Chapter 41

Conduction of Electricity in Solids

In this chapter we focus on a goal of physics that has become enormously important in the last half century. That goal is to answer the question: What are the mechanisms by which a material conducts, or does not conduct electricity?

The answers are complex since they involve applying quantum mechanics not just to individual particles and atoms, but to a tremendous number of particles and atoms grouped together and interacting.

Scientists and engineers have made great strides in the quantum physics of materials science, which is why we have computers, calculators, cell phones, and many other types of solid-state devices.

We begin by characterizing solids that conduct electricity and those that do not.

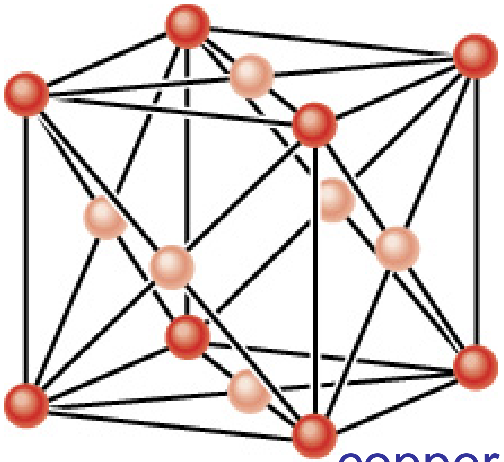
Electrical Properties of Solids

Crystalline solid: solid whose atoms are arranged in a repetitive three-dimensional structure (lattice). Basic unit (unit cell) is repeated throughout the solid.

Basic Electrical Properties

1. **Resisivity ρ:** relates how much current an applied electric field produces in the solid (see Section 26-4). Units ohm meter (Ω m).
2. **Temperature coefficient of resistivity α:** defined as $\alpha=(1/\rho)(d\rho/dT)$. Characterizes how resistivity changes with temperature. Units inverse Kelvin (K^{-1}).
3. **Number density of charge carriers n:** the number of charge carriers per unit volume. Can be determined from Hall measurements (Section 28-4). Units inverse cubic meter (m^{-3})

Face-centered cubic



copper

(a)

Diamond lattice

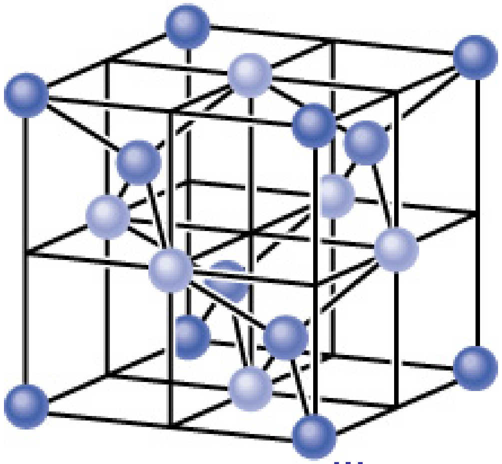


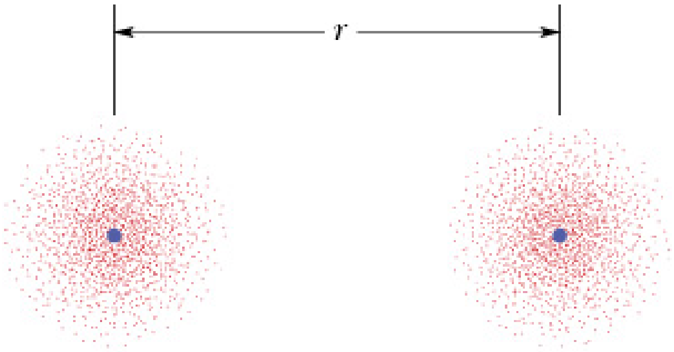
Fig. 41-1 (b) silicon or carbon

Electrical Properties of Solids, cont'd

Table 41-1 Some Electrical Properties of Two Materials

Properties	Unit	Material: Copper	Material: Silicon
Type of conductor		Metal	Semiconductor
Resistivity, ρ	Ω m	2×10^{-8}	3×10^{3}
Temperature Coeff. Of resistivity, α	K^{-1}	$+4 \times 10^{-3}$	-70×10^{-3}
Number density of charge carriers, n	m^{-3}	9×10^{28}	1×10^{16}

Energy Levels in a Crystalline Solid



Electronic configuration of copper atom:

$1s^2\ \ 2s^2 2p^6\ \ 3s^2 3p^6 3d^{10}\ \ 4s^1$

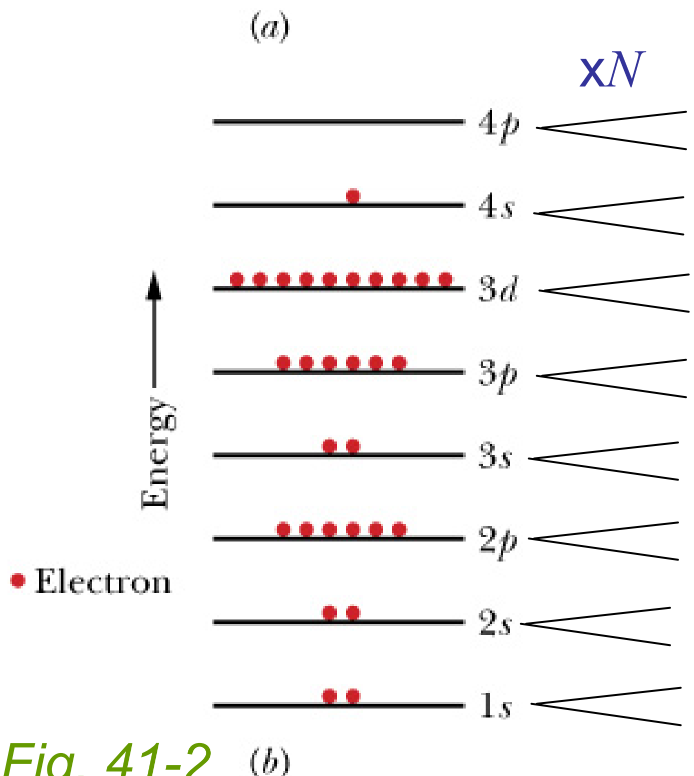


Fig. 41-2

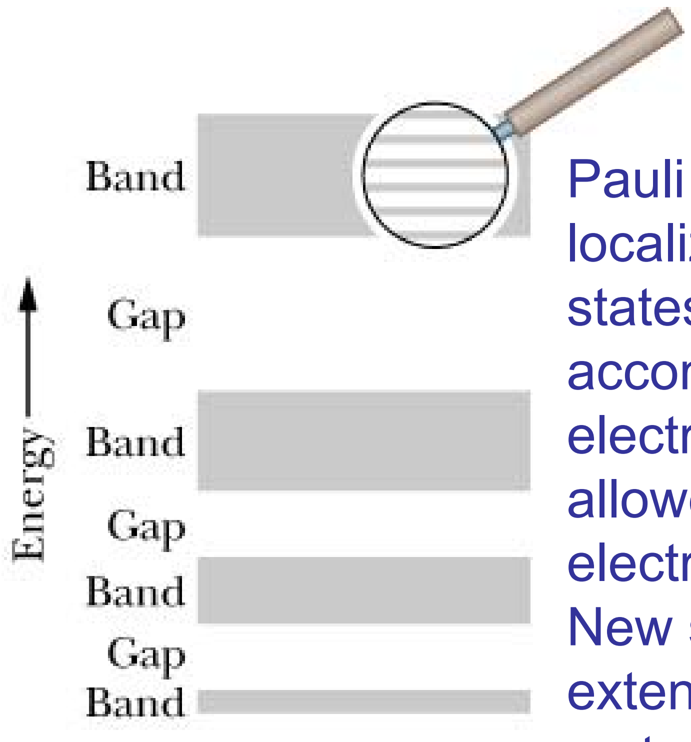


Fig. 41-3

Pauli exclusion→ localized energy states split to accommodate all electrons, e.g., not allowed to have 4 electrons in 1*s* state. New states are extended throughout material.

Insulators and Metals

To create a current that moves charge in a given direction, one must be able to excite electrons to higher energy states. If there are no unoccupied higher energy states close to the topmost electrons, no current can flow.

In metals, electrons in the highest occupied band can readily jump to higher unoccupied levels. These **conduction** electrons can move **freely** throughout the sample, like molecules of gas in a closed container (see free electron model-Section 26-6).

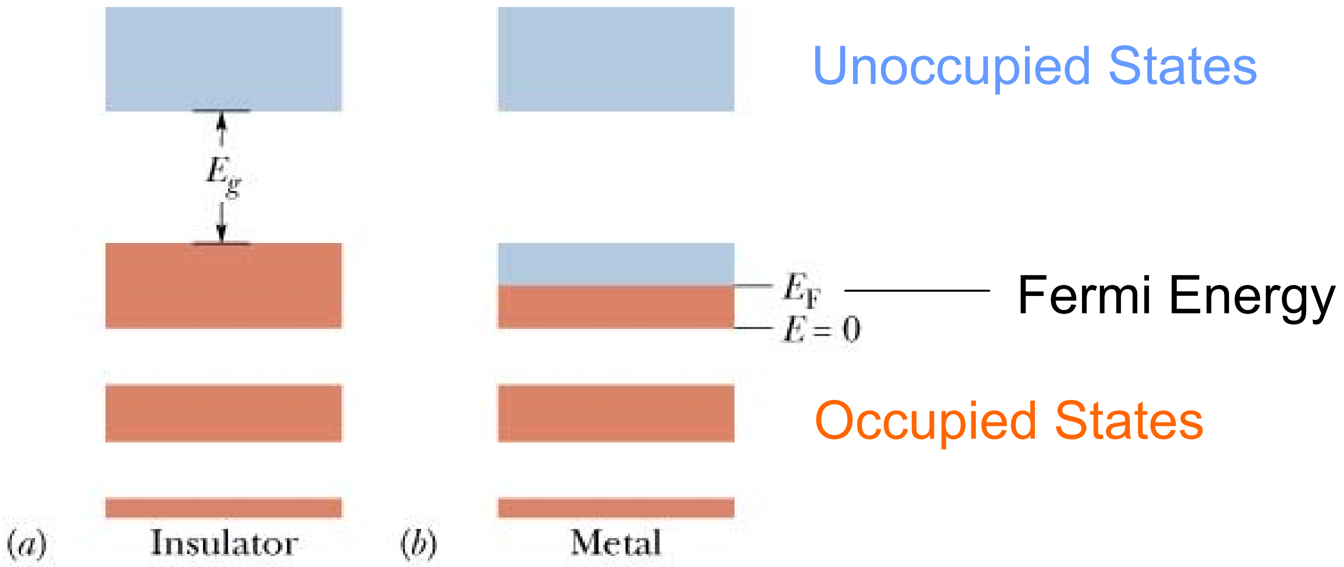


Fig. 41-4

How Many Conduction Electrons Are There?

Not all electrons in a solid carry current. Low energy electrons that are deeply buried in filled bands have no unoccupied states nearby into which they can jump, so they cannot readily increase their kinetic energy. Therefore, only the electrons at the outermost occupied shells (near the Fermi energy) will conduct current. These are called valence electrons, which also play a critical role in chemical bonding by determining the "valence" of an atom.

$$\begin{pmatrix}\text{number of conduction}\\ \text{electrons in sample}\end{pmatrix} = \begin{pmatrix}\text{number of atoms}\\ \text{in sample}\end{pmatrix}\begin{pmatrix}\text{number of valence}\\ \text{electrons per atom}\end{pmatrix}$$

$$n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume V}}$$

$$\begin{pmatrix}\text{number of atoms}\\ \text{in sample}\end{pmatrix} = \frac{\text{sample mass } M_{\text{sam}}}{\text{atomic mass}} = \frac{\text{sample mass } M_{\text{sam}}}{(\text{molar mass } M)/N_A}$$

$$= \frac{(\text{material's density})(\text{sample volume } V)}{(\text{molar mass } M)/N_A}$$

Conductivity Above Absolute Zero

As far as the conduction electrons are concerned, there is little difference between room temperature (300 K) and absolute zero (0 K). Increasing temperature does change the electron distribution by thermally exciting lower energy electrons to higher states. The characteristic thermal energy scale is kT (k is the Boltzmann constant), which at 1000 K is only 0.086 eV. This is a very small energy compared to the Fermi energy, and barely agitates the “sea of electrons.”

How Many Quantum States Are there?

Number of states per unit volume in energy range from E to $E+dE$:

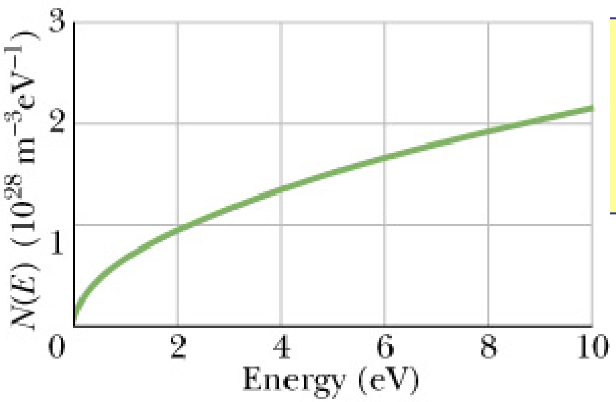


Fig. 41-5

$$N(E)=\frac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}\quad \text{(density of states, m}^{-3}\text{J}^{-1}\text{)}$$

Analogous to counting number of modes in a pipe organ→frequencies f (energies) become more closely spaced at higher f→density (in interval df) of modes increases with f.

Occupancy Probability *P(E)*

Ability to conduct depends on the probability $P(E)$ that available vacant levels will be occupied. At $T = 0$, the $P(E < E_F) = 1$ and $P(E > E_F) = 0$. At $T > 0$ the electrons distribute themselves according to **Fermi-Dirac** statistics:

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1} \quad \text{(occupancy probability)}$$

$$\text{At } T = 0: \quad \text{For } E < E_F,\ e^{(E-E_F)/kT} \to e^{-\infty} \to P(E) = 1$$

$$\text{For } E > E_F,\ e^{(E-E_F)/kT} \to e^{+\infty} \to P(E) = 0$$

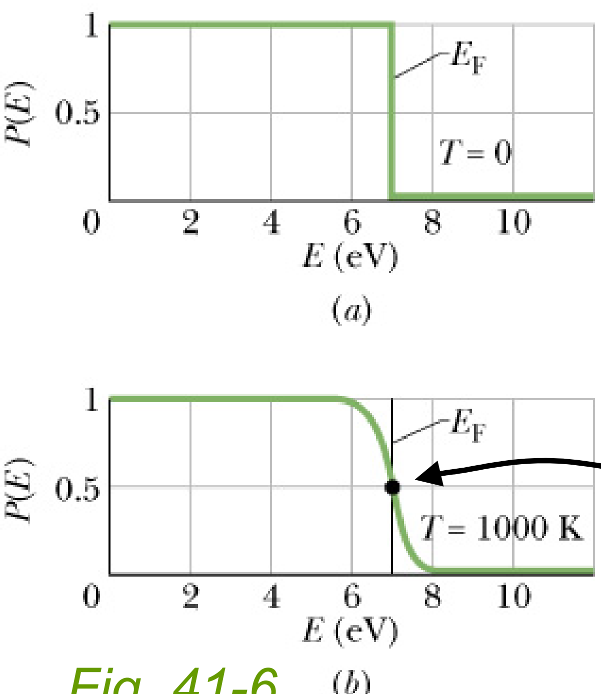


Fig. 41-6

Fermi energy of a material is the energy of a quantum state that has the probability of 0.5 of being occupied by an electron

How Many Occupied States Are There?

Density of occupied states (per unit volume in energy range E to $E+dE$) is $N_O(E)$:

$$\begin{pmatrix}\text{density of occupied states}\\ N_O(E)\text{ at energy }E\end{pmatrix}=\begin{pmatrix}\text{density of states}\\ N(E)\text{ at energy }E\end{pmatrix}\begin{pmatrix}\text{occupancy probability}\\ P(E)\text{ at energy }E\end{pmatrix}$$

$$\text{or}\quad N_O(E)=N(E)P(E)\quad\text{(density of occupied states)}$$

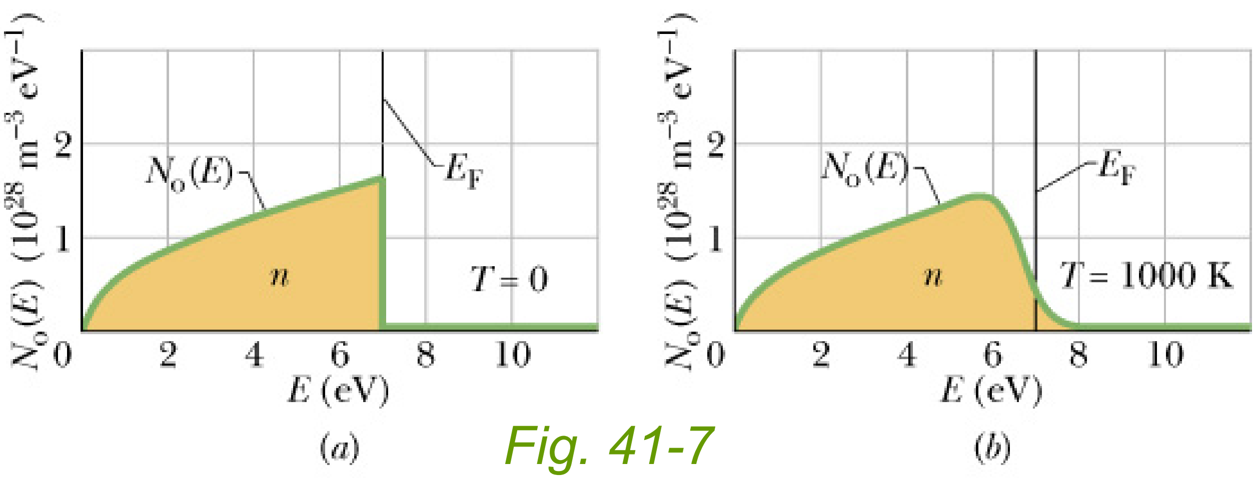


Fig. 41-7

Calculating the Fermi Energy

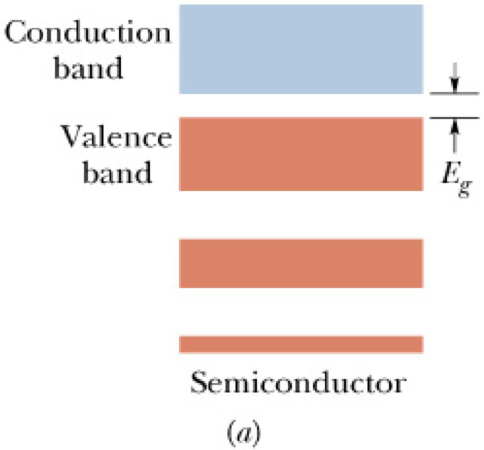
$$\text{At } T = 0,\ n = \int_0^{E_F} N_O(E)\,dE = \int_0^{E_F} N(E)P(E)\,dE = \int_0^{E_F} N(E)\cdot 1\,dE$$

Plugging in for $N(E)$

$$n = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\int_0^{E_F} E^{1/2}\,dE = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\frac{2E_F^{3/2}}{3}$$

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3}\frac{h^2}{m}n^{2/3} = \frac{0.121h^2}{m}n^{2/3}$$

Semiconductors



Semiconductors are qualitatively similar to insulators but with a much smaller (~1.1 eV for silicon compared to 5.5 for diamond) energy gap E_g between top of the valence band and bottom of the conduction band

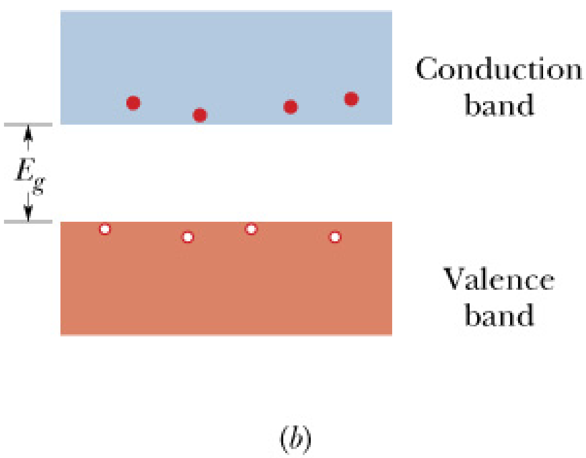


Fig. 41-8

Number density of carriers n: thermal agitation excites some electron at the top of the valence band across to the conduction band, leaving behind unoccupied energy state (holes). Holes behave as positive charges when electric fields are applied.

$n_{Cu} / n_{Si} \sim 10^{13}$.

Resistivity ρ: since $\rho = m/e^2 n \tau$, the large difference in charge carrier density mostly account for the large increase (~10^{11}) in ρ in semiconductors compared to metals

Temperature coefficient of Resistivity α: When increasing temperature, resistivity in metals increases (more scattering off lattice vibrations) while it decrease in semiconductors (more charge carriers excited across energy gap)

Doped Semiconductors

Doping introduces a small number of suitable replacement atoms (impurities) into the semiconductor lattice. This not only allows one to control the magnitude of n, but also its sign!

Pure Si

n-type doped Si

p-type doped Si

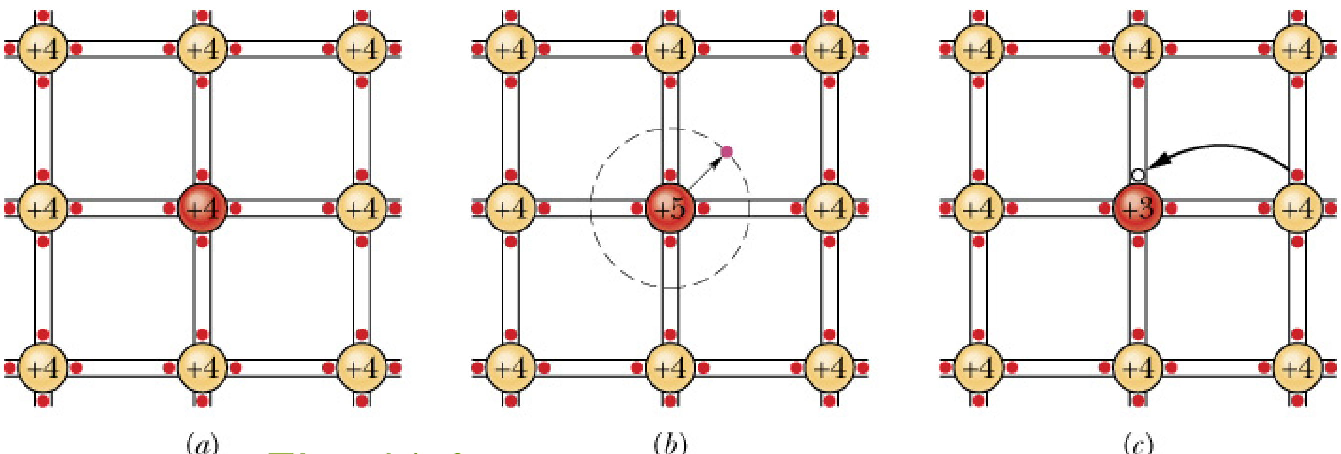


Fig. 41-9

Phosphorous acts as donor

Aluminum acts as acceptor

Doped Semiconductors, cont'd

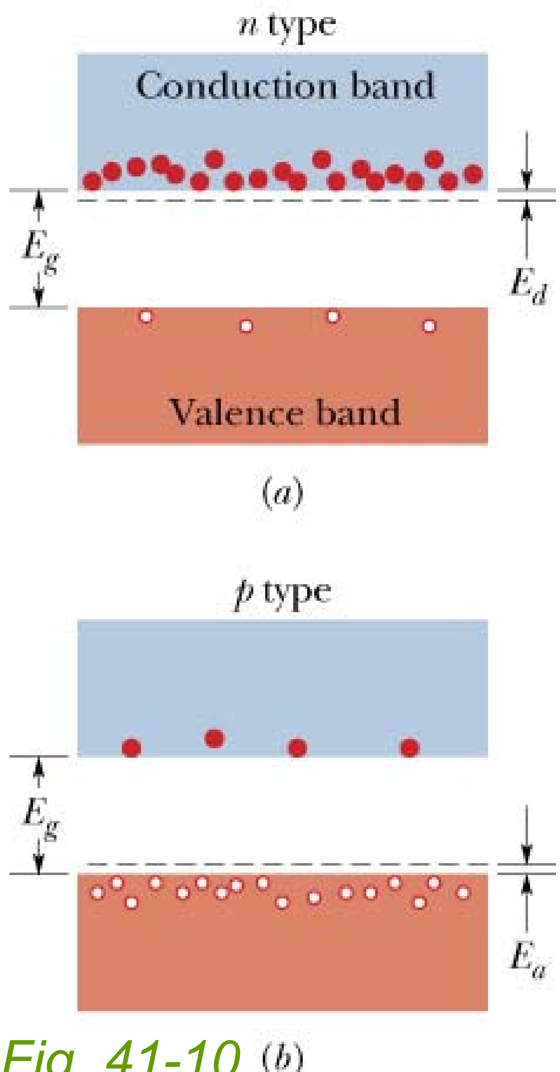


Fig. 41-10

Table 41-2

Properties of Two Doped Semiconductors

Property	Type of Semiconductor *n*	*p*
Matrix material	Silicon	Silicon
Matrix nuclear charge	$+14e$	$+14e$
Matrix energy gap	1.2 eV	1.2 eV
Dopant	Phosphorous	Aluminum
Type of dopant	Donor	Acceptor
Majority carriers	Electrons	Holes
Minority carriers	Holes	Electrons
Dopant energy gap	E_d=0.045 eV	E_a=0.067 eV
Dopant valence	5	3
Dopant nuclear charge	$+15e$	$+13e$
Dopant net ion charge	$+e$	$-e$

The *p-n* Junction

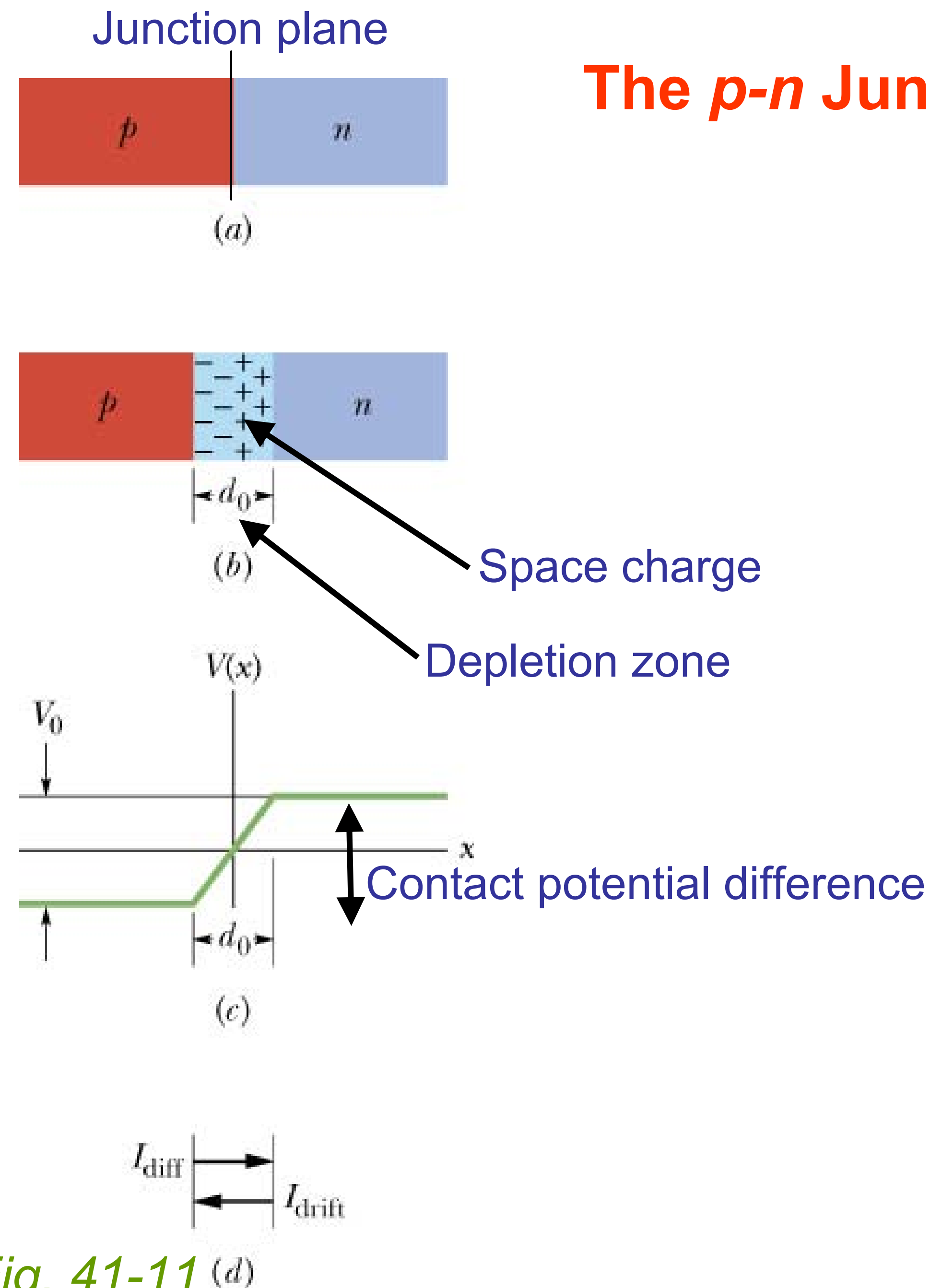


Fig. 41-11

The Junction Rectifier

Allows current to flow in only one direction

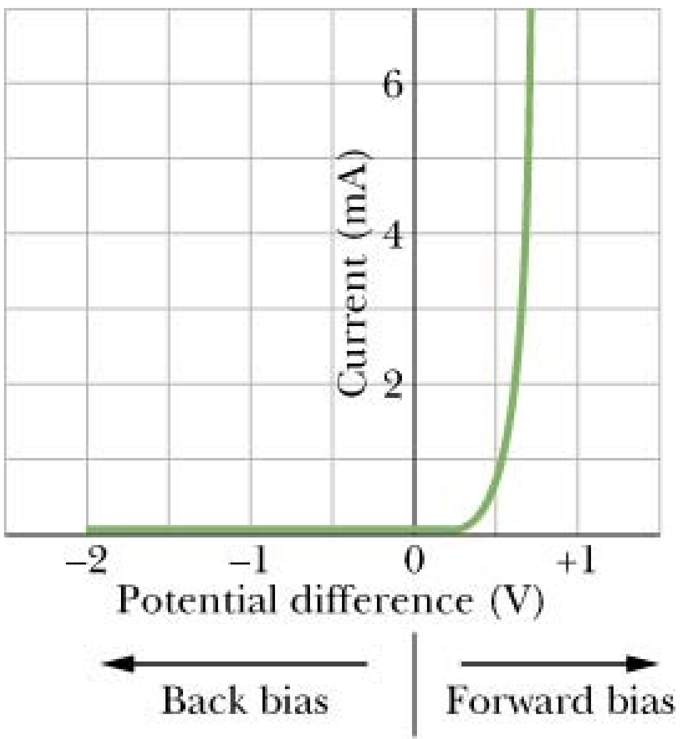


Fig. 41-12

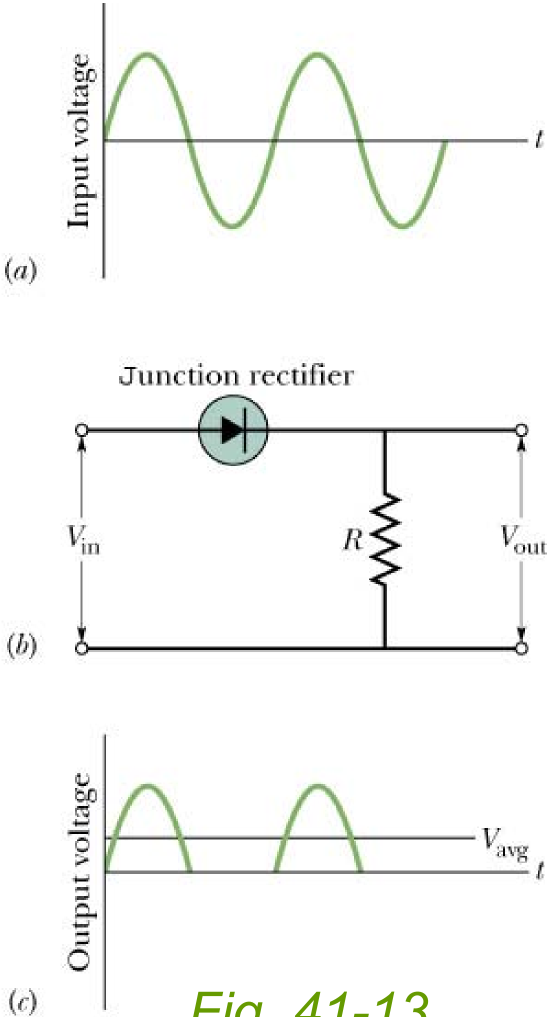


Fig. 41-13

The Junction Rectifier, cont'd

Forward-bias

depletion region shrinks

Current flows

Back-bias

depletion region grows

No current flows,

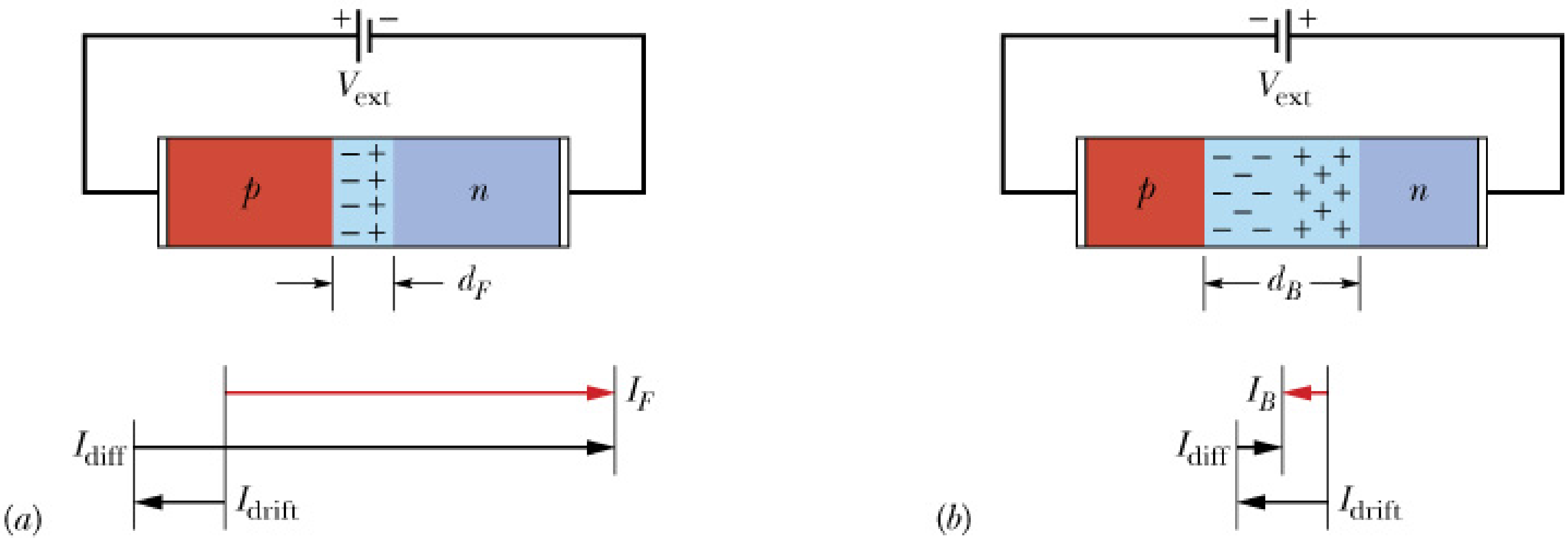


Fig. 41-14

Light Emitting Diode

At junction, electrons recombine with holes across E_g, emitting light in the process

$$\lambda = \frac{c}{f} = \frac{c}{E_g/h} = \frac{hc}{E_g}$$

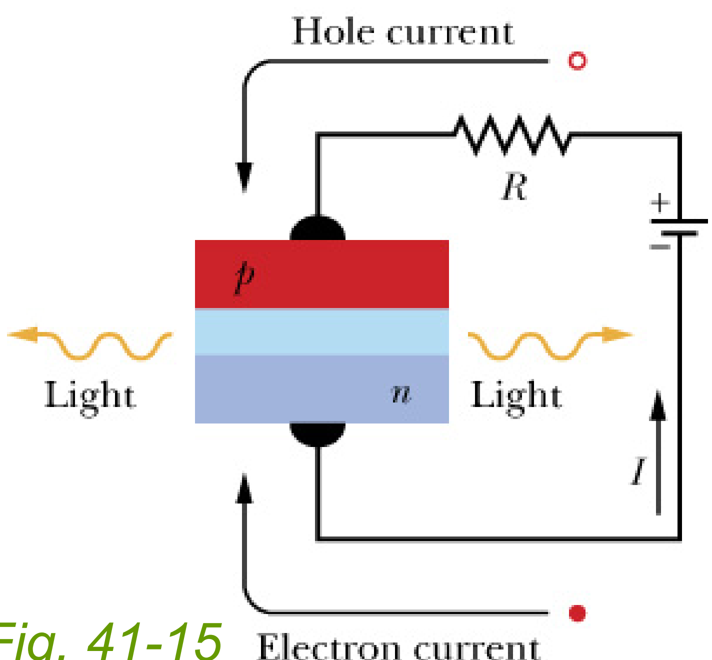


Fig. 41-15

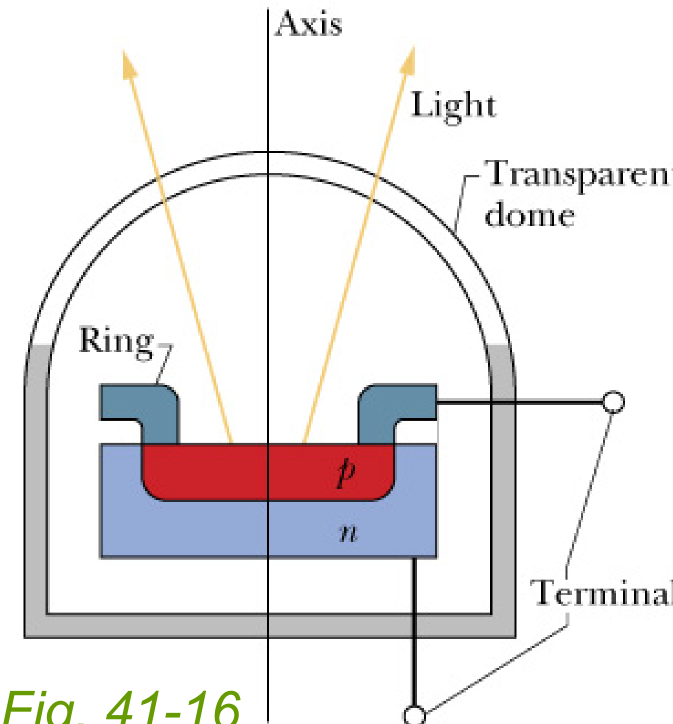


Fig. 41-16

The Photo-Diode

Use a p-n junction to detect light. Light is absorbed at p-n junction, producing electrons and holes, allowing a detectible current to flow.

Junction Laser

p-n already has a population inversion. If the junction is placed in an optical cavity (between two mirrors), photons that reflect back to the junction will cause stimulated emission, producing more identical photons, which in term will cause more stimulated emision.

The Transistor

Transistor is a three terminal device where a small gate (G) voltage/current controls the resistance between the source (S) and drain (D), allowing large currents to flow→power amplification!

Field Effect Transistor: gate voltage depletes (dopes) charge carriers in semiconductor, turning it into an insulator (metal)

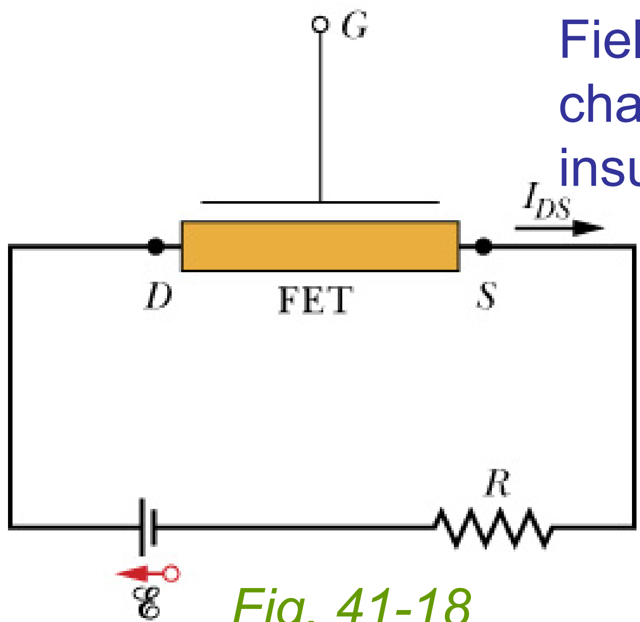


Fig. 41-18

metal-**o**xide-**s**emiconductor-**f**ield-**e**ffect-**t**ransistor (MOSFET)

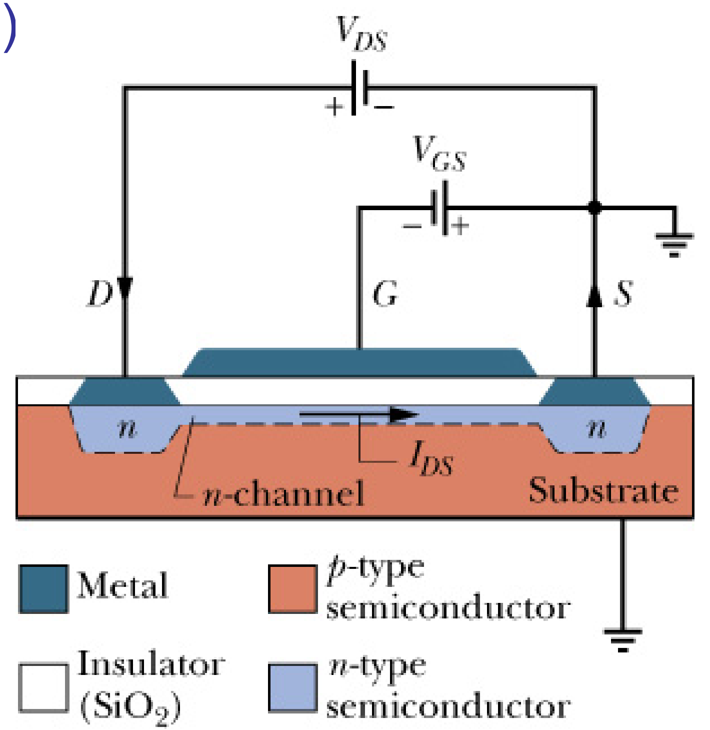


Fig. 41-19

Integrated Circuits

Thousands, even millions of transistors and other electronic components (capacitors, resistors, etc) manufactured on a single chip to make complex devices such as computer processors. Fast, reliable, small, well-suited for mass-production.