In this chapter we focus on a goal of physics that has become enormously important in the last half century. That goal is to answer the question: What are the mechanisms by which a material conducts, or does not conduct electricity?

The answers are complex since they involve applying quantum mechanics not just to individual particles and atoms, but to a tremendous number of particles and atoms grouped together and interacting.

Scientists and engineers have made great strides in the quantum physics of materials science, which is why we have computers, calculators, cell phones, and many other types of solid-state devices.

We begin by characterizing solids that conduct electricity and those that do not.
Electrical Properties of Solids

Crystalline solid: solid whose atoms are arranged in a repetitive three-dimensional structure (lattice). Basic unit (unit cell) is repeated throughout the solid.

Basic Electrical Properties

1. **Resisivity** $\rho$: relates how much current an applied electric field produces in the solid (see Section 26-4). Units ohm meter ($\Omega \text{m}$).

2. **Temperature coefficient of resistivity** $\alpha$: defined as $\alpha=\frac{1}{\rho}\frac{d\rho}{dT}$. Characterizes how resistivity changes with temperature. Units inverse Kelvin ($K^{-1}$).

3. **Number density of charge carriers** $n$: the number of charge carriers per unit volume. Can be determined from Hall measurements (Section 28-4). Units inverse cubic meter ($m^{-3}$).
### Table 41-1 Some Electrical Properties of Two Materials

<table>
<thead>
<tr>
<th>Properties</th>
<th>Unit</th>
<th>Copper</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of conductor</td>
<td></td>
<td>Metal</td>
<td>Semiconductor</td>
</tr>
<tr>
<td>Resistivity, $\rho$</td>
<td>$\Omega$ m</td>
<td>2x10^{-8}</td>
<td>3x10^3</td>
</tr>
<tr>
<td>Temperature Coeff. Of resistivity, $\alpha$</td>
<td>K^{-1}</td>
<td>+4x10^{-3}</td>
<td>-70x10^{-3}</td>
</tr>
<tr>
<td>Number density of charge carriers, $n$</td>
<td>m^{-3}</td>
<td>9x10^{28}</td>
<td>1x10^{16}</td>
</tr>
</tbody>
</table>
Energy Levels in a Crystalline Solid

Electronic configuration of copper atom:

\[ 1s^2 \ 2s^2 2p^6 \ 3s^2 3p^6 3d^{10} \ 4s^1 \]

Pauli exclusion → localized energy states split to accommodate all electrons, e.g., not allowed to have 4 electrons in \( 1s \) state. New states are extended throughout material.

Fig. 41-2

Fig. 41-3
To create a current that moves charge in a given direction, one must be able to excite electrons to higher energy states. If there are no unoccupied higher energy states close to the topmost electrons, no current can flow.

In metals, electrons in the highest occupied band can readily jump to higher unoccupied levels. These conduction electrons can move freely throughout the sample, like molecules of gas in a closed container (see free electron model-Section 26-6).

**Insulators and Metals**

![Diagram of Insulator and Metal with Fermi Energy](image)
How Many Conduction Electrons Are There?

Not all electrons in a solid carry current. Low energy electrons that are deeply buried in filled bands have no unoccupied states nearby into which they can jump, so they cannot readily increase their kinetic energy. Therefore, only the electrons at the outermost occupied shells (near the Fermi energy) will conduct current. These are called valence electrons, which also play a critical role in chemical bonding by determining the “valence” of an atom.

\[
\text{(number of conduction electrons in sample)} = \left( \frac{\text{number of atoms in sample}}{\text{atomic mass}} \right) \frac{\text{sample mass } M_{\text{sam}}}{(\text{molar mass } M)/N_A} = \frac{\text{(material's density)}(\text{sample volume } V)}{(\text{molar mass } M)/N_A}
\]

\[
n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume } V}
\]
**Conductivity Above Absolute Zero**

As far as the conduction electrons are concerned, there is little difference between room temperature (300 K) and absolute zero (0 K). Increasing temperature does change the electron distribution by thermally exciting lower energy electrons to higher states. The characteristic thermal energy scale is $kT$ ($k$ is the Boltzmann constant), which at 1000 K is only 0.086 eV. This is a very small energy compared to the Fermi energy, and barely agitates the “sea of electrons.”

**How Many Quantum States Are there?**

Number of states per unit volume in energy range from $E$ to $E+dE$:

$$N(E) = \frac{8\sqrt{2\pi} m^{\frac{1}{2}}}{h^3} E^{\frac{1}{2}} \quad \text{(density of states, m}^{-3}\text{J}^{-1})$$

Analogous to counting number of modes in a pipe organ→frequencies $f$ (energies) become more closely spaced at higher $f$→density (in interval $df$) of modes increases with $f$.

*Fig. 41-5*
Occupyancy Probability $P(E)$

Ability to conduct depends on the probability $P(E)$ that available vacant levels will be occupied. At $T = 0$, the $P(E < E_F) = 1$ and $P(E > E_F) = 0$. At $T > 0$ the electrons distribute themselves according to Fermi-Dirac statistics:

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

(occupancy probability)

At $T = 0$:

For $E < E_F$, $e^{(E-E_F)/kT} \to e^{-\infty} \to P(E) = 1$  

For $E > E_F$, $e^{(E-E_F)/kT} \to e^{+\infty} \to P(E) = 0$

Fermi energy of a material is the energy of a quantum state that has the probability of 0.5 of being occupied by an electron.
How Many Occupied States Are There?

Density of occupied states (per unit volume in energy range $E$ to $E+dE$) is $N_o(E)$:

$$\left(\begin{array}{c}
\text{density of occupied states} \\
N_o(E) \text{ at energy } E
\end{array}\right) = \left(\begin{array}{c}
\text{density of states} \\
N(E) \text{ at energy } E
\end{array}\right) \left(\begin{array}{c}
\text{occupancy probability} \\
P(E) \text{ at energy } E
\end{array}\right)$$

or $N_o(E) = N(E)P(E)$ (density of occupied states)

Fig. 41-7
Calculating the Fermi Energy

At $T = 0$, $n = \int_0^{E_F} N_0(E) \, dE = \int_0^{E_F} N(E)P(E) \, dE = \int_0^{E_F} N(E) \cdot 1 \, dE$

Plugging in for $N(E)$

$n = \frac{8\sqrt{2\pi}m^{3/2}}{h^3} \int_0^{E_F} E^{1/2} \, dE = \frac{8\sqrt{2\pi}m^{3/2}}{h^3} \frac{2E_F^{3/2}}{3}$

$E_F = \left(\frac{3}{16\sqrt{2\pi}}\right)^{2/3} \frac{h^2}{m} n^{2/3} = \frac{0.121h^2}{m} n^{2/3}$
Semiconductors

Semiconductors are qualitatively similar to insulators but with a much smaller (~1.1 eV for silicon compared to 5.5 for diamond) energy gap $E_g$ between top of the valence band and bottom of the conduction band.

**Number density of carriers $n$:** thermal agitation excites some electron at the top of the valence band across to the conduction band, leaving behind unoccupied energy state (holes). Holes behave as positive charges when electric fields are applied.

$$n_{Cu}/n_{Si} \approx 10^{13}.$$

**Resistivity $\rho$:** since $\rho = m/e^2 n \tau$, the large difference in charge carrier density mostly account for the large increase ($\approx 10^{11}$) in $\rho$ in semiconductors compared to metals.

**Temperature coefficient of Resistivity $\alpha$:** When increasing temperature, resistivity in metals increases (more scattering off lattice vibrations) while it decrease in semiconductors (more charge carriers excited across energy gap).
Doped Semiconductors

Doping introduces a small number of suitable replacement atoms (impurities) into the semiconductor lattice. This not only allows one to control the magnitude of $n$, but also its sign!

Fig. 41-9

- **Pure Si**
- **$n$-type doped Si**
- **$p$-type doped Si**

Phosphorous acts as donor

Aluminum acts as acceptor
### Table 41-2

**Properties of Two Doped Semiconductors**

<table>
<thead>
<tr>
<th>Property</th>
<th>Type of Semiconductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix material</td>
<td>Silicon</td>
</tr>
<tr>
<td>Matrix nuclear charge</td>
<td>+14e</td>
</tr>
<tr>
<td>Matrix energy gap</td>
<td>1.2 eV</td>
</tr>
<tr>
<td>Dopant</td>
<td>Phosphorous</td>
</tr>
<tr>
<td>Type of dopant</td>
<td>Donor</td>
</tr>
<tr>
<td>Majority carriers</td>
<td>Electrons</td>
</tr>
<tr>
<td>Minority carriers</td>
<td>Holes</td>
</tr>
<tr>
<td>Dopant energy gap</td>
<td>$E_d = 0.045$ eV</td>
</tr>
<tr>
<td>Dopant valence</td>
<td>5</td>
</tr>
<tr>
<td>Dopant nuclear charge</td>
<td>+15e</td>
</tr>
<tr>
<td>Dopant net ion charge</td>
<td>+e</td>
</tr>
</tbody>
</table>

**Fig. 41-10 (a)**

**Fig. 41-10 (b)**
The $p$-$n$ Junction

Junction plane

Space charge

Depletion zone

Contact potential difference

Fig. 41-11
The Junction Rectifier

Allows current to flow in only one direction

**Fig. 41-12**

**Fig. 41-13**
The Junction Rectifier, cont’d

Forward-bias
depletion region shrinks
Current flows

Back-bias
deployment region grows
No current flows,

Fig. 41-14
Light Emitting Diode

At junction, electrons recombine with holes across $E_g$, emitting light in the process

$$\lambda = \frac{c}{f} = \frac{c}{E_g/h} = \frac{hc}{E_g}$$
The Photo-Diode

Use a p-n junction to detect light. Light is absorbed at p-n junction, producing electrons and holes, allowing a detectible current to flow.

Junction Laser

p-n already has a population inversion. If the junction is placed in an optical cavity (between two mirrors), photons that reflect back to the junction will cause stimulated emission, producing more identical photons, which in term will cause more stimulated emission.
The Transistor

Transistor is a three terminal device where a small gate (G) voltage/current controls the resistance between the source (S) and drain (D), allowing large currents to flow → power amplification!

Field Effect Transistor: gate voltage depletes (dopes) charge carriers in semiconductor, turning it into an insulator (metal)

metal-oxide-semiconductor-field-effect-transistor (MOSFET)
Integrated Circuits

Thousands, even millions of transistors and other electronic components (capacitors, resistors, etc) manufactured on a single chip to make complex devices such as computer processors. Fast, reliable, small, well-suited for mass-production.