - v_S}$$


$$f' = f\,\frac{v + v_D}{v + v_S}$$

$$\boxed{f' = f\,\frac{v \pm v_D}{v \pm v_S}}$$