Chapter 42

Nuclear Physics

In the previous chapters we have looked at the quantum behavior of electrons in various potentials (quantum wells, atoms, etc) but have neglected what happens at the center of the atom, the nucleus.

For the last 90 years, a principal goal of physics has been to work out the quantum physics of nuclei themselves. In that same period new applications ranging from radiation therapy in cancer treatment to detecting radon gas in basements have been developed.

Discovering the Nucleus

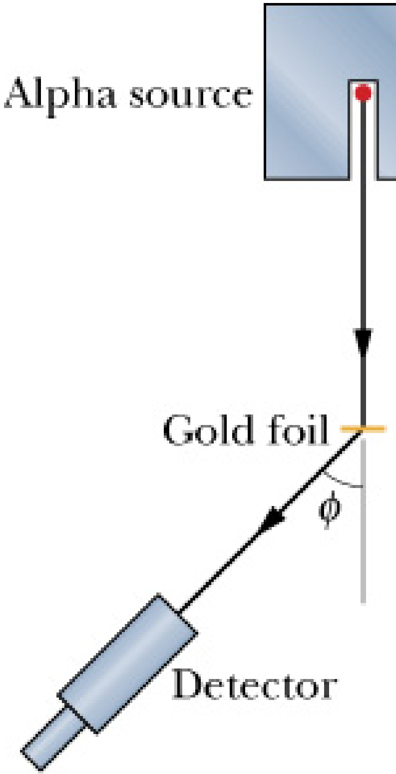


Fig. 42-1

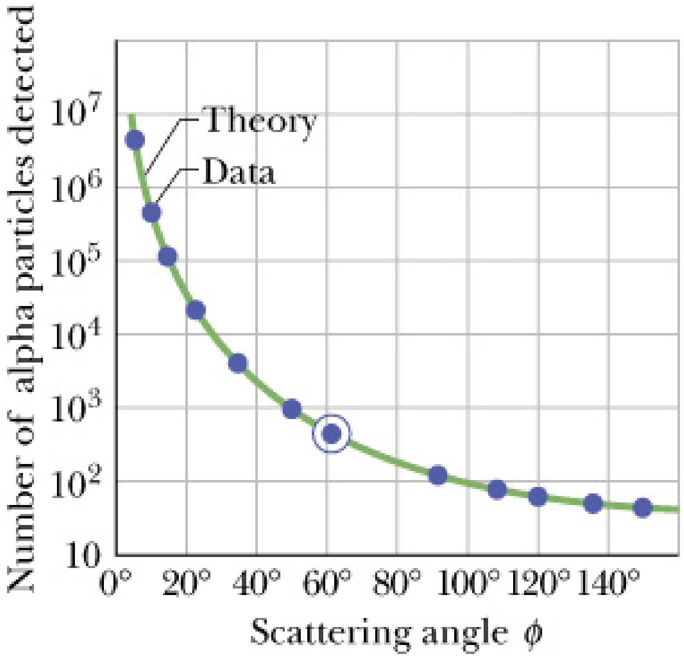


Fig. 42-2

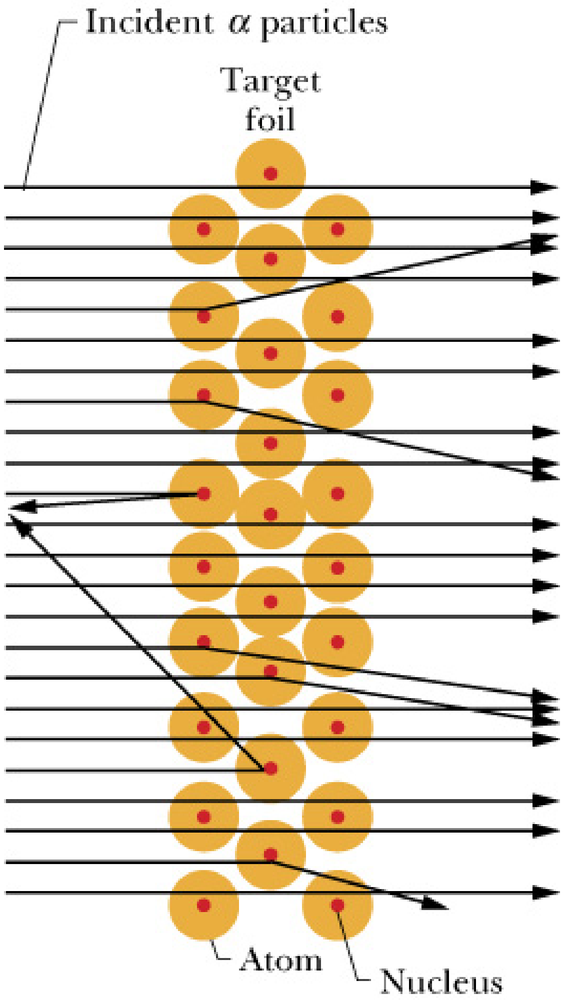


Fig. 42-3

Rutherford, Geiger, Marsden experiments 1911-1913
→nucleus is small in size, massive, and positively charged
Not plum pudding (evenly distributed)!

Some Nuclear Properties

Table 42-1 Some Properties of Selected Nuclides

Nuclide	*Z*	*N*	*A*	Stability	Mass (u)	Spin	Binding Energy (MeV/nucleon)
^{1}H	1	0	1	99.985%	1.007 825	1/2	—
^{7}Li	3	4	7	92.5%	7.016 004	1/2	5.60
^{31}P	15	16	31	100%	30.973 762	1/2	8.48
^{84}Kr	36	48	84	57.0%	83.911 507	0	8.72
^{120}Sn	50	70	120	32.4%	119.902 197	0	8.51
^{157}Gd	64	93	157	15.7%	156.923 957	3/2	8.21
^{197}Au	79	118	197	100%	196.966 552	3/2	7.91
^{227}Ac	89	138	227	21.8 y	227.027 747	3/2	7.65
^{239}Pu	94	145	239	24 100 y	239.052 157	1/2	7.56

Some Nuclear Terminology

Atomic number or **proton number**: Z

Number of neutrons or **neutron number**: N

Total number of neutrons and protons, **mass number**: A

$$A = Z + N \qquad (42\text{-}1)$$

Protons and neutrons are called **nucleons**.

$$^{197}\mathrm{Au}:\ A = 197,\ \mathrm{Au} \rightarrow Z = 79,\ N = A - Z = 197 - 79 = 118$$

Nuclides with same Z but different A are called **isotopes,** e.g., ^{173}Au to ^{204}Au

Radionuclides decay (or **disintegrate**) by emitting a particle, thereby transforming into a different nuclide

Organizing the Nuclides

Nuclidic Chart

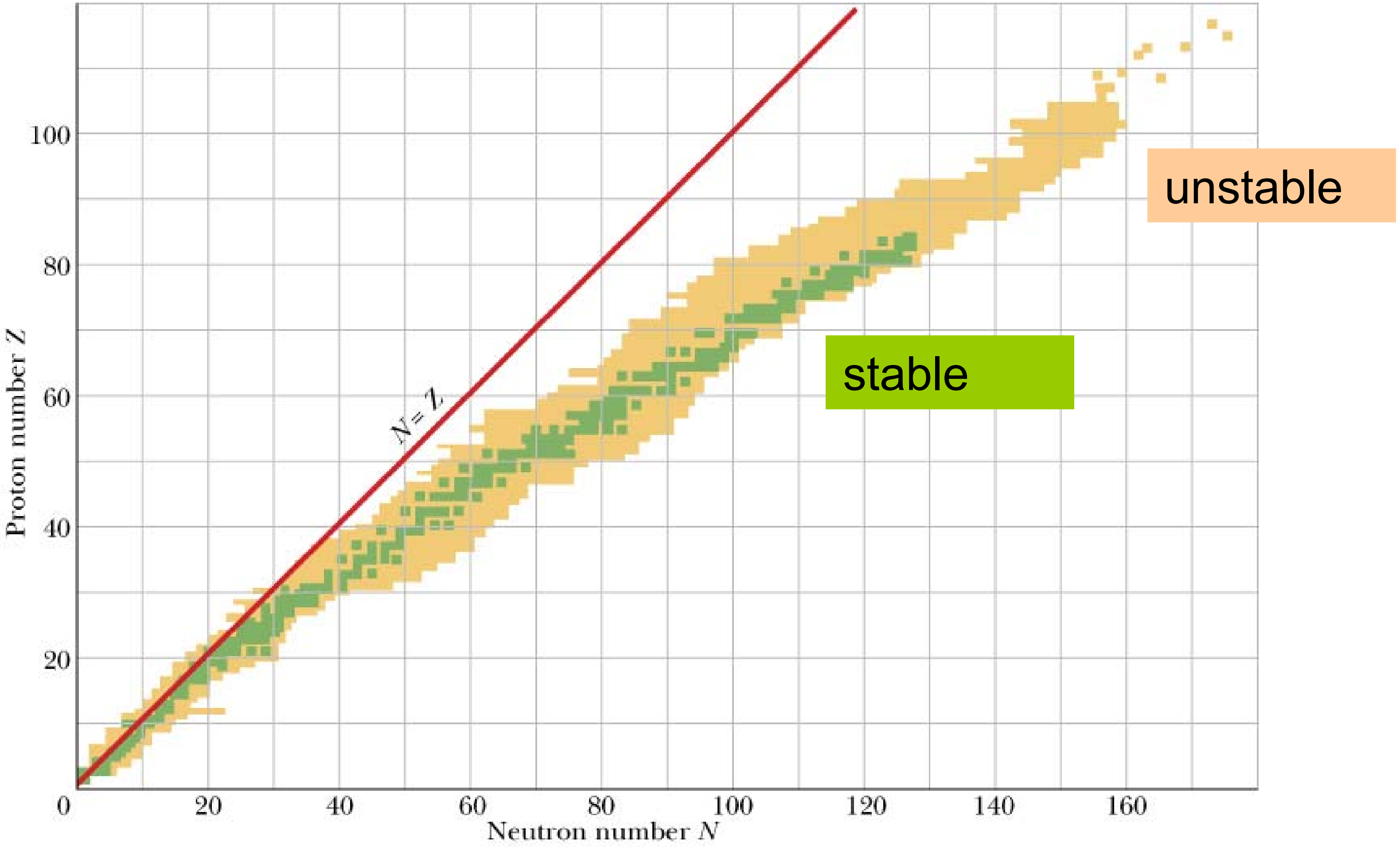
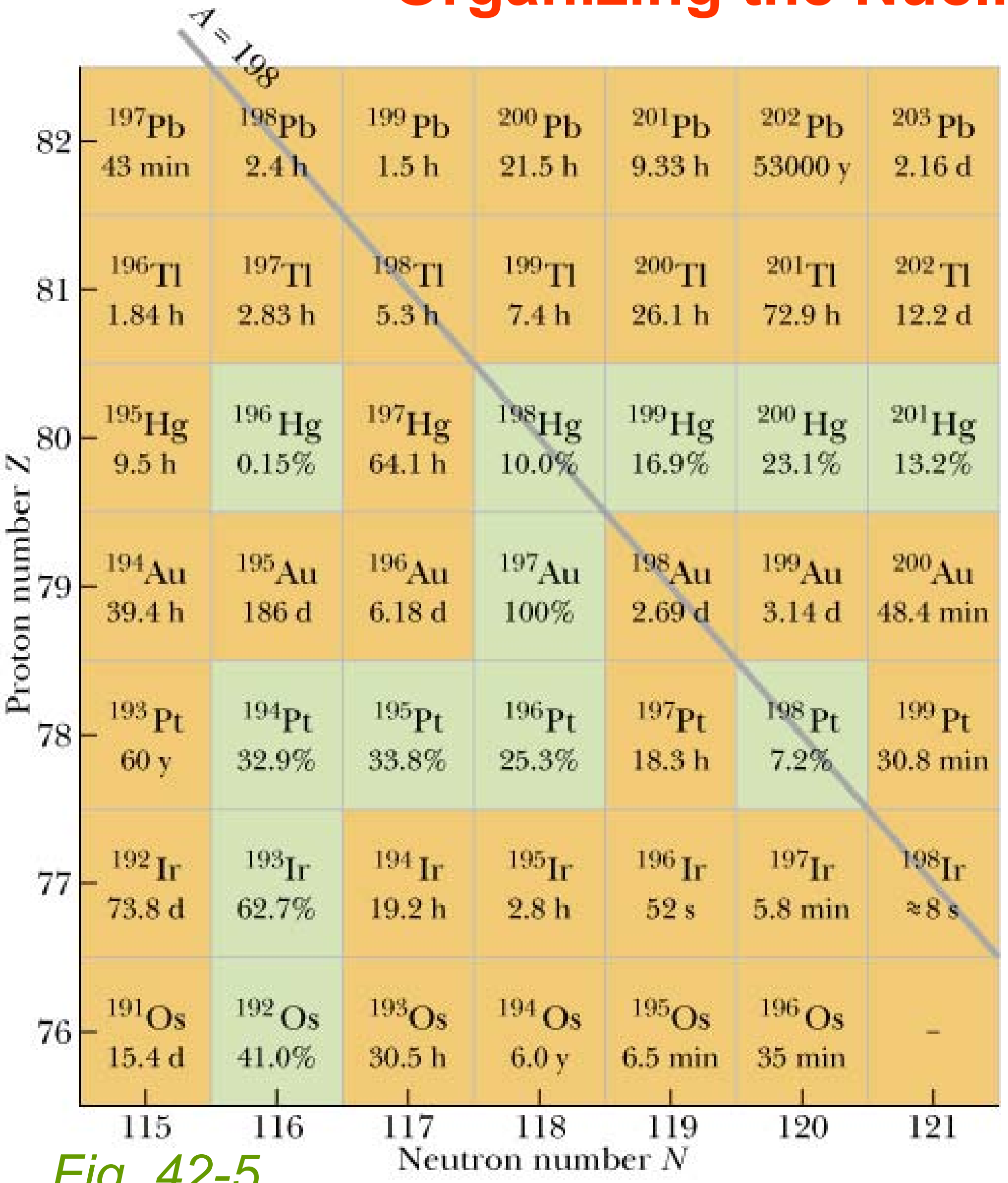


Fig. 42-4

Organizing the Nuclides, cont'd



Z \ N	115	116	117	118	119	120	121
82	^{197}Pb 43 min	^{198}Pb 2.4 h	^{199}Pb 1.5 h	^{200}Pb 21.5 h	^{201}Pb 9.33 h	^{202}Pb 53000 y	^{203}Pb 2.16 d
81	^{196}Tl 1.84 h	^{197}Tl 2.83 h	^{198}Tl 5.3 h	^{199}Tl 7.4 h	^{200}Tl 26.1 h	^{201}Tl 72.9 h	^{202}Tl 12.2 d
80	^{195}Hg 9.5 h	^{196}Hg 0.15%	^{197}Hg 64.1 h	^{198}Hg 10.0%	^{199}Hg 16.9%	^{200}Hg 23.1%	^{201}Hg 13.2%
79	^{194}Au 39.4 h	^{195}Au 186 d	^{196}Au 6.18 d	^{197}Au 100%	^{198}Au 2.69 d	^{199}Au 3.14 d	^{200}Au 48.4 min
78	^{193}Pt 60 y	^{194}Pt 32.9%	^{195}Pt 33.8%	^{196}Pt 25.3%	^{197}Pt 18.3 h	^{198}Pt 7.2%	^{199}Pt 30.8 min
77	^{192}Ir 73.8 d	^{193}Ir 62.7%	^{194}Ir 19.2 h	^{195}Ir 2.8 h	^{196}Ir 52 s	^{197}Ir 5.8 min	^{198}Ir ≈8 s
76	^{191}Os 15.4 d	^{192}Os 41.0%	^{193}Os 30.5 h	^{194}Os 6.0 y	^{195}Os 6.5 min	^{196}Os 35 min	-

Fig. 42-5

unstable

stable

Isobar: nuclides with same mass number

Nuclear Radii

$$1 \text{ femtometer} = 1 \text{ fermi} = 1 \text{ fm} = 10^{-15} \text{ m} \qquad (42\text{-}2)$$

$$\boxed{r = r_0 A^{1/3} \qquad (42\text{-}3)}$$

$$r_0 \approx 1.2 \text{ fm}$$

Eq. 42-3 does not apply to halo nuclides, neutron rich nuclides in which some neutrons form a large halo around a spherical core of protons.

For example ^{8}Li+n→ ^{9}Li, r increases 4%,
but when ^{9}Li+2n→ ^{11}Li (halo nuclide), r increases 30%,

Atomic Masses

$$1\ \mathrm{u} = 1.66053873 \times 10^{-27}\ \mathrm{kg} \qquad (42\text{-}4)$$

The actual mass of a nucleus is not simply the sum of the masses of all its constituent nucleons. Energy ($Q = -\Delta m\, c^2$, which is equivalent to mass) can be released or absorbed in nuclear reaction forming the nucleus.

$$c^2 = 931.494013\ \mathrm{MeV/u} \qquad (42\text{-}5)$$

$$\boxed{\Delta = M - A \qquad \text{(excess mass)} \quad (42\text{-}6)}$$

M is the actual mass of the atom in atomic mass units and A is the mass number for the nucleus.

Nuclear Binding Energies

The mass M of the nucleus is less than the total mass of its individual nucleons $\Sigma m \rightarrow$ nucleus has less energy Mc^2 than all the separated nucleons $\Sigma(mc^2) \rightarrow$ this energy difference (**binding energy**) favors the nucleons binding into a nucleus.

$$\Delta E_{be} = \sum\left(mc^2\right) - Mc^2 \qquad \left(\text{binding energy}\right) \qquad (42\text{-}7)$$

If we could tear apart a nucleus into its separate nucleons, the work required would be ΔE_{be}.

binding energy per nucleon

$$\Delta E_{ben} = \frac{\Delta E_{be}}{A} \qquad \left(\text{binding energy per nucleon}\right) \qquad (42\text{-}8)$$

ΔE_{ben} represents the average energy holding each nucleon into the nucleus.

Nuclear Binding Energies

If weakly bound nuclei transform into more strongly bound nuclei, the total mass can be reduced and the mass energy of the final state is lower than the mass energy of the initial state. Where does the excess initial mass energy go?

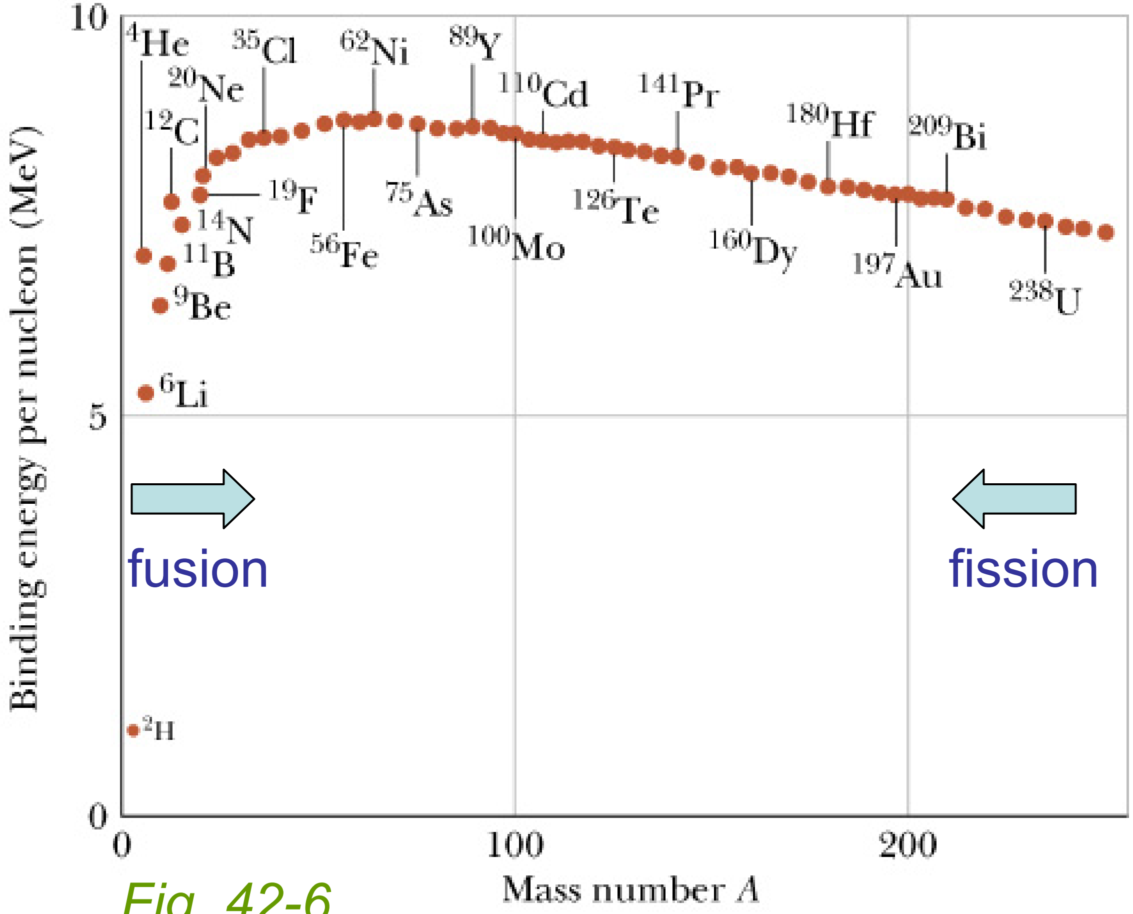


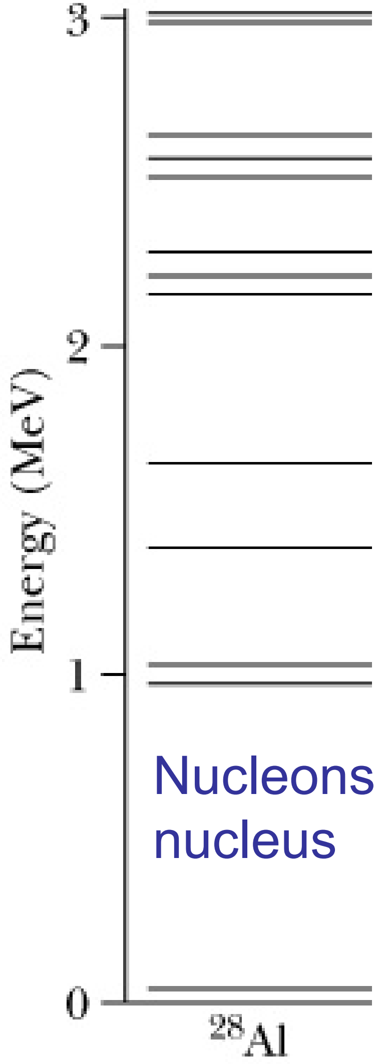
Fig. 42-6

Fission: A nucleus with a larger mass (U, Pu) splits into nuclei with smaller total mass (larger binding energy). Energy is released, e.g., nuclear reactor, nuclear weapons.

Fusion: Two nuclei combine to form a single more tightly bound nucleus, e.g., H+H →He hydrogen bomb and the sun

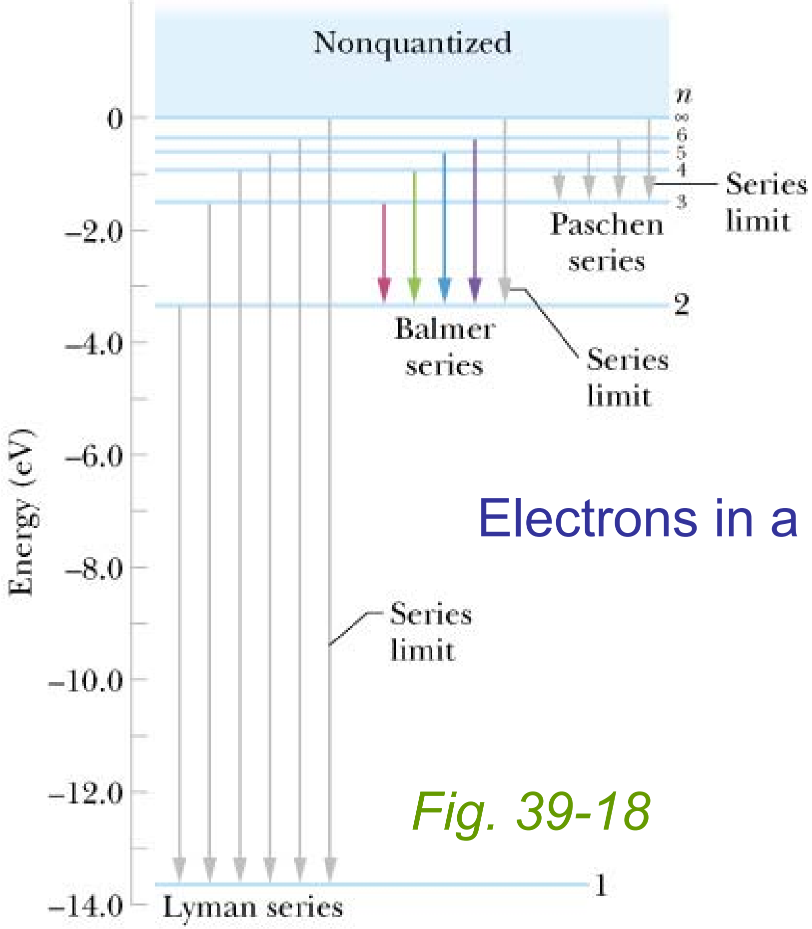
Nuclear Energy Levels

Energy levels of nucleons confined in a nucleus are quantized just as for electrons confined in an atom. Do you notice a major quantitative difference?



Nucleons in an ^{28}Al nucleus

Fig. 42-7



Electrons in a H atom

Fig. 39-18

Nuclear Spin and Magnetism

Many nuclides have an intrinsic *nuclear magnetic moment*, which leads to intrinsic *nuclear angular momentum* or spin. While nuclear angular momentum is similar in magnitude to angular momenta of atomic electrons, nuclear magnetic moments are much smaller than typical atomic magnetic moments.

The Nuclear Force

Attractive, short-range **strong force** binds quarks together to form protons and neutrons. This force "spills over" to bind nucleons in nuclei, overcoming the repulsive Coulomb force between protons

Radioactive Decay

As shown in Fig. 42-4, most known nuclides are unstable/radioactive.

There is absolutely no way to predict whether any given nucleus in a radioactive sample will be among the small number of nuclei that decay during the next second. All have the same chance.

Nuclear decay rate *dN*/*dt* is proportional to the number *N* of nuclei that can decay

$$-\frac{dN}{dt} = \lambda N \qquad (42\text{-}11) \Longrightarrow -\frac{dN}{N} = \lambda dt \qquad (42\text{-}12)$$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt \Longrightarrow \ln N - \ln N_0 = -\lambda\left(t - t_0\right) \qquad (42\text{-}13)$$

$$\ln\frac{N}{N_0} = -\lambda t \qquad (\text{let } t_0 = 0) \quad (42\text{-}14) \Longrightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\Downarrow$$

$$\boxed{N = N_0 e^{-\lambda t} \qquad (\text{radioactive decay}) \quad (42\text{-}15)}$$

Radioactive Decay, cont'd

Radioactive decay rate $R = - dN/dt$:

$$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = \lambda N\left(t\right)$$

$$\boxed{R = R_0 e^{-\lambda t} \qquad \text{(radioactive decay)} \quad \text{(42-16)}}$$

$$R = \lambda N \qquad \text{(42-17)}$$

The total decay rate of one or more nuclides is called the **activity**, with SI units **becquerel**

$$1 \text{ becquerel} = 1 \text{ Bq } = 1 \text{ decay per second}$$

An older unit for activity, the **curie**, is still commonly used

$$1 \text{ curie} = 1 \text{ Ci } = 3.7 \times 10^{10} \text{ Bq}$$

Radioactive Decay, cont'd

Two common time measures of how long any given type of radionuclides lasts.

Half-life $T_{½}$ (1/2 of starting nuclides have decayed) and **mean life** τ (1/e of starting nuclides have decayed).

$$R\left(T_{1/2}\right)=\tfrac{1}{2}R_0=R_0e^{-\lambda T_{1/2}}\quad\Longrightarrow\quad T_{1/2}=\frac{\ln 2}{\lambda}$$

$$R\left(\tau\right)=\frac{1}{e}R_0=R_0e^{-\lambda\tau}\quad\Longrightarrow\quad \tau=\frac{1}{\lambda}$$

$$\boxed{T_{1/2}=\frac{\ln 2}{\lambda}=\tau\ln 2 \qquad (42\text{-}18)}$$

Alpha Decay

When a nucleus undergoes **alpha decay**, it transforms to a different nuclide by emitting an alpha particle (a helium nucleus, ^{4}He).

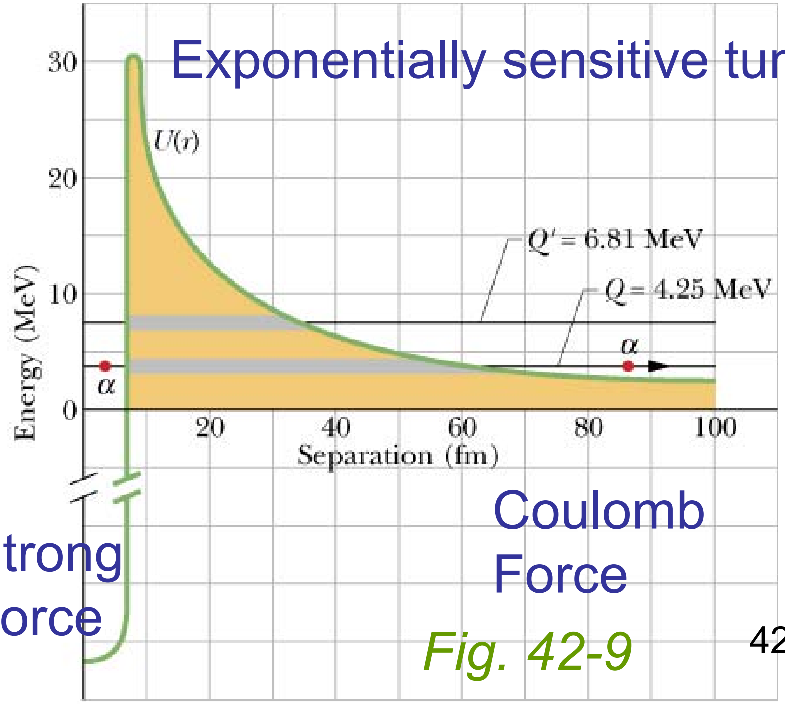
$$^{238}\mathrm{U} \rightarrow {}^{234}\mathrm{Th} + {}^{4}\mathrm{He} \qquad (42\text{-}22)$$

The alpha decay of ^{238}U can occur spontaneously (without an external source of energy) since the mass of ^{238}U is greater than the mass of the total decay products. Disintegration energy $Q=-\Delta Mc^2$. $T_{1/2}$ is 4.5x10^{9} y. Why so long? Why don't all ^{238}U decay immediately?

Table 42-2

Two Alpha Emitters Compared

Radionuclide	Q	Half-Life
^{238}U	4.25 MeV	4.5 x 10^{9} y
^{228}U	6.81 MeV	9.1 min



Exponentially sensitive tunneling

Strong Force

Coulomb Force

Fig. 42-9

Beta Decay

A nucleus that decays spontaneously by emitting an electron or a positron (positively charged particle with mass of an electron) is said to undergo **beta decay**.

Beta-minus (β^-) decay $\quad {}^{32}\mathrm{P} \rightarrow {}^{32}\mathrm{S} + e^- + \upsilon \quad (T_{1/2} = 14.3\ \mathrm{d}) \quad$ (42-24)

Beta-plus (β^+) decay $\quad {}^{64}\mathrm{Cu} \rightarrow {}^{64}\mathrm{Ni} + e^+ + \upsilon \quad (T_{1/2} = 12.7\ \mathrm{h}) \quad$ (42-25)

ν is the symbol for a neutrino, a neutral particle with a very small mass.

Both charge and nucleon number are conserved in beta decay

$$\left.\begin{array}{rl} \text{charge:} & (+15e) = (+16e) + (-e) + (0) \\ \text{nucleon:} & (+32) = (32) + (0) + (0) \end{array}\right\}$$

Beta Decay, cont'd

Beta-minus decay $\mathrm{n} \rightarrow \mathrm{p} + e^{-} + \upsilon$ (42-26)

Beta-plus decay $\mathrm{p} \rightarrow \mathrm{n} + e^{+} + \upsilon$ (42-27)

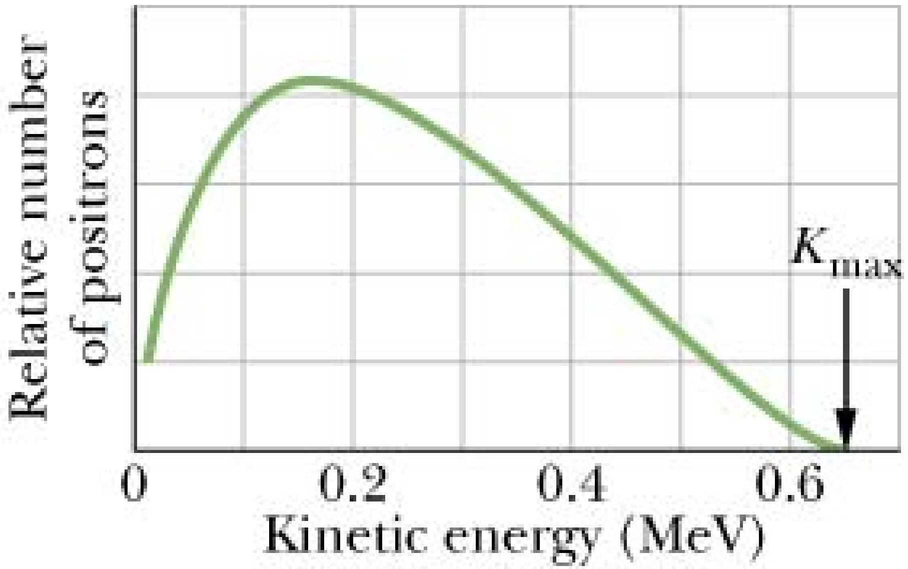


Fig. 42-10

In both alpha and beta decay, the same amount of energy is released in the decay of a particular radionuclide (governed by the mass difference between the initial and final states). In beta-minus (plus) decay the energy is shared between the electron (positron) and the neutrino, so the electron (positron) energy can range from 0 to $K_{\max} = Q$.

$$Q = K_{\max} \qquad (42\text{-}28)$$

The Neutrino

- In 1930 Wolfgang Pauli predicted the existence of the neutrino to 1) explain the wide range of energies for electrons and positrons in beta decay and 2) the missing angular momentum in beta decay measurements.

- Neutrinos are hard to detect; the mean free path of an energetic neutrino in water is several thousand light years! Earth is almost completely transparent to them.

- Neutrinos first detected in laboratory by Reines and Cowan in 1953

- Sun emits large number of neutrinos from its core. Exploding stars (supernovas) emit strong neutrino bursts which have been detected on earth by elaborate detectors located deep underground.

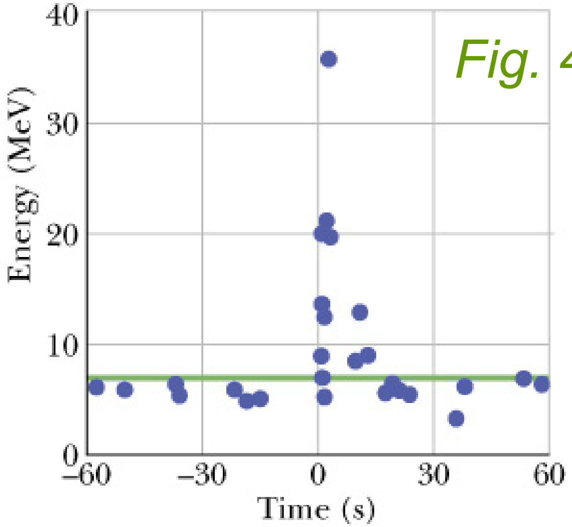
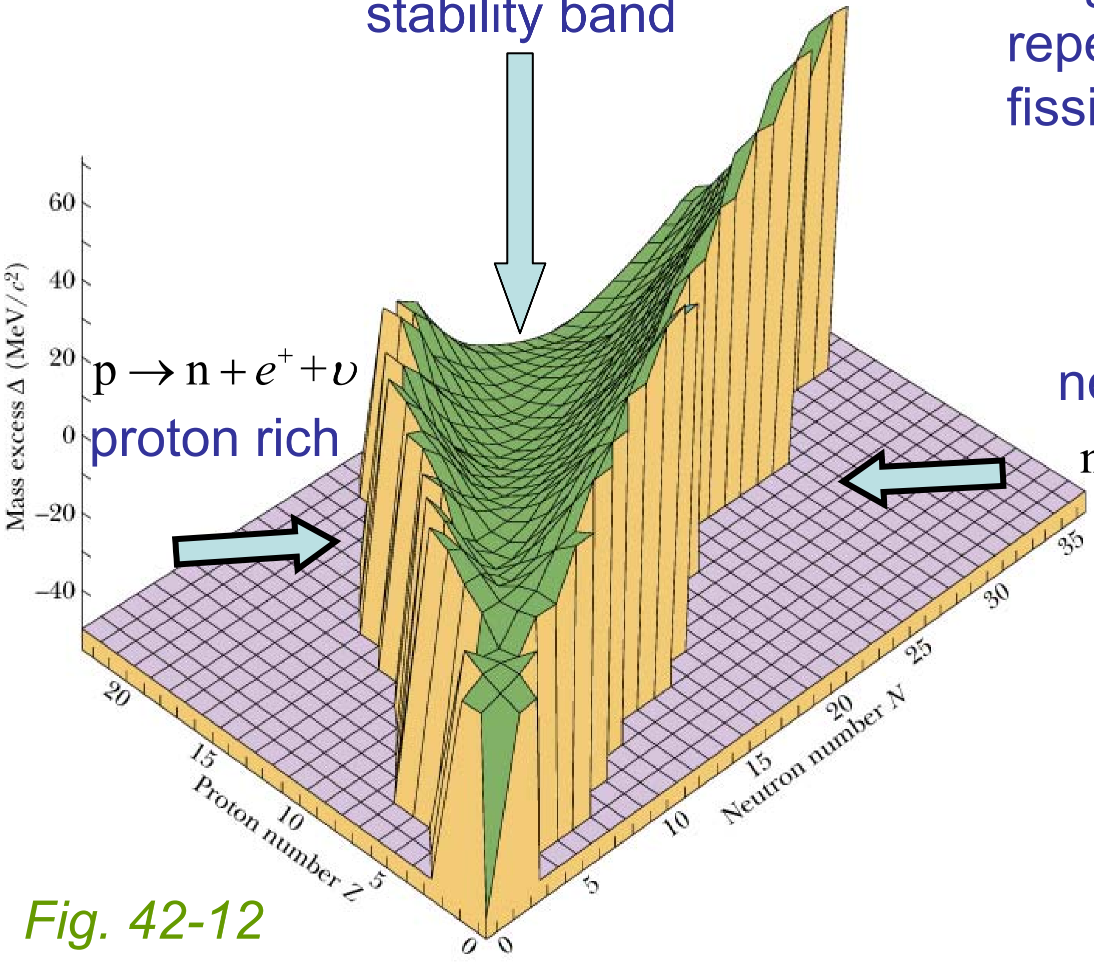


Fig. 42-11

Radioactivity and the Nuclidic Chart

Valley of nuclides stability band

Large A, decay into valley by repeated alpha emission and fission



$p \rightarrow n + e^{+} + \upsilon$

proton rich

neutron rich

$n \rightarrow p + e^{-} + \upsilon$

Fig. 42-12

Radioactive Dating

If you know the half-life of a radionuclide, you can use the decay of that radionuclide as a clock to measure time intervals.

Age of rocks: $^{40}\text{K}\rightarrow{}^{40}\text{Ar}$ with $T_{1/2}=1.25\text{x}10^9$ y, ratio of ^{40}K to ^{40}Ar in a rock can be used to determine when the rock was formed (and the ^{40}K in the rock started transforming into stable ^{40}Ar). This type of technique is used to date the earth and moon with a maximum age of about $4.5\text{x}10^9$ y.

Shorter time intervals (pre-historic and historic dating): $^{14}\text{C}\rightarrow{}^{12}\text{C}$ with $T_{1/2}=5730$ y, ^{14}C is produced at constant rate in upper atmosphere. Living organisms absorb both ^{14}C and ^{12}C while alive, maintaining a constant ratio. Once dead, no more C is absorbed and the remaining ^{14}C begins to decay into stable ^{12}C. By measuring ^{14}C to ^{12}C in organic matter (bones, fossils, parchment) one can determine when the organism that produced the organic matter died. This type of technique is used to date artifacts ranging from the charcoal in ancient campfires to the Dead Sea Scrolls.

Measuring Radiation Dosage

Radiation (cosmic rays, radioactive emission from elements in earth's crust, human activity/industry) can damage living tissue. There are two parts in evaluating the effect of radiation on living tissue.

1. *Absorbed Dose*. Measure of radiation dose (energy per unit mass) actually absorbed by a specific object (for example a patient's hand or chest). SI unit is the **gray** (Gy). Older, commonly used unit is the **rad** (**r**adiation **a**bsorbed **d**ose).

$$1\ \text{Gy} = 1\ \text{J/kg} = 100\ \text{rad} \qquad (42\text{-}32)$$

A whole body, short term gamma-ray dose of 3 Gy (300 rad) will cause death in 50% of the population exposed to it. Typical average dose from natural and human origin is only 2 mGy (0.2 rad) per year.

Measuring Radiation Dosage, cont'd

2. *Dose Equivalent*. Although different types of radiation (gamma rays, neutrons, etc) may deliver same energy to the body, they do not have the same biological effect. Dose equivalent allows us to rescale the absorbed dose to reflect the damage that a particular type of radiation can cause. The scaling factor is the **RBE** (**r**elative **b**iological **e**ffectiveness).

   Dose Equivalent = RBE x Absorbed Dose

   For x-rays and electrons: RBE=1

   For slow neutrons: RBE=5

   For alpha particles: RBE=10

   The SI unit for dose equivalent is the **sievert** (Sv). An earlier unit **rem** (**r**oentgen **e**quivalent **m**an) is still commonly used.

$$1\ \mathrm{Sv} = 100\ \mathrm{rem} \qquad (42\text{-}33)$$

   The National Council on Radiation Protection recommends that no one should receive an equivalent dose greater than 5 mSv per year.

Nuclear Models

Collective Model: nucleus like a drop of liquid. It correlates many facts nuclear masses and binding energies and helps to explain nuclear fission and other nuclear reactions.

$$X + a \to C \to Y + b \tag{42-34}$$

When projectile a enters target nucleus X, they form an excited, quasi-stable intermediate **compound nucleus** C, which after ~10^{-16}s (a long time by nuclear standards) decays into nuclear state Y by emitting particle b. Once in state C the nucleus "forgets" how it got there and hence the decay does not depend on how the nucleus reached state C.

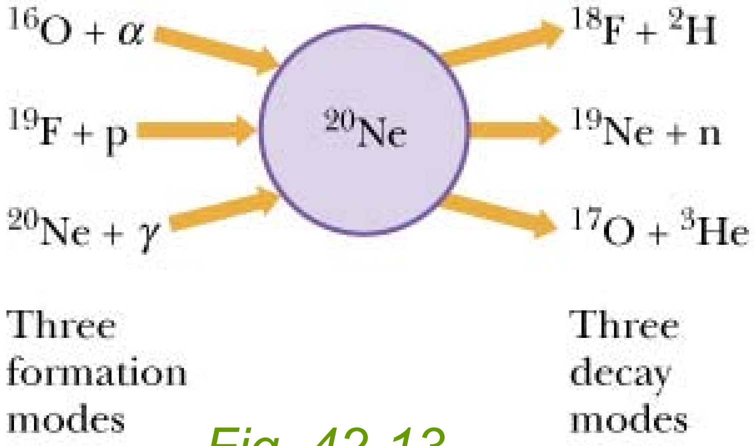


Fig. 42-13

Nuclear Models, cont'd

Independent Particle Model: Unlike collective model, where nucleons move around at random and bump into each other frequently, in this model nucleons remain in well-defined quantum states and hardly collide at all.

Nucleons obey Pauli exclusion principle; no two nucleons in the nucleus may occupy the same quantum mechanical state at the same time. Collisions minimized since nucleons can only scatter into unoccupied states.

In atoms, electrons form shells containing: 2, 10, 18, 36, 54, 86,… electrons

→ *magic electron numbers*

In a nucleus, nucleons form shells containing: 2, 8, 20, 28, 50, 82, 126… nucleons→**magic nucleon numbers**. Nuclei with proton number Z or neutron number N has one of these values has a special stability (like chemical stability of atoms with completely closed electronic shells).

Nuclear Models, cont'd

Independent Particle Model, cont'd:

Magic nuclides include: ^{18}O (Z=8); ^{40}Ca (Z=20, N=20)→doubly magic; ^{92}Mo (N=50); and ^{208}Pb (Z=82, N=126) →doubly magic

Alpha particles ^{4}He are doubly magic and have exceptional stability

Stripping off nucleons from closed shells requires a great deal of energy, while removing a nucleon that is already outside a closed shell is much easier.

For example ^{121}O (Z=51), removing 51st proton (already outside closed shell) only requires 5.8 MeV, removing 50th proton (inside closed shell) requires 11 MeV.

This is analogous to removing electrons from closed atomic shells. Removing the first electron (outside a closed atomic shell) in Na requires 5 eV while removing a second electron (in a closed atomic shell) requires 22 eV.

Nuclear Models, cont'd

A Combined Model: consider a nucleus with a *small number* of neutrons or protons outside a core of closed shells.

Outside nucleons occupy quantized states in potential well formed by core (independent model), but also interact with the core, deforming it and setting up "tidal wave" motions of rotation and vibration within it (collective model).