Chapter 35

Interference

The concept of **optical interference** is critical to understanding many natural phenomena, ranging from color shifting in butterfly wings to intensity patterns formed by small apertures. These phenomena cannot be explained using simple geometrical optics, and are based on the wave nature of light.

In this chapter we explore the wave nature of light and examine several key optical interference phenomena.
Huygen’s Principle: All points on a wavefront serve as point sources of spherical secondary wavelets. After time $t$, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.
Law of Refraction

\[ t = \frac{\lambda_1}{v_1} = \frac{\lambda_2}{v_2} \rightarrow \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \]

\[ \sin \theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle hce)} \]

\[ \sin \theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle hcg)} \]

\[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} \]

Index of Refraction: \[ n = \frac{c}{v} \]

\[ n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2} \]

\[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \]

Law of Refraction: \[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
Wavelength and Index of Refraction

\[ \frac{\lambda_n}{\lambda} = \frac{v}{c} \Rightarrow \lambda_n = \lambda \frac{v}{c} \Rightarrow \lambda_n = \frac{\lambda}{n} \]

\[ f_n = \frac{v}{\lambda_n} = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f \]

The frequency of light in a medium is the same as it is in vacuum.

Since wavelengths in n1 and n2 are different, the two beams may no longer be in phase.

Number of wavelengths in n1: \( N_1 = \frac{L}{\lambda_{n1}} = \frac{L}{\lambda / n_1} = \frac{Ln_1}{\lambda} \)

Number of wavelengths in n2: \( N_2 = \frac{L}{\lambda_{n2}} = \frac{L}{\lambda / n_2} = \frac{Ln_2}{\lambda} \)

Assuming \( n_2 > n_1 \): \( N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_2}{\lambda} = \frac{L}{\lambda} (n_2 - n_1) \)

\( N_2 - N_1 = 1/2 \) wavelength → destructive interference
The geometrical explanation of rainbows given in Ch. 34 is incomplete. Interference, constructive for some colors at certain angles, destructive for other colors at the same angles is an important component of rainbows.
For plane waves entering a single slit, the waves emerging from the slit start spreading out, diffracting.

**Fig. 35-7**

- **(a)** Incident wave \( \text{\textless} | - | \lambda \) and diffracted wave \( \text{\textless} | - | \frac{\lambda}{a} \) (6.0\( \lambda \)).
- **(b)** Incident wave \( \text{\textless} | - | \lambda \) and diffracted wave \( \text{\textless} | - | \frac{\lambda}{a} \) (3.0\( \lambda \)).
- **(c)** Incident wave \( \text{\textless} | - | \lambda \) and diffracted wave \( \text{\textless} | - | \frac{\lambda}{a} \) (1.5\( \lambda \)).
Young’s Experiment

For waves entering a two slit, the emerging waves interfere and form an interference (diffraction) pattern.

Fig. 35-8
Locating Fringes

The phase difference between two waves can change if the waves travel paths of different lengths.

What appears at each point on the screen is determined by the path length difference $\Delta L$ of the rays reaching that point.

Path Length Difference: $\Delta L = d \sin \theta$
Locating Fringes

if $\Delta L = d \sin \theta = (\text{integer})(\lambda) \rightarrow \text{bright fringe}$

**Maxima-bright fringes:**

$$d \sin \theta = m\lambda \quad \text{for} \quad m = 0, 1, 2, K$$

if $\Delta L = d \sin \theta = (\text{odd number})(\lambda) \rightarrow \text{dark fringe}$

**Minima-dark fringes:**

$$d \sin \theta = (m + \frac{1}{2})\lambda \quad \text{for} \quad m = 0, 1, 2, K$$

$m = 2$ bright fringe at: $\theta = \sin^{-1}\left(\frac{2\lambda}{d}\right)$

$m = 1$ dark fringe at: $\theta = \sin^{-1}\left(\frac{1.5\lambda}{d}\right)$
Coherence

Two sources to produce an interference that is stable over time, if their light has a phase relationship that does not change with time: \( E(t) = E_0 \cos(\omega t + \phi) \)

Coherent sources: Phase \( \phi \) must be well defined and constant. When waves from coherent sources meet, stable interference can occur. Sunlight is coherent over a short length and time range. Since laser light is produced by cooperative behavior of atoms, it is coherent of long length and time ranges

Incoherent sources: \( \phi \) jitters randomly in time, no stable interference occurs
Intensity in Double-Slit Interference

\[ E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin(\omega t + \phi) \]

\[ I = 4I_0 \cos^2 \frac{1}{2} \phi \]

\[ \phi = \frac{2\pi d}{\lambda} \sin \theta \]

maxima when: \( \frac{1}{2} \phi = m\pi \) for \( m = 0, 1, 2, K \) \( \rightarrow \phi = 2m\pi = \frac{2\pi d}{\lambda} \sin \theta \)

\( \rightarrow d \sin \theta = m\lambda \) for \( m = 0, 1, 2, K \) (maxima)

minima when: \( \frac{1}{2} \phi = (m + \frac{1}{2})\pi \) \( \rightarrow d \sin \theta = (m + \frac{1}{2})\lambda \) for \( m = 0, 1, 2, K \) (minima)

\[ I_{\text{avg}} = 2I_0 \]
Proof of Eqs. 35-22 and 35-23

**Eq. 35-22**

\[ E(t) = E_0 \sin \omega t + E_0 \sin (\omega t + \phi) = ? \]

\[ E = 2 \left( E_0 \cos \beta \right) = 2E_0 \cos \frac{1}{2} \phi \]

\[ E^2 = 4E_0^2 \cos^2 \frac{1}{2} \phi \]

\[ \frac{I}{I_0} = \frac{E^2}{E_0^2} = 4 \cos^2 \frac{1}{2} \phi \rightarrow I = 4I_0 \cos^2 \frac{1}{2} \phi \]

**Eq. 35-23**

\[ \frac{\text{phase difference}}{2\pi} = \frac{\text{path length difference}}{\lambda} \]

\[ \frac{\text{phase difference}}{2\pi} = \frac{2\pi}{\lambda} \left( \text{path length difference} \right) \]

\[ \phi = \frac{2\pi}{\lambda} (d \sin \theta) \]
Combining More Than Two Waves

In general, we may want to combine more than two waves. For example, there may be more than two slits.

Procedure:

1. Construct a series of phasors representing the waves to be combined. Draw them end to end, maintaining proper phase relationships between adjacent phasors.

2. Construct the sum of this array. The length of this vector sum gives the amplitude of the resulting phasor. The angle between the vector sum and the first phasor is the phase of the resultant with respect to the first. The projection of this vector sum phasor on the vertical axis gives the time variation of the resultant wave.
Interference from Thin Films

\[ \phi_{12} = ? \]

\[ \theta \approx 0 \]

Fig. 35-15
Reflection Phase Shifts

Fig. 35-16

Reflection
Off lower index
Off higher index

Reflection Phase Shift
0
0.5 wavelength
Equations for Thin-Film Interference

Fig. 35-17

Three effects can contribute to the phase difference between \( r_1 \) and \( r_2 \).

1. Differences in reflection conditions
2. Difference in path length traveled.
3. Differences in the media in which the waves travel. One must use the wavelength in each medium \((\lambda / n)\), to calculate the phase.

\[ \frac{1}{2} \text{ wavelength} \quad \text{phase difference} \quad \text{to difference in reflection of} \quad r_1 \quad \text{and} \quad r_2 \]

\[
2L = \frac{\text{odd number}}{2} \times \text{wavelength} = \frac{\text{odd number}}{2} \times \lambda_n \quad \text{(in-phase waves)}
\]

\[
2L = \text{integer} \times \text{wavelength} = \text{integer} \times \lambda_n \quad \text{(out-of-phase waves)}
\]

\[
\lambda_n = \frac{\lambda}{n} \quad \text{for} \quad m = 0, 1, 2, K \quad \text{(maxima-- bright film in air)}
\]

\[
2L = m \frac{\lambda}{n} \quad \text{for} \quad m = 0, 1, 2, K \quad \text{(minima-- dark film in air)}
\]
Film Thickness Much Less Than $\lambda$

If $L$ much less than $l$, for example $L < 0.1\lambda$, than phase difference due to the path difference $2L$ can be neglected.

Phase difference between $r_1$ and $r_2$ will always be $\frac{1}{2}$ wavelength $\rightarrow$ destructive interference $\rightarrow$ film will appear dark when viewed from illuminated side.
Color Shifting by Morpho Butterflies and Paper Currencies

For the same path difference, different wavelengths (colors) of light will interfere differently. For example, \(2L\) could be an integer number of wavelengths for red light but a half integer wavelengths for blue.

Furthermore, the path difference \(2L\) will change when light strikes the surface at different angles, again changing the interference condition for the different wavelengths of light.

Fig. 35-19
Problem Solving Tactic 1: Thin-Film Equations

Equations 35-36 and 35-37 are for the special case of a higher index film flanked by air on both sides. For multilayer systems, this is not always the case and these equations are not appropriate.

What happens to these equations for the following system?

\[ n_1 = 1 \quad n_2 = 1.5 \quad n_3 = 1.7 \]
Michelson Interferometer

$$\Delta L = 2d_1 - 2d_2 \quad \text{(interferometer)}$$

$$\Delta L_m = 2L \quad \text{(slab of material of thickness } L \text{ placed in front of } M_1)$$

$$N_m = \frac{2L}{\lambda_m} = \frac{2Ln}{\lambda} \quad \text{(number of wavelengths in slab of material)}$$

$$N_a = \frac{2L}{\lambda} \quad \text{(number of wavelengths in same thickness of air)}$$

$$N_m - N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n-1) \quad \text{(difference in wavelengths for paths with and without thin slab)}$$

For each change in path by $1\lambda$, the interference pattern shifts by one fringe at $T$. By counting the fringe change, one determines $N_m - N_a$ and can then solve for $L$ in terms of $\lambda$ and $n$. 

35-20