VI.C. Multidimensional Scaling: Models, Methods, and Relations to Delphi

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Multidimensional scaling (MDS) is a general term for a class of techniques that are been developed to deal with problems of measuring and predicting human judgment. These techniques all have in common the fact that they develop spatial representations of psychological stimuli or other complex objects about which people make judgments (e.g., of preference, similarity, relatedness, or the like). This means that the objects are represented as points in a multidimensional vector space. If this space is two- or three-dimensional it can be viewed all at once by various graphical display devices. If it is more than three dimensional, it cannot, of course, be represented physically; but subspaces of two or three dimensions can be viewed, and can often yield very useful information. Furthermore, there are various plotting "tricks" (plotting higher dimensions by size or shading, for example) that make possible representation of higher dimensions (although it is not clear how many such dimensions a human user can comprehend all at once).

The crucial difference between MDS and such related techniques as factor analysis is that in MDS no preconception is necessary as to which factors might underlie, determine, or correspond to which dimensions. The only data actually required are the similarity judgments of one or more individuals, or some other measure of similarity or "proximity" of pairs of the entities under study. Simply put, and realizing there are many refinements of the technique, the basic similarity data are transformed (i.e., inverted in rank order) to measures of dissimilarity which may be viewed as (directly) monotonically related to Euclidean distances in some space of unknown dimensions. The greater the similarity of two objects, the closer they are constrained to be to each other in the multidimensional space or "map"; the more dissimilar, the farther apart. Then given for each object its distances to all other objects, we attempt to see how well the distances can be fitted (in a certain least-squares sense) by one, two, three, or more dimensions. The number of dimensions is increased until the addition of a new dimension makes very little improvement in the correlation (between the data and the distance) or other "goodness-of-fit" measure used. This usually occurs at considerably less than the N-1 dimensions that potentially could be needed to perfectly fit the distances among N objects. Since quantitative values for the coordinates of points in the space are obtained, one may make plots of the results and observe if the dimensions (the X, Y, Z, etc., coordinates) can be interpreted as underlying psychological or judgmental factors, which then are presumed to account for the original subjective judgments. The contrast in trying to comprehend underlying factors based upon the, raw data matrix as opposed to the plots obtained as a result of MDS is aptly demonstrated in the Morse Code example later in this paper.

The first order use of MDS in Delphi is to provide people with a graphical representation of their subjective judgments and see if they can ascribe meaning to the dimensions of the graphs. It is also possible to obtain a separate graphical representation for any subgroup or individual. Therefore the Delphi designer could exhibit for the respondents a graphical representation of how various subgroups view the problem e.g., how much difference is there among the views of politicians, business executives, and social scientists. He could also easily compare results from different time periods, experimental conditions, etc.

A possible limitation of MDS in Delphi or other applications is that there is no guarantee that a meaningful set of dimensions will emerge. However, the results may be useful even if one cannot interpret the dimensions, since the graphical representation can greatly facilitate the comprehension of patterns in the data.

In MDS, as in other techniques which provide scientific insight, the results can be embarrassing or detrimental-that is, hidden psychological factors could be exposed which might disrupt or damage the group process if not handled with some skill. For example, in examining goals of an *organization* one might find out that certain individual goals like prestige constitute a hidden dimension playing a dominant role. The Delphi designer and the group must be of the frame of mind to face up to such possibilities.

Let us suppose that Delphi were being used to study nations. If one were using MDS as an auxiliary procedure, the Delphi respondents might be asked, first of all, to judge the similarity of pairs of nations. For example, each subject might be asked to judge the similarity of every pair of nations by associating a number between zero and ten with each pair (zero meaning "not at all similar" and ten meaning "virtually identical"). It is, of course, subjective similarity, not similarity in any objective sense, that is being measured. Furthermore, people will differ systematically in their judgments of relative similarity (as they will differ in preference and other judgments).

One of us (Wish) has actually undertaken studies in just this area. This particular study used the INDSCAL (for INdividual Differences Multidimensional SCALing) method, developed by Carroll and Chang (1970). This method, which allows for different patterns of weights or perceptual importance of stimulus dimensions for different individuals or judges, will be-, discussed in- detail at a later point In the present case (in which' the various nations were the "stimuli") the analysis revealed three important dimensions, shown in Figs. 1 and 2, which can be described as "Political Alignment and Ideology" (essentially "communist" vs. "noncommunist" with "neutrals" in between), "Economic Development" contrasting highly industrialized or economically "developed" nations with "undeveloped" or "developing" nations (with "moderately developed" countries in between) and "Geography and Culture" (essentially "East" vs. "West").

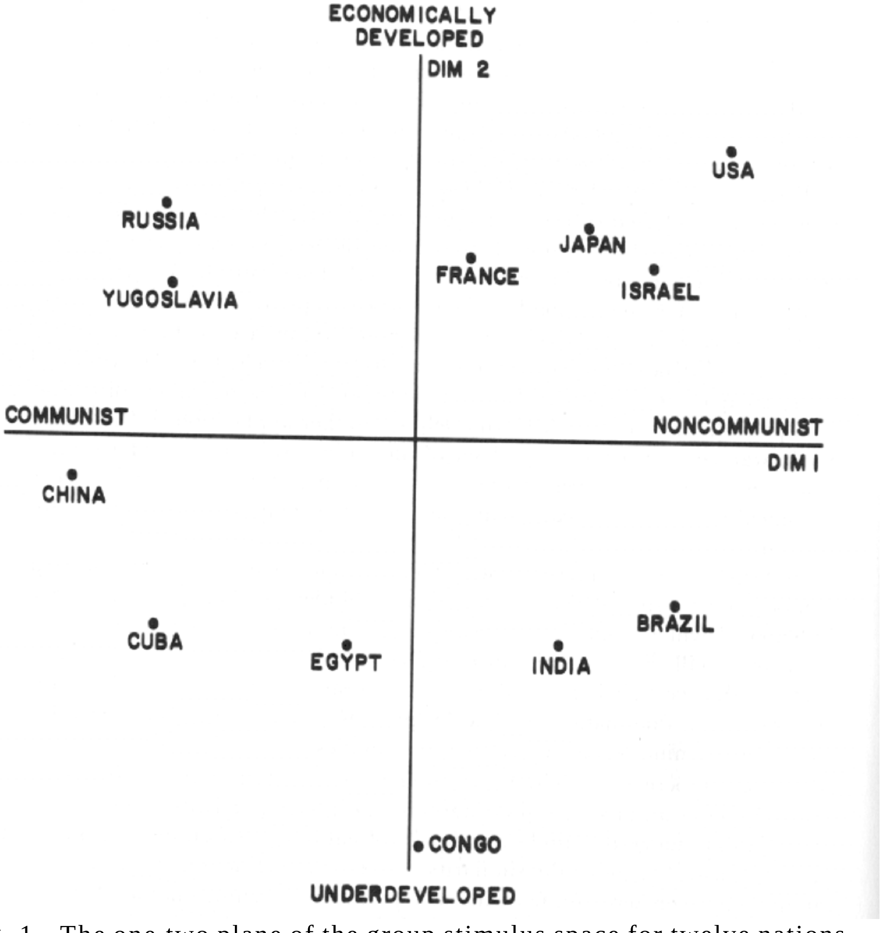

Fig. 1. The one-two plane of the group stimulus space for twelve nations (data dur to Wish), Dimensions one and two were interpreted by Wish as political alignment (communist-noncommunist) and economic development (economically developed underdeveloped) respectively.

The subjects in this study were also asked to indicate their position on the Vietnam War (these data were collected several years ago, it should be noted) and based on this they were classified as "Doves," "Hawks," or "Moderates." As shown in Fig. 3, it was possible to relate political attitudes very systematically to the perceptual importance (as measured by the INDSCAL analysis) of the first two dimensions to these subjects. Specifically, "Hawks" apparently put much more emphasis on the "Political Alignment and Ideology" dimension than on the "Economic Development"

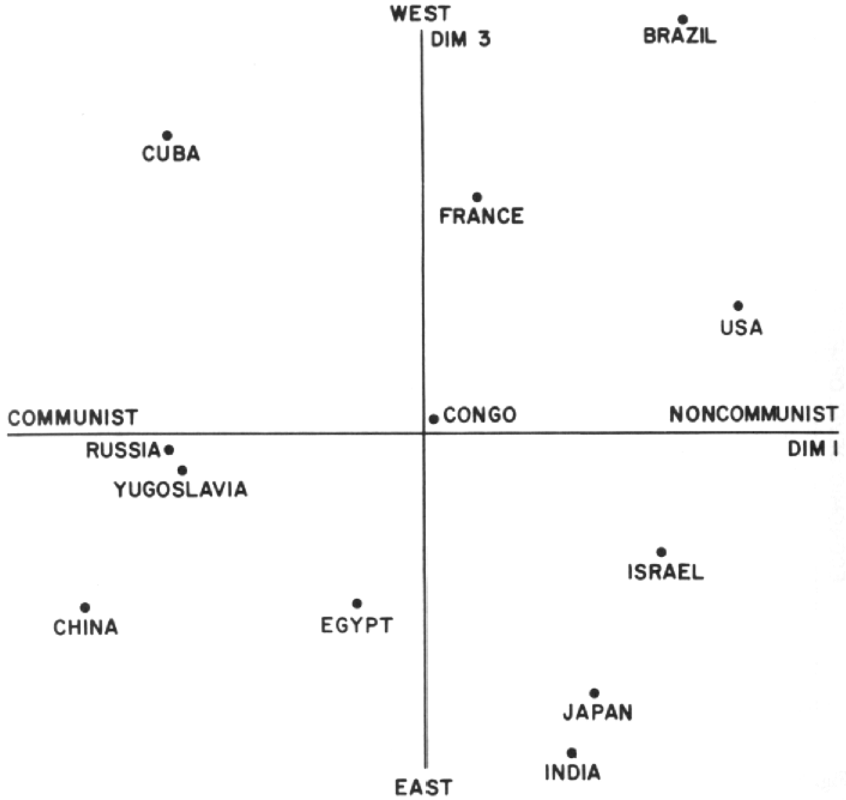

Fig. 2. The one-three plane of the group stimulus space for the Wish data on twelve nations. Wish interpreted dimension three as a geography dimension (east-west).

dimension, while "Doves" reversed this pattern. A "Hawk" would see two nations that differed in ideology but were very close in economic development-e.g., Russia and the U. S.-as very dissimilar, while a "Dove" would see them as rather similar. In contrast, a "Hawk" would view Russia and Cuba (which are close ideologically but different in economic development) as very similar, while a "Dove" would perceive them as quite different. As can be seen in Fig. 4, the third dimension ("East-West") does not discriminate these political viewpoints.

This example serves to illustrate that MDS (particularly the INDSCAL method for individual differences scaling) can simultaneously yield information about the stimuli (nations, in this example) and about the individuals or subjects whose judgments constitute the basic data (similarities or dissimilarities of nations in this case). We have learned that the three most important dimensions underlying subjects' perceptions of nations, at least for

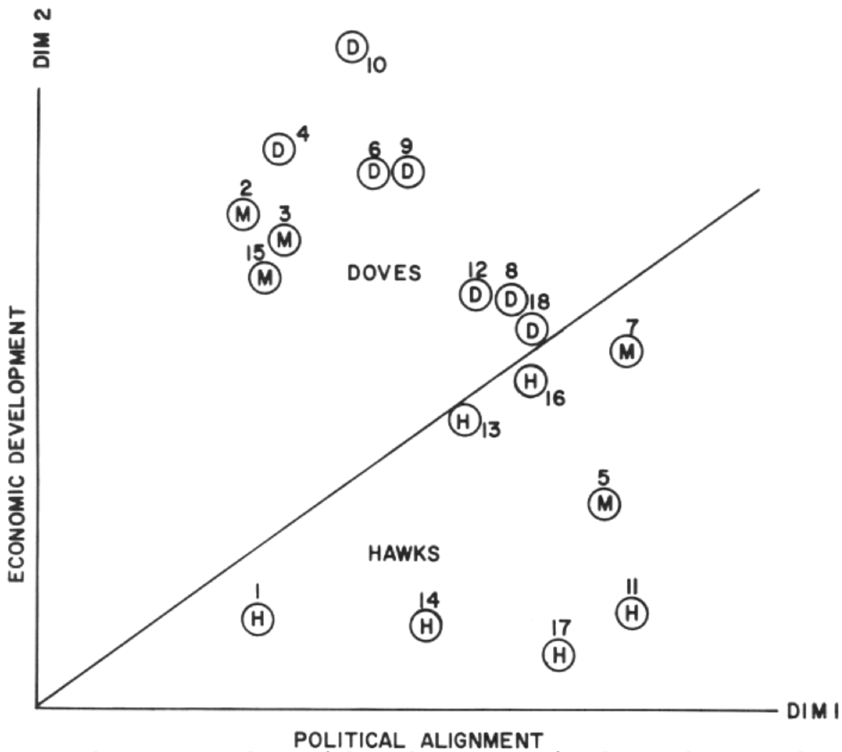

Fig. 3. The one-two plane of the subject space for the Wish nation data. *D, H* and M stand for "dove," "hawk," and "moderate" (as determined by subjects' self-report) vis -a-vis attitudes on Vietnam War. Forty-five-degree line divides "doves" from "hawks," with "moderates" on both sides.

graduate students, are ideology, economic development, and geography and culture (on an East-West dimension). [In later studies (Wish, 1972; Wish and Carroll, 1972), each of these single "dimensions" has shown itself to be multidimensional. To show this, however, required much more data and very high-powered data analytic techniques which are beyond the scope of the present paper.] We also learn of a very interesting correlation between the importance or salience of these dimensions to individual subjects and these subjects' political attitudes (we cannot, of course, infer causality from this correlation, but we can certainly conclude that perception of nations and political attitudes about them are not independent). Finally, the graphical aspect of MDS lets the user (who may also be one of the subjects, especially in Delphi applications) see these relationships (and others) in a clear, concise, intuitively appealing, and highly informative manner. In a real-time computerized Delphi system, this should provide an excellent way of giving

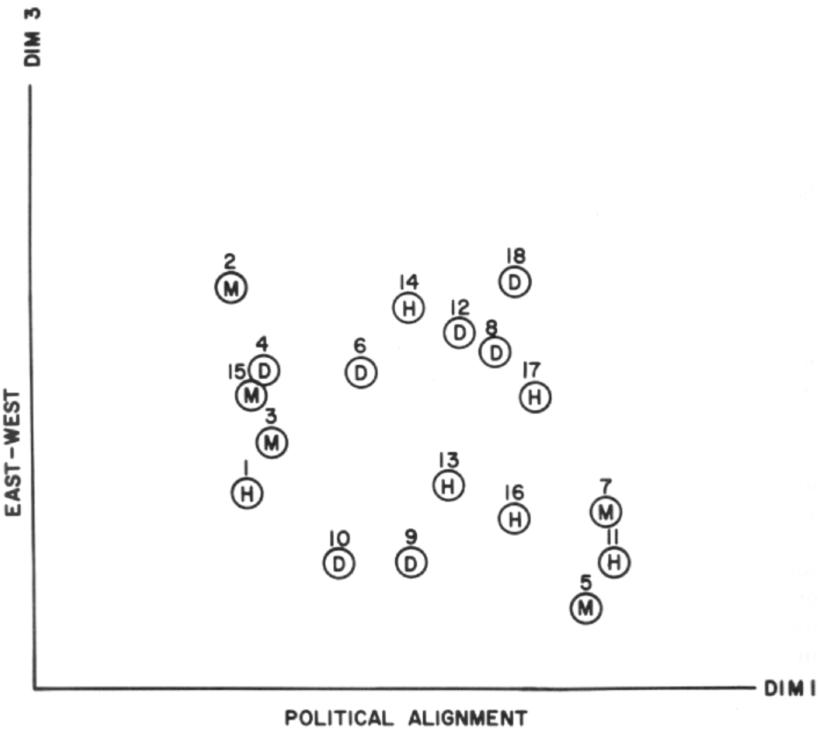

Fig. 4. The one-three plane of the subject space for Wish nation data. Coding of subjects is as in Fig. 3.

feedback about the judgments of a large number of respondents simultaneously (stressing the communalities in their judgments, but at the same time allowing each judge to see clearly the range of individual differences and, in particular, to see where *he* or she lies vis-a-vis the rest of the group). The respondent can also provide the interpretation of dimensions and the group can attempt to arrive at a collective view based upon discussions. In cases requiring special expertise available only to members of the respondent group, this may in fact be necessary.

Similarities or dissimilarities data are inherently *dyadic* data, in that they imply an ordering, or a scale, for the n(n-1)/2 pairs (dyads) of the n stimuli. There are many other kinds of dyadic data than similarities or dissimilarities, however. Usually these can be interpreted in some sense as proximities or antiproximities --- that is, as related in some relatively simple way (e.g., monotonically) to *distance* in a multidimensional space.

To return to our example of nations, other such dyadic data that might be considered include friendliness; amount of trade or communication between; communality in ethnic, cultural, linguistic, or other background (these could broadly be interpreted as "proximity" measures, since a large value implies "closeness"). Such other dyadic measures as probability of war between, or amount of competition between (in trade, vying for third nations' favor, prestige in sports, technological feats, or the like), might be interpreted as antiproximity measures (for which a large value implies distance), although there is a very real sense in which some of the them might be viewed as "mixed" proximity and antiproximity measures (e.g., to be economic competitors may be taken to imply a closeness in size, degree of industrialization, and other economic factors, but distance or even opposition in national goals, ideology, and the like). Dyadic data generally are good "grist" for the MDS mill, particularly when interpretable as proximities or antiproximities. Even "mixed" proximity and antiproximity data can potentially be handled, however. Such data could be analyzed by INDSCAL, for example, if negative as well as positive weights are allowed for subjects (and if certain precautions, which will not be described here, are observed in preprocessing of the data).

A good example of a study using dyadic data which does concern (perceived) futures of nations has been provided by Wish (1971) who asked subjects to make two different dyadic judgments of friendship of pairs of nations. The first was of friendship "now" (i.e., at the time of the study) and the other of friendship "ten years from now." The ideology and economic development dimensions already mentioned also emerged as the principal dimensions underlying these "friendship" dyadic judgments. While ideology was the most important dimension for "present" friendship, economic development turned out to be far more important for "future" friendship. This indicated that (at least as perceived by those subjects) future alliances among nations were judged to be based much more on economic than on ideological considerations, although the opposite is now perceived to be the case. Not too much imagination is needed to see how other kinds of dyadic judgments could be used in this way to get a very general composite picture of the perceived future trajectories of nations (or of other entities of interest). The possible applications of MDS as an adjunct to Delphi are, indeed, virtually unlimited.

Another step in a Delphi application might be to ask the subjects (presumably "expert" judges of some sort) to make various judgments directly relevant to futures of nations. These might include things like expected level of industrialization or economic development in ten (or twenty-five or fifty) years; expected population, military development, probability of involvement in wars, or the like. These are all judgments that can be viewed as ordering the (single) nations on some attribute. As such, they are all formally analogous to preference judgments, analysis of which is discussed at a later point. Coombs (1962) coined the general term *dominance judgment* to cover the general class of judgments that order stimuli in terms of a single attribute (preference, wealth, size, etc.). Unlike data on similarities or other dyadic relationships between stimuli, multidimensional solutions are generally possible for dominance data only when more than one individual is involved, and when there are *individual differences* in their dominance judgments. Multidimensional methods for analyzing dominance data will be discussed at the end of this paper.

Having "whetted the appetite" (we hope) of our reader as to the potential of MDS used in conjunction with Delphi, let us now take a more detailed look (although an admittedly superficial and nontechnical one) at just what MDS is all about. We hope that the annotated bibliography at the end of this paper will satisfy the needs of readers desiring a deeper and more detailed view of the subject:

Ordinary or "Two-Way" Multidimensional Scaling

Multidimensional scaling of similarities, dissimilarities, or other dyadic data is usually based on some form of *distance* model [although not' necessarily always; Ekman (1963) for example has proposed a scalar product model for similarities]. Generally speaking, the dyadic data (similarities, say) are assumed to relate in a simple way-usually via a linear or at least by a *monotonic* function --- to distances in a (hypothesized, but initially unknown) multidimensional space. The multidimensional scaling method is usually called a metric one if this function is assumed to be linear, and *nonmetric* if it is assumed to be merely monotonic (or rank order preserving). The function, of course, will be assumed decreasing or increasing (whether linear or merely monotonic) depending on whether the data are proximities (e.g., similarities) or antiproximities (e.g., dissimilarities). These distances are usually, but not always, assumed to be Euclidean distances -the distances of ordinary physical space. (In the INDSCAL and some other individual differences models, "weighted" or other "generalized" Euclidean distances are assumed, but these can usually be represented as ordinary Euclidean distances in a "transformed" space for a particular individual.) The *process* of multidimensional scaling consists of deriving the multidimensional spatial representation *given* only the dyadic (e.g. similarities) data, It is in this sense the obverse of reading distances from a map. In MDS, we are given the distances (or numbers monotonically related to them) and are required to construct the map! As we shall see, this is quite feasible, especially if one has the aid of a high-speed digital computer.

To get a better feel for this approach, let us look at some data analyzed by multidimensional scaling methods. Fig. 5 shows a matrix of confusions between Morse Code signals, collected by Dr. E. Z. Rothkopf who is now at Bell Laboratories, Subjects who were learning the International Morse Code were asked to listen to pairs of Morse Code signals and judge whether the two were "same" or "different." The elements in this matrix are simply the percentage of subjects in the sample who judged the two signals as "same." If the subjects were making errorless judgments, all the entries on the main diagonal would be 100 and off-diagonal elements would all be zero. We can see by the fact that there are a large number of nonzero off-diagonal elements

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	1	2	3	4	5	6	7	8	9	0
A	92	04	06	13	03	14	10	13	46	05	22	03	25	34	06	06	09	35	23	06	37	13	17	12	07	03	02	07	05	05	08	06	05	06	02	03
B	05	84	37	31	05	28	17	21	05	19	34	40	06	10	12	22	25	16	18	02	18	34	08	84	30	42	12	17	14	40	32	74	43	17	04	04
C	04	38	87	17	04	29	13	07	11	19	24	35	14	03	09	51	34	24	14	06	06	11	14	32	82	38	13	15	31	14	10	30	28	24	18	12
D	08	62	17	88	07	23	40	36	09	13	81	56	08	07	09	27	09	45	29	06	17	20	27	40	15	33	03	09	06	11	09	19	08	10	05	06
E	06	13	14	06	97	02	04	04	17	01	05	06	04	04	05	01	05	10	07	67	03	03	02	05	06	05	04	03	05	03	05	02	04	02	03	03
F	04	51	33	19	02	90	10	29	05	33	16	50	07	06	10	42	12	35	14	02	21	27	25	19	27	13	08	16	47	25	26	24	21	05	05	05
G	09	18	27	38	01	14	90	06	05	22	33	16	14	13	82	52	23	21	05	03	15	14	32	21	23	39	15	14	05	10	04	10	17	23	20	11
H	03	45	23	25	09	32	08	87	10	10	09	29	05	08	08	14	08	17	37	04	36	59	09	33	14	11	03	09	15	43	70	35	17	04	03	03
I	64	07	07	13	10	08	06	12	93	03	05	16	13	30	07	03	05	19	35	16	10	05	08	02	05	07	02	05	08	09	06	08	05	02	04	05
J	07	09	38	09	02	24	18	05	04	85	22	31	08	03	21	63	47	11	02	07	09	09	09	22	32	28	67	66	33	15	07	11	28	29	26	23
K	05	24	38	73	01	17	25	11	05	27	91	33	10	12	31	14	31	22	02	02	23	17	33	63	16	18	05	09	17	08	08	18	14	13	05	06
L	02	69	43	45	10	24	12	26	09	30	27	86	06	02	09	37	36	28	12	05	16	19	20	31	25	59	12	13	17	15	26	29	36	16	07	03
M	24	12	05	14	07	17	29	08	08	11	23	08	96	62	11	10	15	20	07	09	13	04	21	09	18	08	05	07	06	06	05	07	11	07	10	04
N	31	04	13	30	08	12	10	16	13	03	16	08	59	93	05	09	05	28	12	10	16	04	12	04	06	11	05	02	03	04	04	06	02	02	10	02
O	07	07	20	06	05	09	76	07	02	39	26	10	04	08	86	37	35	10	03	04	11	14	25	35	27	27	19	17	07	07	06	18	14	11	20	12
P	05	22	33	12	05	36	22	12	03	78	14	46	05	06	21	83	43	23	09	04	12	19	19	19	41	30	34	44	24	11	15	17	24	23	25	13
Q	08	20	38	11	04	15	10	05	02	27	23	26	07	06	22	51	91	11	02	03	06	14	12	37	50	63	34	32	17	12	09	27	40	58	37	24
R	13	14	16	23	05	34	26	15	07	12	21	37	14	12	12	29	08	87	16	02	23	23	62	14	12	13	07	10	13	04	07	12	07	09	01	02
S	17	24	05	30	11	26	05	59	16	03	13	10	05	17	06	06	03	18	96	09	56	24	12	10	06	07	08	02	02	15	28	09	05	05	05	02
T	13	10	01	05	46	03	06	06	14	06	14	07	06	05	06	11	04	04	07	96	08	05	04	02	02	06	05	05	03	03	03	08	07	06	14	06
U	14	29	12	32	04	32	11	34	21	07	44	32	11	13	06	20	12	40	51	06	93	57	34	17	09	11	06	06	16	34	10	09	09	07	04	03
V	05	17	24	16	09	29	06	39	05	11	26	43	04	01	09	17	10	17	11	06	32	92	17	57	35	10	10	14	28	79	44	36	25	10	01	05
W	09	21	30	22	09	36	25	15	04	25	29	18	15	06	26	20	25	61	12	04	19	20	86	22	25	22	10	22	19	16	05	09	11	06	03	07
X	07	64	45	19	03	28	11	06	01	35	50	42	10	08	24	32	61	10	12	03	12	17	21	91	48	26	12	20	24	27	16	57	29	16	17	06
Y	09	23	62	15	04	26	22	09	01	30	12	14	05	06	14	30	52	05	07	04	06	13	21	44	86	23	26	44	40	15	11	26	22	33	23	16
Z	03	46	45	18	02	22	17	10	07	23	21	51	11	02	15	59	72	14	04	03	09	11	12	36	42	87	16	21	27	09	10	25	66	47	15	15
1	02	05	10	03	03	05	13	04	02	29	05	14	09	07	14	30	28	09	04	02	03	12	14	17	19	22	84	63	13	08	10	08	19	32	57	55
2	07	14	22	05	04	20	13	03	25	26	09	14	02	03	17	37	28	06	05	03	06	10	11	17	30	13	62	89	54	20	05	14	20	21	16	11
3	03	08	21	05	04	32	06	12	02	23	06	13	05	02	05	37	19	09	07	06	04	16	06	22	25	12	18	64	86	31	23	41	16	17	08	10
4	06	19	19	12	06	25	14	16	07	21	13	19	03	03	02	17	29	11	09	03	17	55	08	37	24	03	05	26	44	89	42	44	32	10	03	03
5	08	45	15	14	02	45	04	67	07	14	04	41	02	00	04	13	07	09	27	02	14	45	07	45	10	10	14	10	30	69	90	42	24	10	06	05
6	07	80	30	17	04	23	04	14	02	11	11	27	06	02	07	16	30	11	14	03	12	30	09	58	38	39	15	14	26	24	17	86	69	14	05	14
7	06	33	22	14	05	25	06	04	06	24	13	32	07	06	07	36	39	12	06	02	03	13	09	30	30	50	22	29	18	15	12	61	85	70	20	13
8	03	23	40	06	03	15	15	06	02	33	10	14	03	06	14	12	45	02	06	04	06	07	05	24	35	50	42	29	16	16	09	30	60	89	61	26
9	03	14	23	03	01	06	14	05	02	30	06	07	16	11	10	31	32	05	06	07	06	03	08	11	21	24	57	39	09	12	04	11	42	56	91	78
0	09	03	11	02	05	07	14	04	05	30	08	03	02	03	25	21	29	02	03	04	05	03	02	12	15	20	50	26	09	11	05	22	17	52	81	94

Fig. 5. Matrix of confusabilities of thirty-six Morse Code signals (data from Rothkopf, 1957).

in this matrix that, in fact, subjects were making quite considerable confusions among these stimuli. It is reasonable to assume, as Roger Shepard did in analyzing these data, that the more similar were two signals, the more likely it was that subjects would confuse them. Under this assumption, stimulus confusability, as measured by size of the entry for a pair of signals in this matrix, should be a direct measure of similarity of the pair.

Figure 6 shows the two-dimensional solutions Shepard attained on the basis of this matrix. The computer took the matrix in Fig. 5 as input, and produced the two-dimensional structure in Fig. 6 as output. The analysis was via Kruskal's (1964) nonmetric scaling procedure called MDSCAL, which stands simply for Multi Dimensional SCALing, while the interpretation of this figure, including the curvilinear coordinate system drawn there, is credited to Shepard (1963). The reasoning he used is as follows: it is easy to see by inspection that, as we move from the lower to the upper part of this figure, the aspect of the signals that is changing is number of components, ranging from those signals with one component at the very bottom of the figure to those with five components at the top of the figure. Thus, Shepard called this dimension (slightly curved in this figure) "number of components," As we move from left to right in the figure we can see that what is changing is the ratio of dots to dashes. That is, the figures on the left consist primarily of figures with all or mostly dots; those on the right, signals with all or mostly dashes. Shepard, therefore, labeled this dimension "dot-to-dash ratio." While subsequent analysis of these and related data (Wish, 1967, 1969) indicated additional dimensions relating to the precise sequence of dots and dashes, these two dimensions appear to account for a very large proportion of the variance in the "confusion" data.

It should be pointed out that, in addition to drawing the curvilinear coordinate system depicted in Fig. 6, Shepard had to orient the figure so that the two dimensions would correspond (more or less) with the horizontal and vertical directions. That is, to use the map analogy, it was necessary for him, as part of the interpretation process, to choose North-South and East-West directions on the map. The computer is able to construct the configuration, but not to orient the coordinate axes correctly. Although this may not be too much of a problem in two or perhaps three dimensions, it could be prohibitively problematical in four or more dimensions. This is, in fact, one of the principal obstacles to interpretation and use of MDS solutions, especially high-dimensional ones. As will be pointed out later, however, the INDSCAL method offers a way out of this particular problem, as it is able to capitalize on individual differences among subjects or judges to orient the coordinate axes *uniquely*.

Figure 7 shows the monotone "distance function" that resulted from this analysis. On the abscissa are distances as computed from the two-dimensional solution shown in Fig. 6, while on the ordinate are the similarities data (in this case, the percentage of "same" responses to each pair) that served as input to the multidimensional scaling program. The monotone function is, in this case, roughly a negative exponential, which fits in very nicely with what we know about confusion data. There is good theoretical reason to assume a function of this general negative exponential type relating distance in a multidimensional space to stimulus confusability. The critical thing here is that the nonmetric scaling computer program *simultaneously* determined the form of this function and the structure of the stimuli in the two-dimensional multidimensional space, and without any knowledge about either the structure of Morse Code signals or about the psychology of stimulus confusion!

Looking back for a moment at the data matrix shown in Fig. 5, it can be seen that one can tell very little indeed about the stimuli or about confusability based on mere inspection of this matrix. However, the two-

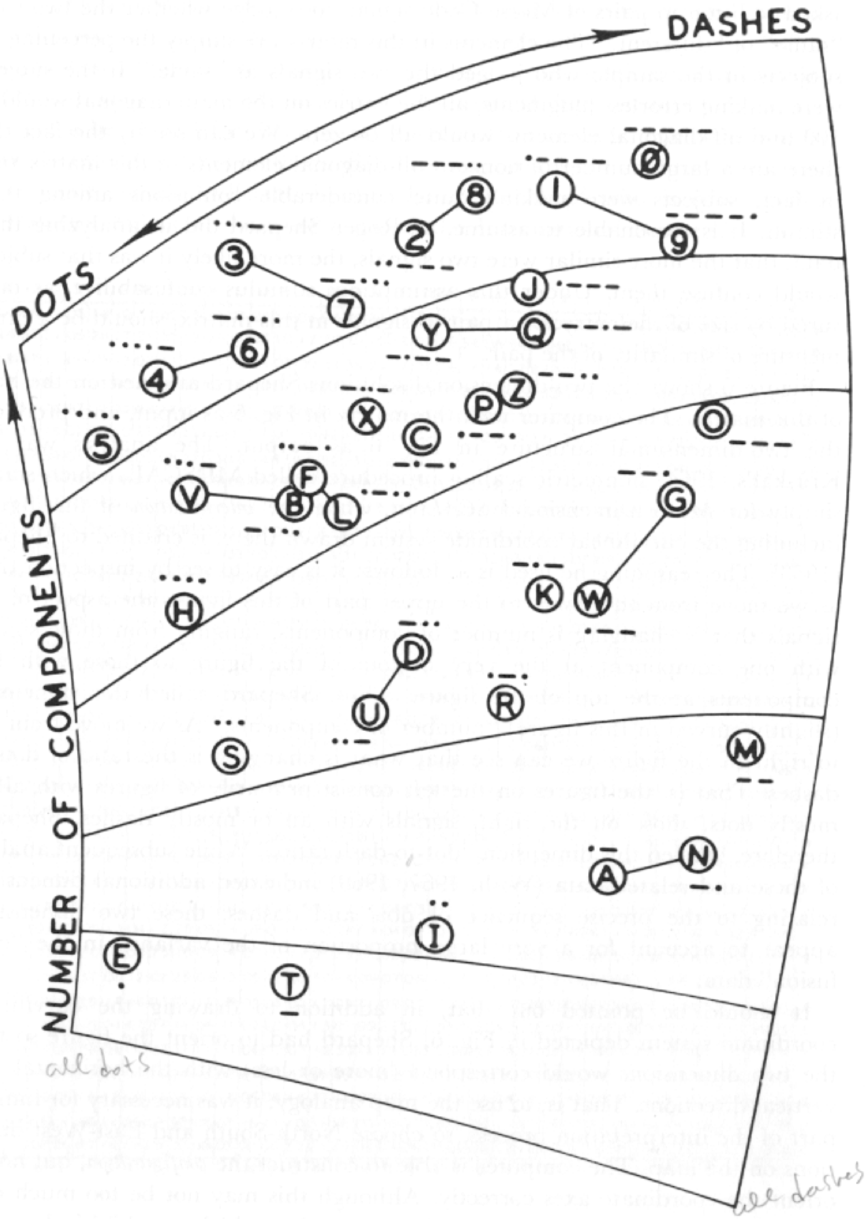

Fig. 6. Two-dimensional configuration resulting from multidimensional scaling analysis of Rothkopf's Morse Code confusabilities (analysis by Shepard, 1963).

dimensional graphical configuration determined from scaling this matrix tells us a great deal. We know immediately that stimuli that are closer together in this two-dimensional space tend to be confused more often Not only does this geometric picture summarize important aspects of the data, but it also tells us the psychological dimensions (or "cues") on which these confusions are base

In this example we see illustrated dramatically two principal advantages of multidimensional scaling: a parsimonious and economical condensation of data, on the one hand, and insight into the psychological dimensions, factors, or "cues," underlying perception of the stimuli on the other. Nonmetric methods allow us to relax our assumptions about the relation between data and distances in the postulated underlying space, and at the same time vastly expand the variety and generality of data that can be considered for multidimensional scaling analysis. This "nonmetric breakthrough" has had a tremendous impact on the field theoretically, even though we are increasingly finding that for purposes of practical data analysis the distinction between metric and nonmetric methods is not as important as it once appeared. The nature of this "practical" insight is that, once one has correctly inferred the

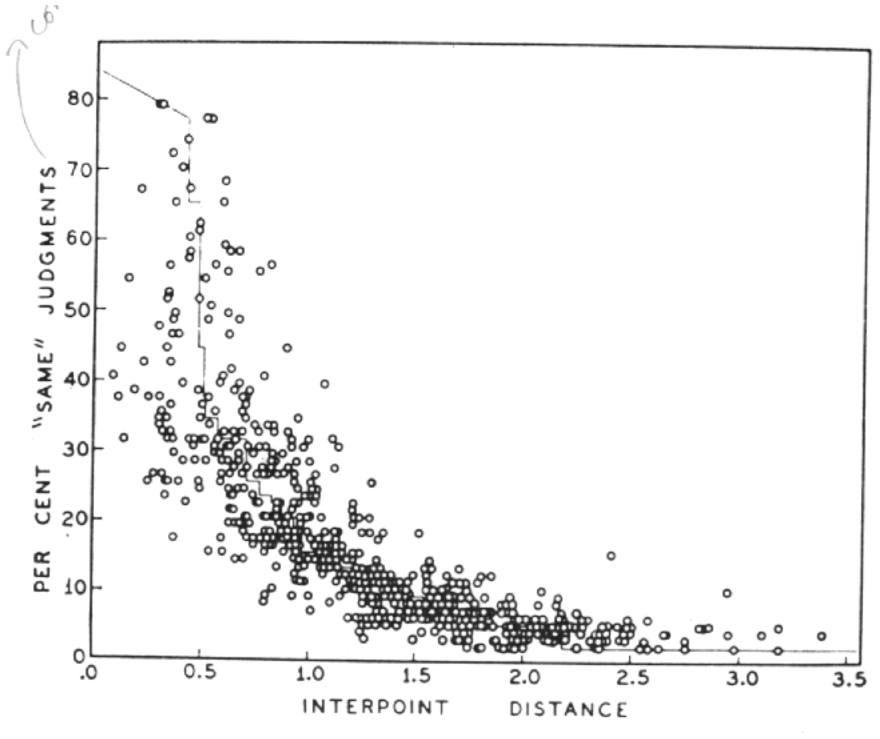

Fig. 7. Relation between mean percent "same" judgment and interstimulus distance in the derived spatial configuration for the Rothkopf data analyzed by Shepard.

appropriate *dimensionality* for the analysis, the difference between metric and nonmetric solutions for the same data is not usually very great. To use statistical terminology, we can say that the metric methods are surprisingly "robust" against a wide variety of violations of the assumptions on which they are based. It required the development of the nonmetric methods, however, to provide the comparison that has made this degree of robustness apparent.

Individual Differences in Perception and the INDSCAL Model

Everyone knows that people are different. If they weren't, the world would certainly be very dull. Worse still, it would not be very functional! That people have different abilities, personalities, attitudes, and preferences is well accepted, and psychologists have been concerned for some time with their measurement. Measurement of individual differences in ability comes under the heading of intelligence, aptitude, or achievement testing. Differences in personality and attitudes are assessed by personality or attitude inventories. Quantitative methods for measuring individual differences in preferences have been developed only recently, and will be described somewhat at a later point.

Most people would agree that people differ in the aspects of human behavior we have so far discussed. One faculty, it would seem, in which people do not differ is that of perception. Surely, if we human beings have anything at all in common, it must be our perceptions, for the way we see the world forms the basis for all the rest of our behavior. If we perceive different worlds, can we in any sense be said to live and behave in the same world? Would not communication between different people be impossible and all social relations reduced to chaos? The answer, however, to which modern psychology is increasingly being led is that people do, indeed, differ in this all-important faculty, although there is necessarily a certain communality cutting across these differences in perception.

To produce an obvious example of individual differences in perception, consider the case of the color-blind individual, who must certainly perceive colors differently from most of the rest of us. A man who is red-green color-blind simply does not perceive a certain dimension of colors along which people with normal vision can clearly discriminate. He would judge two colors as very similar or identical that others would see as highly dissimilar. A man who is weak in red-green perception, but not totally red-green blind, would lie somewhere between the two extremes. To take another example, a tone-deaf individual perceives sounds differently from a musically inclined person. To take these examples to their logical extremes, a blind or deaf person- most certainly perceives differently from a person with normal vision and hearing.

But these examples seem to concern obvious *deficiencies* in the peripheral equipment, the eyes or ears in these cases, which carry sensory signals to the brain. Surely, it will be objected, if no signal, or a diminished signal, gets through to the central nervous system, the individual must perceive-: in an impoverished way. Given the same signal from the peripheral receptors, the eyes, ears, nose, tongue, tactile, and kinesthetic receptors, two individuals must perceive the same way! Or must they?

There is mounting evidence that this last assertion is not true. Granted that it is difficult, and sometimes impossible, to "parcel out" perceptual effects between peripheral receptors and central nervous system processing of received signals (even taking into account complex interactions between the two such as central nervous system feedback to, and control of, incoming. signals from receptor organs), there remains strong evidence for individual differences in perception that are purely a function of central activities.

In the field of comparative linguistics this fact has been embodied in the so-called "Whorfian hypothesis," formulated by the famous linguist Benjamin Lee Whorf. Stated in oversimplified terms, this hypothesis says that many of the differences between distinct languages, of the kind that often make it difficult to find a strictly one-to-one translation of important concepts and propositions from one language to another, are due to the fact that the languages are in fact talking about somewhat different perceptual worlds. This might well be a reflection of the fact that various perceptual dimensions have different relative salience, or perceptual importance, for different linguistic communities or cultural groups. An example of this was reported by the anthropologist H. C. Conklin a few years ago. A Pacific tribal group called the Hanunóo, at first thought to be color-blind because they named colors quite differently from Westerners, proved on closer observation not to be color-blind at all, but to be emphasizing different dimensions in making color judgments. Their judgments rely much more heavily on brightness, for example, than do ours, so that a light green and a bright yellow are given the same name, but both are named differently from darker shades of the same hues. On the other hand, their color-naming behavior seems to reflect hardly at all what we usually think of as hue differences (that is, differences related to the physical spectrum). This is so even though there appears to be no physiological impairment of vision among members of this tribe.

We might suppose that such differences in naming of colors or other classes of stimuli, whether at the cultural or at the individual level, are correlated with differences in perception. Both kinds of differences could result in part from particular aspects of individual or cultural histories that have made some dimensions relatively more important for one person or group; than for another. In the case of the Hanunóo, for example, their idiosyncrasies in color-naming seem to be related to the tribe's food-seeking behavior. Possibly some dimensions of color are more important than others for discriminating food from nonfood items, or differentiating among different kinds of foods in their island environment. Our interest, however, will center not so much on how perceptual differences may have developed, but on their general nature and how to_measure such differences, assuming they exist.

Our approach to measuring individual differences in perception assumes that the differences are reflected in the way people make judgments of relative similarity of stimuli. It thus involves a model for individual differences, the INDSCAL model, which falls in the class of multidimensional scaling models for similarity (or other dyadic) judgments. As a matter of fact, the INDSCAL procedure was the one used to analyze the data on similarities between nations.

The INDSCAL model (Carroll and Chang, 1970) shares with other multidimensional scaling models the assumption that for each individual, similarities judgments are (inversely) related to distance in the individual's perceptual space. Each individual is assumed to have his own "private" perceptual space, but this is not completely idiosyncratic. Rather, a certain communality of perceptual dimensions is assumed, but this communality is balanced by diversity in the pattern of relative salience, or weights of these common dimensions. The salience of a perceptual dimension to an individual can be defined in terms of how much of a difference (perceptually) a difference (or change) on that dimension makes to the individual in question. For our hypothetical red-green color-blind individual a change all the way from red to green makes little or no perceptual difference, while a normal individual would easily detect a much smaller change, say from a brilliant red to a slightly less saturated reddish-pink. Perhaps a color-weak individual could not distinguish this difference, but could distinguish between a bright red and a very desaturated pink (that is, a greyish-pink).

The **INDSCAL** model handles this notion of differential salience of common dimensions by assuming that each individual's "private" space is derived from a common, or "group," space by differentially stretching (or shrinking) the dimensions of the group space in proportion to the subject's weights for the dimensions] The weights of the dimensions for each subject can be plotted in another "space," called the "subject space," in which the value plotted for a given individual on a particular dimension is just the stretching or shrinking factor, for that individual on that dimension (for technical reasons it is actually the square of that stretching factor that is plotted). A value of zero means that the dimension is shrunk literally to the vanishing point, which is to say that he just doesn't perceive the attribute corresponding to that dimension, or, in any case, "acts like" he doesn't perceive it.

After applying the stretching and shrinking transformations, as defined by the "subject space," each individual is assumed to "compute" psychological distances in this transformed space. His similarity (or dissimilarity) judgments are then assumed to be monotonic with the distances thus "computed." Of course, all of these statements about the INDSCAL model should be regarded merely as "as if" statements; that is, the individual acts "as if" he had gone through these steps. We do not literally believe, for example, that distances are computed and a monotone transformation applied to derive similarity values, only that there are psychological processes that have the same final effect "as if" these operations were performed.

Since we have been frequently invoking the example of color blindness in discussing individual differences in perception, it might be of interest to see how the INDSCAL method actually deals with data on color vision. Fortunately, data of the appropriate kind were collected by Dr. Carl E. Helm, now of the City University of New York, who asked subjects directly to judge psychological distances between ten colors. They did so by arranging triples of color chips into triangles so that the lengths of the three sides were proportional to the psychological distances between the colors on the chips at the vertices. By having subjects do this for every triple, Helm was able to construct, for each subject, a complete matrix of psychological distances. Helm's sample included four subjects who were deficient (to a greater or less extent) in red-green color vision. We applied the INDSCAL method to Helm's data, and obtained interesting results that serve very nicely to illustrate the INDSCAL model. The "group" stimulus space in Fig. 8 nicely reproduced the well-known "color circle" (as would be expected, since Helm used Munsell color chips selected to be constant in brightness, and very nearly constant in saturation) going from the violets and blues through the greens and yellows to the oranges and reds, with the "nonspectral" purples between the reds and the violets. Basically, the fact that colors of constant brightness and saturation are represented on a circle rather than on a straight line, as in the case of the physical spectrum, reflects the fact that violet and red, which are at opposite ends of the physical continuum, are more similar psychologically than red and green, two colors much closer on the physical spectrum, The circular representation also permits incorporation of the so-called "nonspectral" purples, which, psychologically, lie between violet and red. Of course, not all hues were actually represented in Helm's set of color chips, but it is fairly clear where the missing hues would fit into the picture. The precise locations of the two coordinate axes describing this two-dimensional space are of particular interest, since, as we have said, the orientation of axes is not arbitrary in INDSCAL. The extreme points of dimension 1 are a purplish blue at one end and yellow at the other. This dimension, then, could be called a blue-yellow dimension. Dimension 2 extends from red to a bluish green, and thus could be called a red-green dimension.

The "subject space" from the analysis of Helm's data shows that the color-deficient subjects (including one who appears twice because he repeated the experiment on two occasions) all have smaller weights for the red-green dimensions than do the normal subjects. Furthermore, the order of the weights is consistent with the relative degrees of deficiency, as reported by Helm, except for one small reversal (subject CD-3 was least deficient by Helm's measures, and CD-4 second least deficient, while the weight for CD-4 on dimension 2 is slightly higher than that of CD-3).

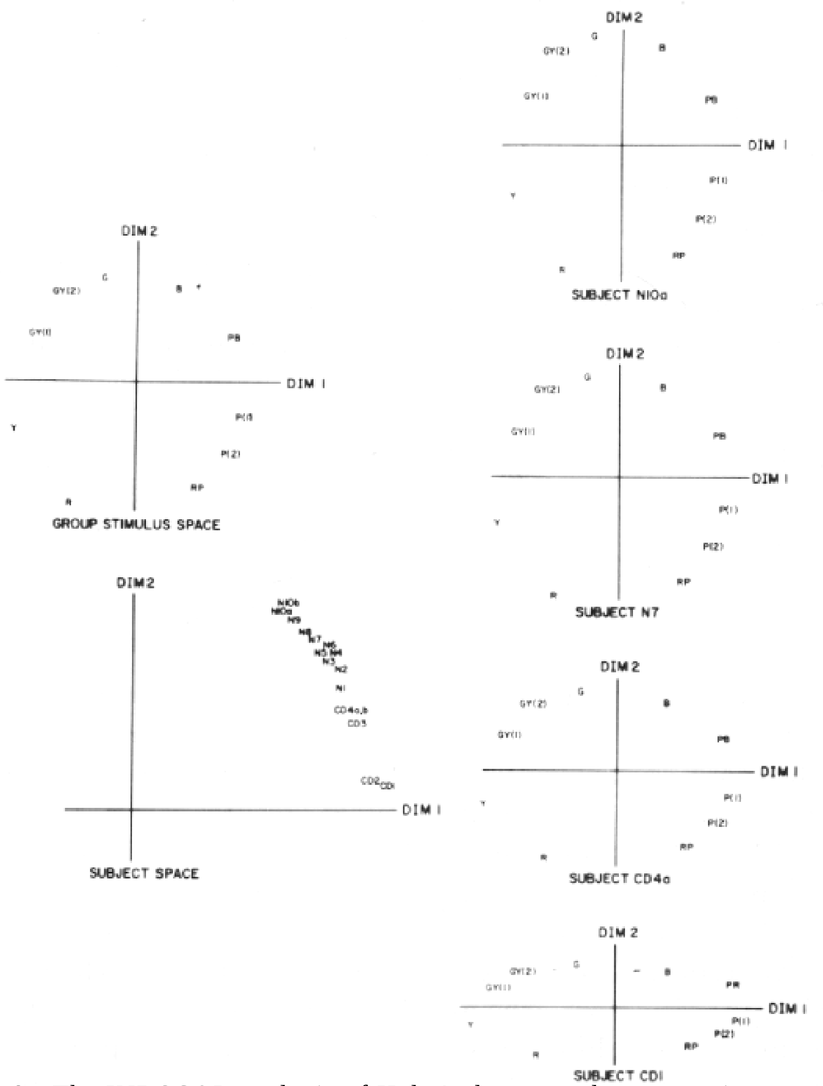

Fig. 8. The INDSCAL analysis of Helm's data on color perception produced the "group" stimulus space shown in A, which conforms quite well with the standard representation of the color circle, and the "subject space" shown in B. The coding of the stimuli is as follows: R=Red, Y=Yellow, GY(1)=Green Yellow, GY(2)=Green Yellow containing more green than GY(1), G=Green, B=Blue, PB=Purple Blue, P(l)=Purple, P(2)=a Purple containing more red than P(1), RP=Red Purple.

INDSCAL also allows construction of a "private" perceptual space for each individual. If we construct such a space for each of these subjects, we find that, while for normal subjects the "color circle" looks pretty much like a circle, for the color-deficient subjects it looks more like an ellipse whose major axis is the blue-yellow dimension and whose minor axis is the red-green dimension (the more deficient the subject the more elliptical --- that is, the smaller the ratio of minor and major axes). Of course, there is considerable variation among the "normal" subjects themselves, with the more extreme (such as N-10) bordering on blue-yellow color deficiency (which is much rarer than red-green deficiency). For such an extreme individual the color circle is deformed into an ellipse whose major axis corresponds to the red-green dimension. The strength of the INDSCAL model and method as revealed in this analysis is that it accommodates all these perceptual variations among subjects in a very parsimonious way, and at the same time tells us a great deal about color vision, which, if not precisely new and unexpected, confirms in an elegant way what has been worked out over the years by vision theorists in much slower and more tedious ways.

Another example which illustrates particularly well the uniqueness property of INDSCAL dimensions (that is, the fact that one does not generally have to rotate axes to obtain an interpretable solution in terms of presumably fundamental perceptual dimensions) involves some data on perception of acoustical "tone" stimuli collected by Bricker and Pruzansky. Interest in the particular stimuli used in this experiment was generated in part by the fact that they were among possible candidates for "tone ringers," which will eventually replace the mechanical "bell ringers" of the present day in the all-electronic telephone of the future. This particular study is part of a larger project in which the perceptual dimensions of a wide range of acoustical stimuli are being studied and related to such other qualities as subjective preference, "attention-getting" properties, and the like. This project is bringing to bear a large number of multidimensional scaling and related techniques on the question of optimal selection of the sound which will ultimately become as familiar and omnipresent as today's telephone bell. It will help guide the decision on the "tone ringer" ultimately chosen, and also that of whether controls should be provided to allow individual telephone users to modify the sound of their ringers, and, if so, of what controls should be provided (i.e., what variables or dimensions should be modifiable). The stimuli in the present case were tones that varied in a systematic way in three well-defined physical dimensions. These were modulation frequency, for which there were four values (5; 10, 20, and 40 hertz), modulation percentage, with three values (3 percent, 10 percent, and 25 percent), and modulation waveform, which took on two values (sine wave vs. square wave). All combinations of values of these three physical dimensions were used, making a total of 4 x 3 x 2 = 24 stimuli.

Since there were three physical dimensions, it should not be too surprising if there turned out to be three psychological dimensions corresponding more or less in a one-to-one fashion (or forming the "psychological concomitants") of the physical dimensions. It would be surprising if the "psychophysical mapping" from physical to psychological dimensions were perfectly linear, but on the other hand, it would be even more surprising if there were no correspondence at all between the two domains.

In fact an INDSCAL analysis did reveal three psychological dimensions underlying Bricker and Pruzansky's data, and these corresponded in a highly regular fashion to the physical dimensions (although the structure of the psychological space did reveal some small but systematic distortions reflecting interactions between these dimensions). The high degree of correspondence between the two kinds of dimensions was revealed by use of a mathematical technique for finding lines defining directions in the three-dimensional psychological space corresponding optimally to each of the physical dimensions. Although this technique placed no constraint at all on the locations of these lines, they tumed out to correspond in an essentially one-to-one fashion to the coordinate axes of the "group" stimulus space from INDSCAL. This correspondence could be measured quantitatively by the fact that the angles between the "psychological" coordinate axes and the lines corresponding to the best physical concomitant of each psychological axis were all less than 15 degrees (the exact angles were 8.1, 5.7, and 14.5 degrees for modulation frequency, percentage, and waveform respectively), while the degree of correspondence of each physical dimension to its psychological concomitant was very high (even though the relations were somewhat nonlinear, the linear correlation coefficients were .956, .969, and .854 respectively, where a value of 1.0 represents a perfect linear relation). A more detailed discussion of this study, including graphical displays of the results, can be found in Carroll and Chang (1970).

This property of unique orientation of axes in the case of INDSCAL solutions is very important, for it means that the problem of rotation of axes that usually arises in interpreting multidimensional scaling solutions can often be avoided by reliance on individual differences to orient the "psychological axes" uniquely.

In these two examples, the perceptual dimensions of the stimuli were pretty well known beforehand, either through a long history of past study of the stimuli (in the case of the colors) or because the stimuli were actually generated by systematically manipulating some well-defined physical dimensions (in the case of the acoustical stimuli) It should be emphasized, however, that in neither case was this a priori knowledge about the dimensions "communicated to" the computer program that performed the INDSCAL analysis The program "saw" only the raw data on similarity judgments of the various subjects, and constructed both the stimulus and subject spaces on the basis of this information alone. We will now discuss an example by Jones and Young (like the application involving nations) in which much less was known about the stimuli on an a priori basis.

The Jones and Young (1972) application should be of great interest to potential Delphi users, since it can be viewed as paving the way to a new approach to sociometry. Jones and Young were both at the Psychometric Laboratory of the University of North Carolina in 1969 and 1970, when this study was carried out. An interesting aspect of the study is that the "stimulus set" was the same as the "subjects"; namely, members of the Psychometric Laboratory (faculty, students, and staff). A subset of these members, acting as subjects or judges, were asked to judge the similarity of another subset of these members, and also to complete various rating scales relating to perceived interests, status, and the like, as well as to provide standard sociometric information. (The stimulus subset was actually contained in the subject subset, so that all stimulus persons were also judges, but not vice versa.) INDSCAL analyses were done separately for the similarity data collected in 1969 and 1970, as well as a composite analysis for the combined data from the two years, (restricted to those stimulus persons included in the stimulus sample both years). The rating scale and similarities data were used as aids, in interpreting the resultant dimensions.

The main results were exceptionally clear and consistent. Three dimensions emerged from the data for both years, and these agreed remarkably in the independent analyses of the data from the two years. The first dimension in both cases was, unambiguously, a status dimension (it correlated in the mid-to-high nineties with both rated "perceived status" and with academic rank). The second dimension was called "Political Persuasion"-it correlated highly with perceived position along a "left-right" or liberal-conservative political scale, and also with a lifestyle variable contrasting "unconventional" with "conventional" modes of dress, hair, behavior patterns, and the like. A third dimension had to do with "Professional Interests," and primarily contrasted those whose interests were statistical or methodological with those who were primarily concerned with specific content areas.

As with Wish's nation study, a very interesting pattern of subject weights emerged from this study. The subjects, as displayed in the subject (weight) space, fell into two clearly discernible clusters --- one consisting of faculty members and the other of students. The faculty members tended to put much more weight on the "Status" dimension than on the "Political Persuasion" dimension, while the students put about equal weight on the two. The student and faculty groups did not differ in the weight for the "Professional Interests" dimension.

One notable aspect is that those who *had* greater status (i.e., were higher on the "Status" dimension in the stimulus space) put more weight on that dimension (i.e,, were higher on the "Status" dimension in the *subject* space). Thus, in this sociometric application, where the stimulus and subject spaces may contain the "same" objects, there is reason to believe that the positions of these objects on at least this dimension (and quite possibly on other dimensions as well) will be correlated. The other aspect of this study that bears notice is the fact that "Status" did, indeed, come out as the first (and by far the strongest) dimension. While it may be a bit speculative to assume this will always be so, if it were this might suggest a very "neutral" way to measure status in a group-without ever asking for a direct judgment of status! That is, simply ask group members to judge similarity among themselves, and then take the first dimension from an INDSCAL analysis as a direct measure of status (we stress that this is only speculative at the moment, and would certainly not want to assume this would work in general without a great deal of additional study).

As was mentioned at the beginning of the paper, multidimensional methods can also be used to analyze preferences of other kinds of "dominance" data. There are two different general kinds of multidimensional preference analysis, called by Carroll (1972) *internal* and *external.* In an internal analysis the positions of stimulus points and the directions of "subject" vectors are determined from the preference data alone. In an external analysis the stimulus points are given a priori (say from an MDS analysis of similarity or other dyadic data) and the vectors, ideal points, or other entities characterizing subjects are "mapped" into this predefined stimulus space. Such an "external" analysis called for mapping dominance judgments (vis-á-vis, say, probable futures of nations) into MDS spaces derived, say, from similarities data For a complete discussion of multidimensional preference analysis, the reader is referred to the 1972 paper by Carroll (in the book edited by Shepard, Romney, and Nerlove [1972]).

A study by McDermott (1969) illustrates an internal analysis of preference data. In this study thirty-one subjects listened to speech transmitted over twenty-two simulated "telephone circuits" that had been subjected to various linear and nonlinear distortions. A preference ordering for the twenty-two circuits was determined for each subject on the basis of preference comparisons involving all possible pairs of circuits. The data were analyzed in terms of a "vector model" whose solution simultaneously provides information about the psychological factors underlying the stimuli and about the extent and nature of individual differences among the subjects. This model assumes that the stimuli (the circuits) can be represented as points in a multidimensional space, and that the subjects can be represented as vectors in the same space. A subject's preference ordering of the stimuli is assumed to correspond to the rank order of the projection of the stimulus points onto that subject's vector. The axes of a properly chosen coordinate system describing this multidimensional space can be thought of as corresponding to the psychological factors, or dimensions, along which the stimuli vary. The cosines of angles of a subject's vector with these coordinate axes measure the importance of that dimension for that subject's preference judgments. This vector model is illustrated in Fig. 9, in which the preference

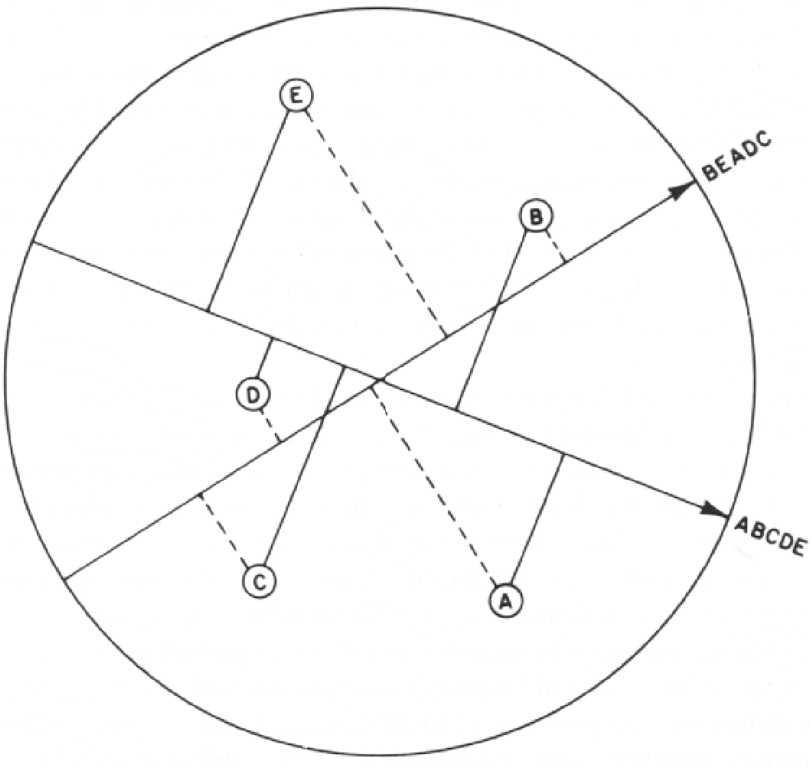

Fig. 9. Illustration of the vector model for preferences in a simple two-dimensional case. Five stimulus points are represented by the five letters A through E, and two subjects are represented by two vectors. Perpendicular projections of stimulus points onto a subject's vector are assumed to generate the preference order for that subject. Thus the first subject, for whom the lines of projection are represented as solid lines, has preference order ABODE (that is, he prefers A to B, prefers B to C, and so on). The second subject, whose projection lines are dotted lines, has preference BEADC. Many more preference orders could be accommodated, even in this simple two-dimensional case, by allowing additional vectors pointing in still other directions.

orderings of two hypothetical subjects for five hypothetical stimuli are shown.

Three dimensions, which were interpreted as "Degree of distortion in signal", "Degree of distortion in background," and "Loudness level" accounted very well for the preference judgments of all subjects. Although subjects preferred the circuits that had the least distortion, they differed with

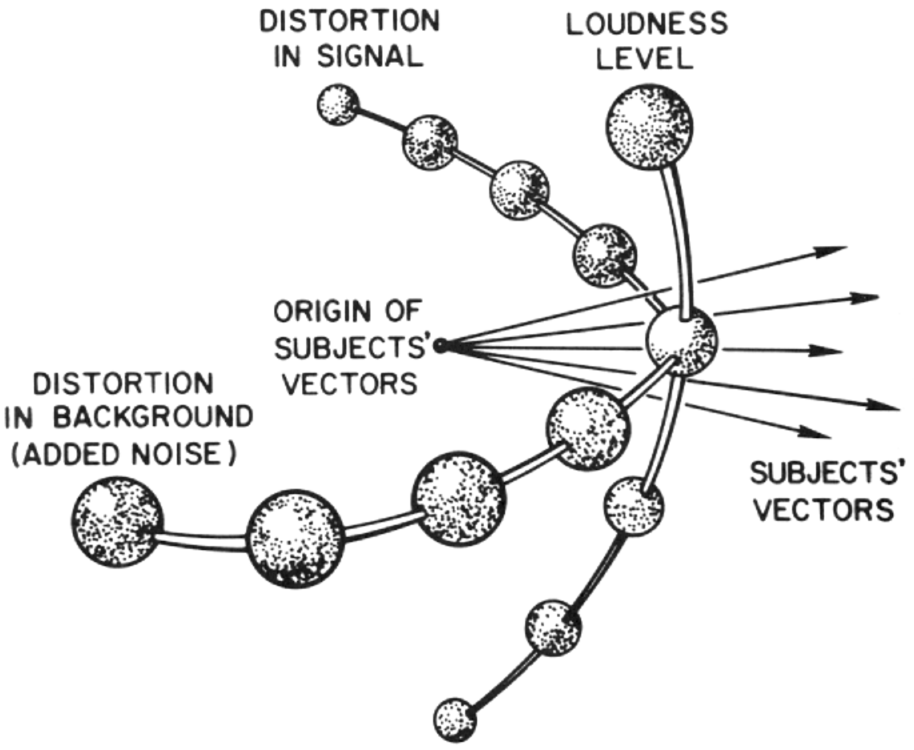

Fig. 10. A schematic representation of a subset of stimuli and subjects from the three-dimensional "vector model" analysis of McDermott's circuit quality data. The order of perpendicular projections of the stimulus points on a subject's vector reflects that subject's preference ordering.

regard to the relative importance of signal and background distortion as well as in their preferred loudness level. The schematic diagram of the three-dimensional solution in Fig. 10, which shows only a subset of the twenty-two stimuli and only a few of the subject vectors, should make this interpretation clearer.

This "vector model" is only one of a large class of possible models for individual differences in preference, and is, in fact, the simplest model in a number of ways. Another model, often called the "unfolding model," postulates a different "ideal" stimulus for each subject, and assumes that preference decreases monotonically with increasing Euclidian distance from the "ideal." (The "vector" model" is actually a special case of this, since it is equivalent to an "unfolding model" with "ideal" points infinitely distant from the origin.) Other models generalize the "unfolding model" by assuming, for example, different saliences for dimensions as well as different "ideal" points, or even that there is more than' one "ideal," or highly preferred, region.

One thing that should be emphasized in summarizing analyses discussed, in this paper is that the solutions were obtained by use of computer programs that were *in no* explicit way informed or instructed as to what the psychological (or physical) dimensions were. In the case of McDermott's analysis, for example, the computer was merely programmed to construct a three-dimensional solution involving stimulus points and subject vectors so as to account as well as possible, in a well-defined statistical sense, for the preference data from the thirty-one subjects (via the computer program, by Carroll and Chang, called MDPREF). As is true of all multidimensional scaling solutions, the computer produces the "picture" while the user furnishes the interpretation. This process of interpretation often involves what is termed "rotation of axes," which simply means that, out of the infinite number of coordinate systems that could be used to describe the same multidimensional configuration, that particular one is sought that most nearly corresponds to interpretable or "meaningful" psychological dimensions. This process of rotation is analogous to orienting a map in terms of a north-south and east-west (rather than, say, a northeastsouthwest and southeast-northwest) coordinate system. The problem of rotation to achieve maximum interpretability is one that often enters into, and sometimes complicates, applications of multidimensional scaling. As we saw earlier, Shepard had to rotate the two-dimensional Morse Code solution to achieve the interpretable solution he found. McDermott put in a great deal of time and effort to find the best orientation of axes for the three-dimensional circuit quality solution just discussed (in fact, this was by far the most difficult aspect of her analysis). It is, in fact, one of the principal virtues of INDSCAL that this rotational problem can usually be circumvented when this analytic technique is used. INDSCAL capitalizes on individual differences among subjects in similarity or other dyadic judgments to orient the coordinate axes uniquely. This property, we believe, makes it the method of choice whenever data of the right kind are available.

It would appear that both the Delphi approach and multidimensional scaling bear enormous promise for the future. While these two methodologies have much to offer separately, *in combination* they portend no less than a revolution in the study and rationalization of individual and group information processing and decision analysis. A great many of the fundamental topics examined in Delphi studies lend themselves to the MDS treatment. How similar or related are various alternative goals, objectives, values, plans, and programs? In essence, MDS can be used to determine if there exists for the Delphi group a morphology which clarifies the factors underlying heir views on the subject at hand. The exposing of these factors, it is hoped, would allow such groups to come to a better collective understanding of their problem and thereby promote results of greater utility.

ANNOTATED BIBLIOGRAPHY[1]

I. Books

I. 1 Torgerson, W. *S. Theory and methods of scaling.* New York: Wiley, 1958 (particularly Chapter 11). This book is devoted to scaling in very general sense; and most of it would be of primary interest to experimental or theoretical psychologists. Chapter 11, however, deals exclusively with the "classical" metric method of multidimensional scaling (based in large measure on papers published. circa 1938 by Eckart, Young, Householder, and Richardson) with which Torgerson's name is associated.

I. 2 Coombs, C. H. A *theory of data.* New York: Wiley, 1964. This book is primarily of interest to theoretical psychologists and other social scientists. It deals with models more than methods, and many of the actual methods discussed have now been supplanted by more advanced computer-implemented procedures. It discusses the earliest form of nonmetric multidimensional scaling, which was invented by Coombs and his colleagues (in which not only the data, but also the solution is nonmetric). It also describes such highly useful models as the "unfolding" model of personal preference. The "Theory of Data" to which the title refers is a classification scheme for categorizing geometrically based measurement models of the kind assumed in MDS. This classification system may provide a useful conceptual tool for many readers. It is especially recommended to those interested in the theoretical and geometrical underpinnings of MDS models.

I. 3 Green, P. E., and Carmone, F. J. *Multidimensional scaling and related techniques in marketing analysis*. Boston: Allyn and Bacon, 1970.

I. 4 Green, P. E., and Rao, V. R. *Applied multidimensional scaling.* New York: Holt, Rinehart and Winston, 1972.

I. 5 Green, P. E., and Wind, Y. (with contributions by J. D. Carroll). *Multi-attribute decisions in marketing.* New York: Dryden Press, 1973.
These three books, all having Paul E. Green of the Wharton School as first author, are fairly nontechnical, with emphasis on marketing applications. The first (Green and Carmone) may be the best book presently available to provide a general overview of the field to the nonspecialist (particularly one in marketing research or other fields of business or finance). It has a very long and helpful bibliography (although both the book and the bibliography may be somewhat out of date by now, owing to the rapid pace of developments in this field). The second (Green and Rao) is a kind of practitioner's guide to selection and comparison of algorithms. It uses the pedagogic device of applying virtually all the (then available) methods

[1] We thank Dr. Joseph B. Kruskal for his permission to borrow extensively from a previous jointlyauthored unpublished annotated bibliography on MDS for which he was principally responsible.

to a single core data set, so as to show directly the comparative virtues and weaknesses of the various methods. The third book (Green and Wind) focuses on "conjoint measurement" approaches, particularly to studies of "multiattribute decision making," but also uses a considerable amount of MDS methodology. It includes an appendix by Carroll that provides mathematical and technical descriptions of many of the most important MDS, conjoint measurement, clustering, and related multivariate analysis techniques (including descriptions of relevant computer programs).

L6 Shepard, R. N., Romney, A. K., and-Nerlove, S. *Multidimensional scaling:Theory and applications in the behavioral sciences. Vol. I: Theory*. New York: Seminar Press, 1972.

L7 Romney, A. K., Shepard, R. N., and Nerlove, S. *Multidimensional scaling: Theory and applications in the behavioral sciences. Vol. II: Applications.* New York: Seminar Press, 1972.

This two-volume set contains fairly up-to-date contributions in both MDS theory and applications to many fields. Theoretical or methodological papers (in Vol. I) that should be particularly of interest include: Shepard's introduction and chapter on taxonomy of scaling models and methods; Lingoes' paper surveying the Guttman-Lingoes, series of programs for nonmetric analysis; Young's paper on "polynomial conjoint analysis of proximities data"; Carroll's paper on individual differences, which discusses both the INDSCAL model for individual differences in similarities and other dyadic data, and a wide variety of models and methods for individual differences in preferences or other dominance judgments; Degerman's paper on hybrid models in which discrete clustering-like structure is combined with continuous spatial structure. Some of the applications chapters of major interest include: Wexler and Romney on kinship structures in anthropology; Rapoport and Fillenbaum on semantic structures; Rosenberg and Sedlak on perceived trait relationships; two different approaches (by Green and Carmone, on the one hand, and Stefflre, on the other) to marketing applications and the work of Wish, Deutsch and Biener on nation perception (which takes up where the "nation" study discussed here leaves off).

L8 Carroll, J. D., Green, P. E., Kruskal, J. B., Shepard, R. N., and Wish, M. Book in preparation, based generally but not slavishly on Workshop on MDS held at University of Pennsylvania in June 1972. While book is not yet available, handout material for workshop can be obtained by writing to authors. Title and publisher not yet certain.

II. Basic Theory (Published Papers)

II. 1 Torgerson, W. S. Chapter 11 of Theory and Methods of Scaling. (See Books: L1. for description.)

II. 2 Shepard, R. N. "The analysis of proximities: Multidimensional scaling with an unknown distance function. I." Psychometrika, 27, (1962),

pp. 125-39(a). II. 3 Shephard, R.N. "The analysis of proximities: Multidimensional scaling with an unknown distance function. II." Psychometrika 27, (1962), pp. 219-46. (b).

These two articles by Shepard focus on the rank-order relationship between data and distances, and show it is often enough to determine the solution. They introduce what Guttman later called the "antirank-image principle" for estimating the relationship between distances and similarities.

II. 4 Kruskal J. B. "Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis." Psychometrika, 29, (1964), pp. 1-27. (a)

II. 5 Kruskal, J. B. "Nonmetric multidimensional scaling: A numerical method." Psychometrika 29, (1964), pp. 115-29. (b)

These two articles focus on goodness-of-fit to explicit assumptions. They introduce computational methods from numerical analysis for systematically minimizing such a function. They introduce least-squares monotone regression for estimating the relationship between distances and similarities.

II. 6 Guttman, L. "A general nonmetric technique for finding the smallest coordinate space for a configuration of points." Psychometrika, 33, (1968), pp. 469-506. Introduced rank-image principle and goodness-of-fit functions based on it. Introduced new computational methods.

II. 7 Carroll, J. D. and Chang, J. J. "Analysis of individual differences in multidimensional scaling via an N-way generalization of 'Eckart-Young' decomposition." Psychometrika, 35, (1970), pp. 283-319. Introduced the generalization of multidimensional scaling called "individual differences scaling" (now called INDSCAL) which permits different data matrices to reflect different weights for the various axes. Introduced computational method which generalizes classical scaling, and is the basis for the INDSCAL method. Also includes discussions of analyses of the Bricker and Pruzansky "tone-ringer" data and of the Wish nation data described in the present paper.

III. Other Theoretical, Methodological, and Practical Contributions (Published Papers)

III. 1 Shepard, R. N. "Analysis of Proximities as a technique for the study of information processing in man." *Human Factors, 5, (1963), pp. 33-48. Dis*cusses among other applications of MDS, the analysis of Rothkopf's Morse Code data used as an illustration of two-way MDS in the present paper.

III. 2 Tucker, L. R. and Messick, S. "An individual differences model for multidimensional scaling." *Psychometrika, 28, (1963), pp. 333-67.* Given several matrices of dissimilarities, this approach uses factor analysis to cluster the matrices, and then analyzes an average matrix for each cluster.

The relationship of this method to Carroll and Chang's individual differences scaling is discussed in their paper. (See Basic Theory II.7.)

III. 3 McGee, V. W. "The multidimensional analysis of "elastic" distances." *British Journal of Mathematical and Statistical Psychology, 19, (1966), pp. 181-96.* Introduces a kind of scaling similar to the Shepard-Kruskal approach, but with the squared differences multiplied by weights. Very similar results can be accomplished with MDSCAL, using the weights as permitted there.

III. 4 Shepard, R. N. "Metric structures in ordinal data." *Journal of Mathwwtical Psychology, 3, (1966), pp. 287-315.* Discusses general approach to deriving metric (i.e., numerical) information from nonmetric (i.e., merely ordinal) data. Nonmetric scaling and nonmetric factor analysis provide illustrations. Some early, but quite useful, Monte Carlo results are included.

III. 5 Shepard, R. N., and Carroll, J. D. "Parametric representation of nonlinear data structures." In P. R. Krishnaiah (ed.), *Multivariate analysis.* New York: Academic Press, *1966, pp. 561-92.* Introduces two new ideas. One, called "parametric mapping," relies on smoothness of mapping: the other is scaling based on a merely local monotonicity.

III. 6 Krantz, D. H. "Rational distance functions for multidimensional scaling." *Journal of Mathematical Psychology 4, (1967), pp. 226-45.*

III. 7 Beals, R. W., Krantz, D. H., and Tversky, A. "The foundations of multidimensional scaling." *Psychological Review, 75, (1968), pp. 127-42.*

III. 8 Tversky, A. and Krantz, D. H. "The dimensional representation and the metric structure of similarity data." *Journal of Mathematical Psychology, 1, (1970), pp. 572-96.*

These three articles are concerned with theoretical examination of the abstract mathematical foundations of multidimensional scaling.

III. 9 Roskam, E. I. *Metric analysis of ordinal data in psychology.* Voorschoten, Holland: University of Leiden Press, 1968. A general overview and comparison of MDS methods and models up to 1968, with some original methodological contributions.

III. 10 Kruskal, J. B. and Carroll, J. D. "Geometrical models and badness-of-fit functions." In P. R. Krishnaiah (ed.), *Multivariate Analysis. II.* New York: Academic Press, 1969, pp. 639-70. Properties and alternative choices for badness-of-fit, for scaling, parametric mapping, multidimensional unfolding, and tree structure. New theoretical results on stress function for scaling.

III. 11 McDermott, B. J. "Multidimensional analyses of circuit quality judgments." *The Journal of the Acoustical Society of America,* 45, (1969), p. 774. Contains a full description of the circuit quality study described very superficially in this paper.

III. 12 Carroll, J. D. "An overview of multidimensional scaling methods emphasizing recently developed models for handling individual differences." In C. W. King and D. Tigert (eds.), *Attitude Research*

Reaches New Heights. Chicago: American Marketing Association, 1971, pp. 235-62. A nontechnical overview of MDS models and methods up to 1971, with emphasis on individual differences in perception and preference. Covers much of the same ground as the 1972 Carroll paper "Individual Differences and Multidimen- sional Scaling" in volume I of the Shepard, Romney, and Nerlove twovolume set, but in a much less technical way.

III. 13 Jones, L. E. and Young, F. W. "Structure of a social environment: longitudinal individual differences scaling of an intact group." *Journal of Personality and Social Psychology,* 24, (1972), pp. 108-21. Describes in detail the "sociometric" study using INDSCAL discussed in the present paper.

III. 14 Tucker, L. R. "Relations between multidimensional scaling and three-mode factor analysis." *Psychometrika 37,* (1972), pp. 3-27. Shows how threemode factor analysis can be applied to analysis of individual differences in multidimensional scaling. The implied model is a generalization of the INDSCAL model, but lacks the dimensional uniqueness property of INDSCAL.

III. 15 Carroll, J. D. and Wish, M. "Multidimensional perceptual models and measurement methods." In E. C. Carterette, and M. P. Friedman (eds.), *Handbook* of *Perception.* New York: Academic Press, 1974. A discussion of the geometrical models of MDS viewed as general theories of perception, and of a MDS methods as instruments for measurement of parameters of perceptual systems. Stresses the philosophical and mathematical foundations of MDS, relations to measurement theory, and to the basic theoretical underpinnings of psyehology in general and perception in particular.

III. 16 Indow, T. "Applications of multidimensional scaling in perception." In E. C. Carterette and M. P. Friedman (eds.), *Handbook of Perception.* New York: Academic Press, 1973. Provides a number of applications of two-way MDS to data, principally on color perception and perception of "visual space."

III. 17 Wish, M. and Carroll, J. D. "Applications of INDSCAL to studies of human perception and judgment." In E. C. Carterette and M. P. Friedman (eds.), *Handbook of Perception.* New York: Academic Press, 1974. Provides a wide variety of applications of the INDSCAL technique to areas of perception ranging from perception of color (including the analysis of the Helm data discussed in this paper), consonant phonemes, and "stress patterns" in English, through a very thorough study of nation perception.

III. 18 Carroll, J. D. and Wish, M. Models and methods for three-way multidimensional scaling. In D. H. Krantz, R. C. Atkinson, R. D. Luce, and P. Suppes (eds.), Contemporary Developments in Mathematical Psychology. (Vol. II.: Measurement, Psychophysics and Neural

Information Processing). San Francisco: W. H. Freeman, 1974, pp. 57-105.

Provides a general overview of INDSCAL, IDIOSCAL, Three-Mode Scaling, PARAFAC-2 and other three-way and higher way MDS models and methods, emphasizing the interrelationships among these methods, as well as their relationships to older methods for dealing with individual differences in similarities data, such as the Tucker-Messick "points of view" approach. Also discusses use of these methods for more general multi-way data analysis.