V.B. An Elementary Cross-Impact Model[#]

NORMAN C. DALKEY*

Abstract

Cross-impact analysis is a method for revising estimated probabilities of future events in terms of estimated interactions among those events. This Report presents an elementary cross-impact model where the cross-impacts are formulated as relative probabilities. Conditions are derived for the consistency of the matrix of relative probabilities of n events. An extension also provides a necessary condition for the vector of absolute probabilities to be consistent with the relative probability matrix. An averaging technique is formulated for resolving inconsistencies in the matrix, and a nearest-point computation derived for resolving inconsistencies between the set of absolute probabilities and the matrix.

Although elementary, the present model clarifies some of the conceptual problems associated with cross-impact analysis, and supplies a relatively sound basis for revising probability estimates in the limited case where interactions can be approximated by relative probabilities.

Introduction

One of the more promising new tools for long-range forecasting is cross-impact analysis. The general notion was first suggested by Helmer and Gordon with the game "Futures" created for the Kaiser Corporation. Cross-impact analysis has now been expanded and applied to a number of forecasting areas by Gordon and others at The Institute for the Future [1]. The motivation for cross-impact analysis arises from a basic aspect of long-range forecasting. There are usually strong interactions among a set of potential technological events or among a set of potential social developments. In assessing the likelihood that any given event or development will occur, the interactions with other events are clearly relevant. However, the number of first-order potential interactions increases as the square of the number of events. Even if a matrix describing the interactions is available-say from estimates furnished by a panel of experts-the task of

* DR. DALKEY is Senior Mathematician at the RAND Corporation, Santa Monica, California. This research is supported by the Advanced Research Projects Agency under Contract No. DAHC1567C0141. Views or conclusions contained in this study should not be interpreted as representing the official opinion or policy of Rand or of ARPA.

thinking through the implications rapidly gets out of hand. Some computational aid is required to take account of the large number of interdependencies.

Gordon and others at The Institute for the Future have developed two major approaches to the computational program.[1] Both approaches involve (1) preliminary estimates of the absolute probabilities (i.e., the probabilities of the individual events), (2) estimates of the interdependencies in terms of a cross-impact matrix, (3) a Monte Carlo sampling of chains of events in which the probability of an event in the chain is modified by the cross-impact of the previously occurring event in the chain, and (4) reestimation of the absolute probability of each event in terms of the relative frequency of the occurrence of that event in the sample of chains. The difference between the two approaches lies in the mode of modification of the probabilities. In the first approach, the basic method, the modification is effected by a heuristic algorithm. Cross-impacts are rated on a nominal scale of -10 to +10. Modification of successive probabilities is computed via a family of quadratic "adjustments," based on the cross-impact rating and the unmodified probability. The second approach, the likelihood ratio method, defines cross-impacts in terms of a factor by which the odds favoring the target event are to be multiplied, given the occurrence of the impacting event. The second approach is conceptually clearer than the first, and removes some of the arbitrariness associated with it.[2]

Both approaches suffer from a lack of clarity concerning the purpose of the computation. The notion of "taking account of the interactions" is not adequate to answer questions such as "are the revised probabilities in fact more accurate estimates than the original ones?" In addition, as will be seen below, the Monte Carlo computation contains implicit assumptions concerning higher-order interactions that are not defined, and are surprisingly difficult to state precisely. (To say they are implicit is not to say they are not recognized by the developers of the method, only that the nature of the assumptions is not clearly stated.)

In this article an elementary model of probability cross-impacts is formulated that clarifies the notion of "taking account of interactions," and does this without requiring any assumptions concerning higher-order interactions. The model is based on fundamental postulates and theorems of probability. The model does not take into account all of the aspects of an interacting system that are pertinent-in particular, it neglects time as a parameter. More generally, it neglects nonprobabilistic order effects.

The approach is elementary in two respects. First, the type of probability system and the form of the cross-impacts (interdependence) assumed are of a very

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[1] They actually have worked with four variations of the cross-impact technique. Two of these variations, the dynamic model and the scenario model, involve aspects of the interdependency problem (namely, strict time or order relationships) which are beyond the scope of the present paper.

[2] There are a number of considerations which suggest that, for purposes of long-range planning, the absolute probabilities are of secondary interest, whereas the "scenario" probabilities, i.e., the probabilities of joint occurrence or nonoccurrence of many events, are more directly relevant. This topic will be discussed in the text in greater detail.

simple probabilistic form. Second, the notion of taking account of the interactions is elementary. It can be described as follows: If an individual or a group estimates a set of probabilities of events and this set contains interactive terms, then the set may be inconsistent. The purpose of computation, then, is to test the consistency of the set of estimates, and if the set is not consistent, to perform the smallest reasonable perturbation on the original estimates to create a set that is consistent. As soon as the consistent set is achieved-from this elementary point of view-the interactions have been "taken into account."

It might not hurt to amplify this point slightly. If we assume that the purpose of cross impact analysis is to arrive at the best possible estimate of the separate probabilities of each of the events? then regardless of how the original estimates are obtained, they should already include the interactions among the events. The basic assumption of cross-impact analysis is that the separate probabilities do take the interactions into account, but incompletely so that some modification is needed.

Another assumption is that the cross-impact estimates are more "solid" than the absolute probability estimates. There are several motivations behind this assumption. First, there is widespread, and probably generally sound, opinion that relative probabilities are clearer and easier to estimate than absolute probabilities. I do not know of any experimental data to support this, but it does appear that the more limited reference of a relative probability makes it psychologically easer to deal with. Second, there is an argument (which may or may not have logical justification) that narrowing the reference class in some way makes the probability more "correct." This is the basis for Reichenbach's recommendation [Ref. 2, p. 374] that in practical applications of probabilities (decisions) the narrowest reference class for which there is reliable information should be used. Finally, and probably most important for cross-impact analysis, the notion of cross-impacts is new, and should receive greater emphasis.

None of the foregoing justifies the assumption that the cross-impacts are more solid than the absolute probabilities, but they do lend some heuristic weight to the computational structure. These considerations suggest that adjustments should be made in the absolute probabilities, not the cross-impacts. It will be shown below that this point of view cannot be maintained strictly. It is possible that inconsistencies appear in the cross-impacts as well as in the original estimates of absolute probabilities. However, this assumption can be maintained in a weaker sense if the cross-impacts can be adjusted without making use of the absolute probabilities, and then the absolute probabilities can be adjusted with fixed cross-impacts.

The results to be presented in this article, then, can be summed up by saying that given a set of estimates of absolute probabilities and cross-impacts in the form of relative probabilities, simple tests exist for determining the consistency of the cross-impact matrix, and for determining the consistency of the absolute probabilities given that the cross-impact matrix is consistent. If the set of estimates is consistent, then no further computation is required. If the set is not consistent, then a number of steps may be taken to adjust the set, ranging from simplified averaging techniques to reiteration of the estimates, given a display of the inconsistencies involved. The adjustment procedure used should depend on the nature of the inconsistencies and the opportunities for querying the estimators again.

The consistency condition derived below takes a particularly elegant form when the cross-impact matrix is expressed as a set of relative probabilities, that is, the probability of event e_i is p, given that event e_j occurs. With cross-impacts of this form, the consistency condition can be stated as follows: The n events define an n-dimensional probability space (strictly speaking an n-dimensional hypercube, since each probability can vary only between 0 and 1). If the cross-impacts are mutually consistent, they define a single line in this hypercube, which passes through the origin. A set of absolute probabilities consistent with the cross-impacts will then define a point lying on this line. As it turns out, the condition is relatively easy to test and allows a fairly simple description of methods of resolving inconsistencies if the estimates do not pass the test.

II. Consistent Systems of Probabilities

We will be concerned with an elementary system, of probabilities:

A. A set of n events $e_1, e_2, \ldots, e_i \ldots, en$:
B. A set of n absolute probabilities of these events, $P(e_1), P(e_2), \ldots, P(e_i), \ldots, P(en)$.
C. A set of $n^2 - n$ relative probabilities[3] of the form $P(e_1/e_2), \ldots, P(e_i/e_j), \ldots$ (read as "the probability of e_i given that e_j occurs").
D. A set of higher-order probabilities, illustrated by $P(e_i \cdot e_j \cdot e_k)$, where the period indicates joint occurrence (read as "the probability that e_i, e_j, and e_k all occur). There is a large family of probabilities of type D. Since we will not deal formally with this set, they are not listed in detail.

The set of $n^2 - n$ relative probabilities of type C will be referred to as the cross-impact matrix.

The set of events $\{e_i\}$ could be a list of potential technological developments, a list of social or political events, a combination of these, or something entirely different, such as a set of symptoms of diseases among the total U.S. population.

It is easy to show that for a complete specification of a system, $2^n - 1$ probabilities must be given; these are independent except for a set of inequalities illustrated by $P(e_i \cdot e_j) = P(e_i)$. In general, the n absolute probabilities and the $n^2 - n$ relative probabilities are quite insufficient to completely specify the system.

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[3] The notion of an absolute probability is sometimes misunderstood. For the purpose of this discussion, we simply assume a common universe of discourse for the events, and refer the probabilities to that. In this respect, the absolute probabilities of each event should reflect (in a completely buried form) all of the interactions between that event and all others. In particular, the absolute probability of an event is not interpreted in the Bayesian sense of an a priori probability.

Given a set of probabilities of the forms B and C, a simple question can be asked; namely, are they a consistent set? "Consistent" here means compatible with the usual calculus of probabilities. The question is meaningful because the set of probabilities in the forms B and C is redundant.

There are two kinds of redundancy. The first involves the probabilities of type C. All of the $n^2 - n$ relative probabilities in the cross-impact matrix can be replaced by the $n(n - 1)/2$ joint probabilities of the form $P(e_i \cdot e_j)$. (The factor 1/2 comes from the fact that joint occurrence is commutative.) In short, there are twice as many entries in the cross-impact matrix as are needed to specify all probabilities involving no more than two events.

The second type of redundancy involves the interrelationship of the absolute and relative probabilities. Even with a consistent set of relative probabilities, the absolute probabilities may not combine with the relative probabilities in accordance with the rules of the calculus of probabilities.

We will use three elementary postulates of the calculus of probabilities and one theorem. The three elementary postulates are:

Pl. Normalization. $0 = p = 1$ for any probability p.
P2. Rule of the product. $P(e_i \cdot e_j) = P(e_i) \cdot P(e_j / e_i) / = P(e_j) \cdot P(e_i / e_j)$.
P3. Rule of addition. $P(e_i \text{ or } e_j) = P(e_i) + P(e_j) - P(e_i \cdot e_j)$.

The theorem is one which I derived in my Ph.D. thesis, and is referred to in Reichenbach [Ref. 2, p. 112]. It will not be proved here, but simply stated.

THEOREM. Rule of the triangle.

$$P(e_i / e_j) \cdot P(e_k / e_i) \cdot P(e_j / e_k) = P(e_j / e_i) \cdot P(e_k / e_j) \cdot P(e_i / e_k) \cdot \qquad (l)$$

The theorem states that for a set of three events, the product of the relative probabilities multiplying around the triangle in one direction is equal to the product of the relative probabilities multiplying in the other direction. (See Fig. 1.) The theorem is easily extended to four or more events, but the same effect can be achieved by-treating the larger set in subsets of three.

Since we can assume that all the probabilities-given in C and D are already between zero and one, the only role of P1 is to combine with P2 and P3 to give the weak inequality.

$$\begin{aligned} P(e_i) + P(e_j) - P(e_i) \cdot P(e_j/e_i) < 1, \\ P(e_i) + P(e_j) - P(e_j) \cdot P(e_i/e_j) < 1. \end{aligned} \qquad (2)$$

The rule of the product has an immediate consequence.

$$P(e_j) = P(e_i) \cdot P(e_j/e_i) / P(e_i/e_j). \qquad (3)$$

Eq. (3) can be interpreted as asserting that if $P(e_j/e_i)$ and $P(e_i/e_j)$ are fixed, then $P(e_i)$ and

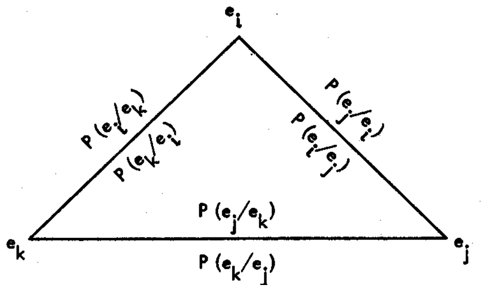

Fig 1. Rule of the Triangle

$P(e_j)$ must lie on a straight line through the origin in the $P(e_i)$, $P(e_j)$ unit square, as illustrated in Fig. 2.

Similarly, for all other pairs of events, the rule of the product implies that these must lie on a straight line through the origin in their respective unit squares when the relative probabilities are given. Now, if we consider the space of all the absolute probabilities the unit hypercube-the rule of the product in conjunction with the rule of the triangle assures that all of the absolute probabilities must lie on a single straight line within the hypercube. To illustrate this theorem, we first consider the unit cube defined by three events. Eq. (3) asserts that the probabilities must lie on the intersection of the planes defined by the straight lines in the respective unit squares.

Figure 3 illustrates the intersection of the two planes defined by the pairs $P(e_i)$, $P(e_k)$ and $P(e_j)$, $P(e_k)$. There is one additional plane defined by the pair $P(e_i)$, $P(e_j)$; and the intersection of it with the first two produce two additional lines. If the relative probabilities are consistent (by the rule of the triangle and the rule of the product) the three lines will coincide. Figure 3 portrays an inconsistent case.

To present the general consistency condition for cross-impact matrices, it is convenient first to establish a lemma concerning lines in n-dimensional Euclidean spaces. A line is defined by two points $X = (x_1, x_2, \ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$. Any other point Z on the line is a linear combination of these two; i.e., $Z = aX + (1-a)Y$, $-8 = a = 8$. It is convenient to shift the origin to Y, in which case $Z - Y = a(X - Y)$. Renaming $Z-Y=Z'$ and $X-Y= X'$, we have $Z'=aX'$.

LEMMA 1. *A necessary and sufficient condition that a matrix* $S = \{s_{ij}\}$, *where* $s_{ji} = 1/s_{ij}$, $s_{ii}=1$,[4] *define a line aX through the origin in n-dimensional Euclidean space, such that the slope* x_i/x_j *of the line projected on the two-dimensional subspace (i, j) is* s_{ji}, *is the triangle rule*

[4] To eliminate inessential special cases, s_{ij} is also assumed to be nonzero.

$$S_{ij} \cdot S_{jk} \cdot S_{ki} = 1. \qquad \text{(a)}$$

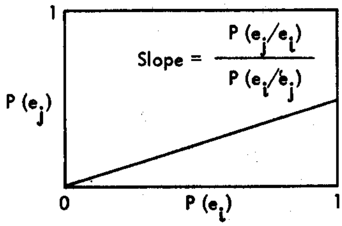

Fig. 2. Geometric representation of the rule of the product

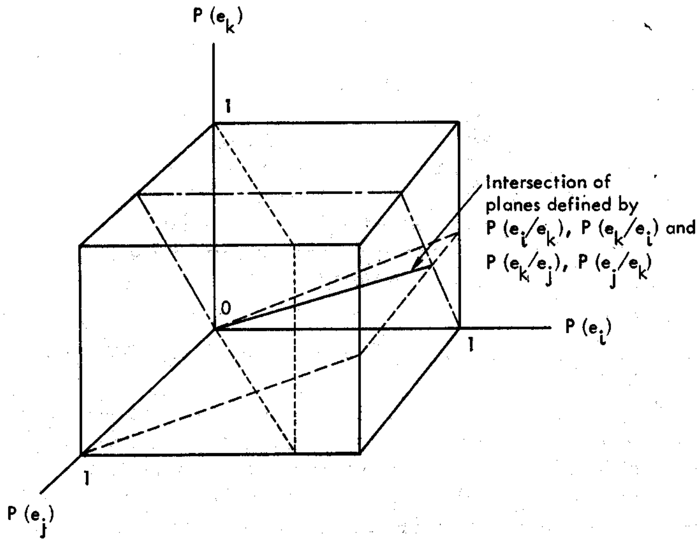

Fig. 3. Inconsistent relative probabilities *for* three events.

Proof. Necessity. Assume there is a line aX with the hypothesized properties, then

$$\frac{x_j}{x_i} \cdot \frac{x_k}{x_j} \cdot \frac{x_i}{x_k} = 1 = s_{ij} \cdot s_{jk} \cdot s_{ki}.$$

Sufficiency. Consider a matrix S that fulfills the triangle rule. Define a point X by $x_i = s_{1j}$. The triangle rule of Eq. (a) implies that $s_{1j} \cdot s_{ji} \cdot s_{i1} = 1$, where sji = 1 / $s_{1j} \cdot s_{i1}$ and since $1/s_{i1} = s_{1i}$,

$$s_{ji} = \frac{s_{1i}}{s_{1j}} = \frac{x_i}{x_j}. \qquad \text{(b)}$$

Since for any other point Y on the line aX, $y_i/y_j = ßx_i/ßx_j = x_i/x_j$, Eq. (b) is completely general.

Lemma 1 is applicable to a cross-impact matrix by defining a matrix S, $s_{ij} = P(e_j/e_i)/P(e_i/e_j)$. The conditions $s_{ji} = 1/s_{ij}$ and $s_{ii} = 1$ follow immediately from the definition, and the triangle rule Eq. (a) follows from the rule of the triangle for relative probabilities.

The rule of addition can be invoked by determining the intersection of the lines defined by setting the inequalities to equalities in Eq. (2). Thus,

$$\left.\begin{aligned} P(e_i) + P(e_j) - P(e_i)\cdot P(e_j/e_i) = 1, \\ P(e_i) + P(e_j) - P(e_j)\cdot P(e_i/e_j) = 1. \end{aligned}\right\} \quad (4)$$

Solving these two for $P(e_i)$ and $P(e_j)$ gives

$$\left.\begin{aligned} P(e_i) = \frac{P(e_i/e_j)}{P(e_i/e_j) + P(e_j/e_i) - P(e_i/e_j)\cdot P(e_j/e_i)}, \\ P(e_j) = \frac{P(e_j/e_i)}{P(e_i/e_j) + P(e_j/e_i) - P(e_i/e_j)\cdot P(e_j/e_i)}. \end{aligned}\right\} \quad (5)$$

It is simple to verify that the pair $P(e_i)$, $P(e_j)$ lies on the line defined by Lemma 1.

The question still remains whether the limits imposed by different applications of Eq. (5) with different pairs result in the same limit. Thus, for example, we have

$$P(e_i) = \frac{P(e_i/e_k)}{P(e_i/e_k) + P(e_k/e_i) - P(e_i/e_k)\cdot P(e_k/e_i)}$$

In general, this is not the same limit as expressed by Eq. (5) above, thus the minimum of all the limits determined by all pairs must be selected. The minimum fixes a point L on the consistency line in the hypercube; any acceptable point is lower than or equal to that point.

This completes the set of consistency conditions. In sum, the consistency conditions define a line passing through the origin in the unit hypercube and a point on that line. To be consistent, the absolute probabilities must lie on the segment of that line between the origin and the given point.

The foregoing demonstrates the necessity, but not the sufficiency, of the conditions. A direct consequence of the rule of addition (P3) is that the probability of the disjunction of any subset of the absolute probabilities must be less than or equal to one. The disjunctive probabilities for subsets larger than two cannot be computed from the absolute probabilities and binary relative probabilities since the disjunctive probabilities of sets larger than two involve higher-order interactions.

III. Resolution of Inconsistencies

In previous sections we defined a set of conditions to be met for a set of absolute probabilities and a cross-impact matrix of relative probabilities to be consistent. The simplest application of the conditions for the matrix is to test all triplets of relative probabilities by the triangle rule. This test is tedious. There are $n(n-1)(n-2)/6$ triplets for n events. For 50 events there are 19,600 triplets.

However, it is necessary to check only a subset of the triplets, a convenient subset .being the set of triangles having one event in common. The number of triangles in such a 'subset is $(n-1)(n-2)/2$. Thus, the number of independent triangles increases as the square of n, rather than as the cube which is the case for the total number of triangles. This happy situation is guaranteed by the following lemma.[5]

> LEMMA 2. If *the rule of the triangle holds for all triangles that have one event in common, then it holds for all triangles.*

Proof. Consider any event e_l, and any triangle e_i, e_j, e_k, not including e_l. Assume that the rule of the triangle holds for all triangles containing e_l. Then, abbreviating $P(e_i/e_j)$ to P(i/j) gives

$$P(l/i)\cdot P(i/j)\cdot P(j/l) = P(i/l)\cdot P(j/i)\cdot P(l/j),$$
$$P(k/l)\cdot P(j/k)\cdot P(l/j) = P(l/k)\cdot P(k/j)\cdot P(j/l),$$
$$P(i/l)\cdot P(k/i)\cdot P(l/k) = P(l/i)\cdot P(i/k)\cdot P(k/l).$$

Multiplying the three expressions on the left side of the equations and the three expressions on the right side, the terms containing 1 cancel and we arrive at

$$P(i/j)\cdot P(j/k)\cdot P(k/i) = P(j/i)\cdot P(k/j)\cdot P(i/k)$$

which was to be proved. --

For the absolute probabilities, assuming the matrix is consistent, the simplest testis first to calculate the limits imposed by the rule of addition. Assuming that test is passed, the individual probabilities can be tested by starting with $P(e_1)$ and computing the others by the rule $P(e_i)=P(e_1)\cdot P(e_i/e_1)$.

It appears likely that in most practical applications the sets of probabilities will not be consistent, and a question arises as to the method-of proceeding. There is no "correct" method of resolving the inconsistencies. There are several directions that can be taken, depending on the interest of the study manager, on the availability of respondents for reestimation, and the like.

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[5] I would like to thank T. Brown and J. Spencer for helpful suggestions concerning this simplification of the consistency test.

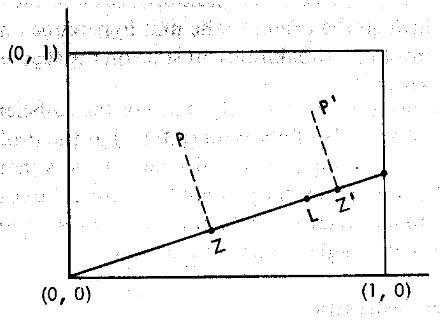

Fig. 4. Adjustment of absolute probabilities by computing the nearest point on the line defined by the cross-impact matrix.

Assuming no restrictions on reestimation, it appears desirable to present the information concerning inconsistencies to the respondents, and obtain reestimates from them. This poses the question of how the information is to be presented to be useful to the respondents.

One reasonable display can be obtained by computing the adjustments described below, and feeding back the original and adjusted estimates.

Assuming that the cross-impact matrix is consistent, but the set of absolute probabilities does not lie on the line, a natural and simple adjustment is to choose the point on the line that is nearest to the estimated point, as illustrated for two dimensions in Fig: 4. The computation of the nearest point is simple, given the line in parametric form. If P is the estimated point and aX is the line, then the desired nearest point on the line Z is given by

$$\mathbf{Z} = \beta \mathbf{X}, \beta = \frac{\sum p_i x_i}{\sum x_i^2}. \tag{6}$$

If Z lies above the limit L imposed by the rule of addition (Z' in Fig. 4), the most natural rule is to select L as the adjusted set of probabilities.

The situation is not quite as neat if the cross-impact matrix itself is inconsistent. The problem here is that inconsistencies result in a set of lines, which can become very large rapidly, and there is no obvious way of weighting these in the computation of a representative line. A relatively simple adjustment, and one that appears sufficient for the purpose - especially if the group is expected to reestimate - is the following

$$x_i = \frac{1}{n} \sum_j \frac{P(e_i/e_j)}{[\sum_k P(e_k/e_j)^2]^{1/2}}. \tag{7}$$

This computation arises from allowing each row in the matrix in turn to generate a potential line, and averaging these lines by summing their intersection with the unit sphere. Although Eq. (7) does not arise from any optimization rule, it does assure that in adjusting X the contributions of all other events to e, will be "taken into account," and the contribution of e, to all the other events enters in the normalization to the unit sphere.

A more satisfying variant of the procedure in Eq. (6) would be to make use of group weights on the probabilities. That is, each member of the group can be requested to evaluate how confident he is of his estimate (with some suitable rating scale) and a group measure of confidence-e.g., the average of the individual ratings-can be used to weight the absolute probability estimates.

If the weights w, are normalized *so* that 0 = w_i = 1, where 1 indicates certainty and 0 indicates sheer guess, then a reasonable weighted form of Eq. (6) would be

$$\beta = \frac{\sum p_i x_i w_i}{\sum x_i^2 w_i}. \tag{8}$$

A weighted adjustment for the relative probabilities *is* much more complex; a reasonable form has not yet been derived. One of the diffculties is that the consistency condition furnished by the rule of the triangle deals only with the ratios of the relative probabilities and not with the probabilities themselves. Ratios of estimated weights are somewhat awkward to use here.

IV. Scenarios

The cross-impact computation developed at the Institute for the Future not only produces a Monte Carlo estimate of revised absolute probabilities, but also gives a Monte Carlo estimate of the probabilities of joint occurrence of the total set of events. Each chain of events is a sample out of a large population of potential chains. The number of potential chains is $n!2^n$. The factor 2^n arises from the total number of joint occurrences (positive or negative) of n events, and the factor $n!$ from the manner in which the *next* event in the chain is selected, namely, by considering each remaining event equally likely to be next.

The computation involves a strong independence assumption, namely, that the change in the likelihood of occurrence of a given event is influenced only by the preceding event that has occurred. (In some forms of the computation the change is assessed in terms of occurrence or nonoccurrence of the preceding event.) In general probability systems, this assumption is not only not fulfilled; the assumption has as a consequence that the system is degenerate. A simple example may suffice to show this. If we make an assumption that all higher-order interactions are determined *solely* by the first-order interactions, then we would have

$$P(e_k/e_i \cdot e_j) = P(e_k/e_i) = P(e_k/e_j). \tag{9}$$

Applying the assumption to all interactions would imply that all the cross-impacts for a given target event are equal. The same result follows from other forms of the same assumption, for example,

$$P(e_k \cdot e_j / e_i) = P(e_k \cdot e_j), \tag{10}$$

$$\frac{P(e_i \cdot e_j \cdot e_k)}{P(e_i \cdot e_j \cdot \bar{e}_k)} = \frac{P(e_k)}{P(\bar{e}_k)}, \tag{11}$$

where the bar over an event indicates negation.

The Monte Carlo computation developed **by Gordon** et al. [1] for the likelihood method involves a slightly weaker assumptionthan Eq. (9). However, it is more complicated, and rather than try to deal with it analytically, a simple illustration will indicate the effect of the assumption. Consider the following cross-impact matrix:

	e_1	e_2	e_3
e_1	1	0.5	0.5
e_2	0.5	1	0.5
e_3	0.5	0.5	1

$$P(e_1) = P(e_2) = P(e_3) = 0.25.$$

Let

$$l_{ij} = \frac{1 - P(e_j)}{P(e_j)} \cdot \frac{P(e_j/e_i)}{1 - P(e_j/e_i)},$$

that is, l_{ji} is the factor by which the odds for e_j are multiplied to generate the odds for e_j, given that e_i has occurred. Then

$$l_{12} = \frac{0.75}{0.25} \cdot \frac{0.5}{0.5} = 3 = l_{ij} \text{ for all } i \neq j.$$

Each event has a positive impact on the others.

If the Monte Carlo computation is carried through, $P(e_i)$ is adjusted from 0.25 to 0.32, The computation suggests that in terms of the positive impacts the absolute probabilities should be higher. However, the original probabilities are fully consistent with the cross-impacts; the increase is, in this sense, unwarranted.[6]

For purposes of evaluating policies, the probabilities of the chains (where each chain can be considered one scenario) are of greater interest than the absolute probabilities of the separate events. We have indicated that to make the scenario computation useful it **will** be necessary either to obtain estimates of the higher-order interactions or find a more logically correct assumption concerning them. The first alternative is rather dis couraging. The number of higher-order probabilities becomes somewhat astronomical with large sets of events. The second alternative - although more attractive in terms of the feasibility of obtaining estimates - is subject to the danger of artificiality.

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[6] Although I have not made the calculation, it appears likely that most of the adjustment in the cases presented in Ref. [1] are of this sort.

Conclusion

This article presents an elementary approach to the estimation of complex probability systems. Above all, it contains no consideration of the structural properties of the subject matter involved. Generally speaking, it can be expected that a given application, whether it is technological events or health categories, will have certain underlying characteristics that will affect both the binary cross-impact probabilities and the higher-order interactions (e.g., with the health categories, it is very unlikely that any individual will simultaneously be afflicted with a large number of pathological conditions). Convenient methods of expressing these structural properties have not, to my knowledge, been defined for the kinds of systems in which cross-impact analysis would appear to be applicable. My first impression is that such structural properties may furnish the required assumption concerning higher-order probabilities.

The resolution of inconsistencies in the cross-impact matrix expressed in Eq. (7) can probably be developed further and appears to be an interesting mathematical problem in its own right.

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