University Physics with Modern Physics

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Chapter 35 Interference



Learning Outcomes

In this chapter, you'll learn...

- what happens when two waves combine, or interfere, in space.
- how to understand the interference pattern formed by the interference of two coherent light waves.
- how to calculate the intensity at various points in an interference pattern.
- how interference occurs when light reflects from the two surfaces of a thin film.
- how interference makes it possible to measure extremely small distances.

Introduction

- Why do soap bubbles show vibrant color patterns, even though soapy water is colorless?
- What causes the multicolored reflections from DVDs?



• We will now look at optical effects, such as interference, that depend on the wave nature of light.



Principle of Superposition

- The term **interference** refers to any situation in which two or more waves overlap in space.
- When this occurs, the total wave at any point at any instant of time is governed by the principle of superposition:

When two or more waves overlap, the resultant displacement at any point and at any instant is found by adding the instantaneous displacements that would be produced at the point by the individual waves if each were present alone.



Wave Fronts From a Disturbance

- Interference effects are most easily seen when we combine sinusoidal waves with a single frequency and wavelength.
- Shown is a "snapshot" of a single source S₁ of sinusoidal waves and some of the wave fronts produced by this source.

Wave fronts: crests of the wave (frequency f) separated by one wavelength λ



The wave fronts move outward from source S_1 at the wave speed $v = f\lambda$.



In optics, sinusoidal waves are characteristic of monochromatic light (light of a single color). While it's fairly easy to make water waves or sound waves of a single frequency, common sources of light *do not* emit monochromatic (single-frequency) light. For example, incandescent light bulbs and flames emit a continuous distribution of wavelengths. By far the most nearly monochromatic light source is the *laser*. An example is the heliumneon laser, which emits red light at 632.8 nm with a wavelength range of the order of ± 0.000001 nm, or about one part in 10⁹. In this chapter and the next, we'll assume that we are working with monochromatic waves (unless we explicitly state otherwise).

Constructive and Destructive Interference

- Shown are two identical sources of monochromatic waves, S₁ and S₂.
- The two sources are permanently in phase; they vibrate in unison.
- Constructive interference occurs at point *a* (equidistant from the two sources).





Figure 35.2 (a) A "snapshot" of sinusoidal waves spreading out from two coherent sources S_1 and S_2 . Constructive interference occurs at point *a* (equidistant from the two sources) and (b) at point *b*. (c) Destructive interference occurs at point *c*.





(b) Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integer number of wavelengths: $r_2 - r_1 = m\lambda$.

 $r_{1} = 7\lambda$ S_{1} $r_{2} - r_{1} = 2\lambda$ S_{2}

(c) Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integer number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$.



$$r_2 - r_1 = m\lambda$$
 ($m = 0, \pm 1, \pm 2, \pm 3, ...$) (constructive
interference, (35.1)
sources in phase)

(35.2)

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$$
 $(m = 0, \pm 1, \pm 2, \pm 3, ...)$ (destructive interference, sources in phase)



Conditions for Constructive Interference (1 of 4)

- The distance from S₂ to point b is exactly two wavelengths greater than the distance from S₁ to b.
- The two waves arrive in phase, and they reinforce each other.
- This is called **constructive interference**.

(b) Conditions for constructive interference: Waves interfere constructively if their path lengths differ by an integer number of wavelengths: $r_2 - r_1 = m\lambda$. $= 2\lambda$



Conditions for Constructive Interference (2 of 4)

- The distance from S₁ to point c is a half-integral number of wavelengths greater than the distance from S₂ to c.
- The two waves cancel or partly cancel each other.
- This is called **destructive interference**.

(c) Conditions for destructive interference: Waves interfere destructively if their path lengths differ by a half-integer number of wavelengths: $r_2 - r_1 = (m + \frac{1}{2})\lambda$. S_1 $= -2.50\lambda$



Conditions for Constructive Interference (3 of 4)

- Shown are two identical sources of monochromatic waves, S₁ and S₂, which are in phase.
- The red curves show all positions where constructive interference occurs; these curves are called antinodal curves.
- Not shown are the nodal curves, which are the curves that show where destructive interference occurs.

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CAUTION Interference patterns are not standing waves In the standing waves described in Sections 15.7, 16.4, and 32.5, the interference is between two waves propagating in opposite directions; there is *no* net energy flow in either direction (the energy in the wave is left "standing"). In the situations shown in Figs. 35.2a and 35.3, there is likewise a stationary pattern of antinodal and nodal curves, but there is a net flow of energy *outward* from the two sources. All that interference does is to "channel" the energy flow so that it is greatest along the antinodal curves and least along the nodal curves.

Conditions for Constructive Interference (4 of 4)

 The concepts of constructive interference and destructive interference apply to these water waves as well as to light waves and sound waves.





Figure 35.5 (a) Young's experiment to show interference of light passing through two slits. A pattern of bright and dark areas appears on the screen (see Fig. 35.6). (b) Geometrical analysis of Young's experiment. For the case shown, $r_2 > r_1$ and both y and θ are positive. If point P is on the other side of the screen's center, $r_2 < r_1$ and both y and θ are negative. (c) Approximate geometry when the distance R to the screen is much greater than the distance d between the slits.

(a) Interference of light waves passing through two slits



Two-Source Interference of Light (1 of 2)

- Shown below is one of the earliest quantitative experiments to reveal the interference of light from two sources, first performed by Thomas Young.
- The interference of waves from slits S_1 and S_2 produces a pattern on the screen.



(a) Interference of light waves passing through two slits



Two-Source Interference of Light (2 of 2)

- (b) shows the actual geometry of Young's experiment.
- If the distance *R* to the screen is much greater than the distance *d* between the slits, we can use the approximate geometry shown in (c).



the distance *d* between the slits ...

(c) Approximate geometry



Interference From Two Slits

- Constructive interference (reinforcement) occurs at points where the path difference is an integral number of wavelengths, $m\lambda$.
- So the bright regions on the screen occur at angles θ for which



 Similarly, destructive interference (cancellation) occurs, forming dark regions on the screen, at points for which the path difference is a halfintegral number of wavelengths.



Video Tutor Solution: Example 35.1

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Figure 35.6 Photograph of interference fringes produced on a screen in Young's double-slit experiment. The center of the pattern is a bright band corresponding to m = 0 in Eq. (35.4); this point on the screen is equidistant from the two slits.

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m (constructive interference, bright regions)	m + 1/2 (destructive interference, dark regions)
	←11/2
$5 \rightarrow$	
	←9/2
4 /	←7/2
3>	← 5/2
$2 \rightarrow$	
1	← 3/2
1-2	← 1/2
$0 \rightarrow$	< -1/2
-1>	< 1/2
	<−3/2
-2→	←-5/2
-3→	
-4->	←-1/2
	←-9 /2
$-5 \rightarrow$	←-11/2
	Concession of the second se

CAUTION Equation (35.6) is for small angles only While Eqs. (35.4) and (35.5) are valid for any angle, Eq. (35.6) is valid for small angles only. It can be used only if the distance R from slits to screen is much greater than the slit separation d and if Ris much greater than the distance y_m from the center of the interference pattern to the mth bright band.

$$y_m = R \tan \theta_m$$

In such experiments, the distances y_m are often much smaller than the distance *R* from the slits to the screen. Hence θ_m is very small, $\tan \theta_m \approx \sin \theta_m$, and

$$y_m = R \sin \theta_m$$

Combining this with Eq. (35.4), we find that for small angles only,

Constructive	Position of <i>m</i> th bright band Wavelength	
interference,	ma	
Young's experiment (small angles only):	$y_m = R \frac{m\pi}{d} (m = 0, \pm 1, \pm 2, \dots)$	(35.6)
(Distance from slits to screen Distance between slits	

We can measure *R* and *d*, as well as the positions y_m of the bright fringes, so this experiment provides a direct measurement of the wavelength λ . Young's experiment was in fact the first direct measurement of wavelengths of light.

EXAMPLE 35.1 Two-slit interference

Figure 35.7 shows a two-slit interference experiment in which the slits are 0.200 mm apart and the screen is 1.00 m from the slits. The m = 3 bright fringe in the figure is 9.49 mm from the central bright fringe. Find the wavelength of the light.

Figure 35.7 Using a two-slit interference experiment to measure the wavelength of light.



IDENTIFY and SET UP Our target variable in this two-slit interference problem is the wavelength λ . We are given the slit separation d = 0.200 mm, the distance from slits to screen R = 1.00 m, and the distance $y_3 = 9.49$ mm on the screen from the center of the interference pattern to the m = 3 bright fringe. We may use Eq. (35.6) to find λ , since the value of R is so much greater than the value of d or y_3 .

EXECUTE We solve Eq. (35.6) for λ for the case m = 3:

$$\lambda = \frac{y_m d}{mR} = \frac{(9.49 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{(3)(1.00 \text{ m})}$$
$$= 633 \times 10^{-9} \text{ m} = 633 \text{ nm}$$

EVALUATE This bright fringe could also correspond to m = -3. Can you show that this gives the same result for λ ?

EXAMPLE 35.2 Broadcast pattern of a radio station

It is often desirable to radiate most of the energy from a radio transmitter in particular directions rather than uniformly in all directions. Pairs or rows of antennas are often used to produce the desired radiation pattern. As an example, consider two identical vertical antennas 400 m apart, operating at 1500 kHz = 1.5×10^6 Hz (near the top end of the AM broadcast band) and oscillating in phase. At distances much greater than 400 m, in what directions is the intensity from the two antennas greatest?

IDENTIFY and SET UP The antennas, shown in **Fig. 35.8**, correspond to sources S_1 and S_2 in Fig. 35.5. Hence we can apply the ideas of two-slit interference to this problem. Since the resultant wave is detected at distances much greater than d = 400 m, we may use Eq. (35.4) to give the directions of the intensity maxima, the values of θ for which the path difference is zero or a whole number of wavelengths.

Figure 35.8 Two radio antennas broadcasting in phase. The purple arrows indicate the directions of maximum intensity. The waves that are emitted toward the lower half of the figure are not shown.



EXECUTE The wavelength is $\lambda = c/f = 200$ m. From Eq. (35.4) with $m = 0, \pm 1$, and ± 2 , the intensity maxima are given by

$$\sin \theta = \frac{m\lambda}{d} = \frac{m(200 \text{ m})}{400 \text{ m}} = \frac{m}{2}$$
 $\theta = 0, \pm 30^{\circ}, \pm 90^{\circ}$

In this example, values of *m* greater than 2 or less than -2 give values of $\sin \theta$ greater than 1 or less than -1, which is impossible. There is *no* direction for which the path difference is three or more wavelengths, so values of *m* of ± 3 or beyond have no meaning in this example.

EVALUATE We can check our result by calculating the angles for *minimum* intensity, using Eq. (35.5). There should be one intensity minimum between each pair of intensity maxima, just as in Fig. 35.6. From Eq. (35.5), with m = -2, -1, 0, and 1,

$$\sin \theta = \frac{\left(m + \frac{1}{2}\right)\lambda}{d} = \frac{m + \frac{1}{2}}{2} \qquad \theta = \pm 14.5^{\circ}, \pm 48.6^{\circ}$$

These angles fall between the angles for intensity maxima, as they should. The angles are not small, so the angles for the minima are *not* exactly halfway between the angles for the maxima.

Phasor Diagram for Superposition

- To add the two sinusoidal functions with a phase difference, we can use the same phasor representation that we used for simple harmonic motion (Chapter 14) and for voltages and currents in ac circuits (Chapter 31).
- Each sinusoidal function is represented by a rotating vector (phasor) whose projection on the horizontal axis at any instant represents the instantaneous value of the sinusoidal function.





Electric Field in Interference Patterns

- To find the intensity at any point in a two-source interference pattern, we have to combine the two sinusoidally varying fields (from the two sources) at a point *P*.
- If the two sources are in phase, then the waves that arrive at *P* differ in phase by an amount φ that is proportional to the difference in their path lengths,
- If we further assume the amplitudes of the two waves are both approximately equal to *E* at point *P*, the combined amplitude is:





Poynting vector
$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$
 Magnetic field



$$I = S_{\rm av} = \frac{E_P^2}{2\mu_0 c} = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_P^2 = \frac{1}{2} \epsilon_0 c E_P^2$$
(35.8)

The essential content of these expressions is that *I* is proportional to E_P^2 . When we substitute Eq. (35.7) into the last expression in Eq. (35.8), we get

$$I = \frac{1}{2}\epsilon_0 c E_P^2 = 2\epsilon_0 c E^2 \cos^2 \frac{\phi}{2}$$
(35.9)

In particular, the *maximum* intensity I_0 , which occurs at points where the phase difference is zero ($\phi = 0$), is

$$I_0 = 2\epsilon_0 c E^2$$

$$\frac{\text{Maximum intensity}}{\text{Intensity in source interference}} I = I_0 \cos^2 \frac{\phi}{2} \frac{\phi}{between waves}$$
(35.10)

Phase Difference and Path Difference

$$\frac{\phi}{2\pi} = \frac{r_2 - r_1}{\lambda}$$

Phase difference in the source interference
$$\phi = \frac{2\pi}{\lambda} (r_2 - r_1) = k(r_2 - r_1)$$
 (35.11)
Wavelength Distance from source 1

Intensity in Interference Patterns

• The intensity at any point in a two-source interference pattern is:



- Here I₀ is the maximum intensity, which is four times as great as the intensity from each individual source.
- The phase difference is:

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Video Tutor Solution: Example 35.3





EXAMPLE 35.3 A directional transmitting antenna array

Suppose the two identical radio antennas of Fig. 35.8 are moved to be only 10.0 m apart and the broadcast frequency is increased to f = 60.0 MHz. At a distance of 700 m from the point midway between the antennas and in the direction $\theta = 0$ (see Fig. 35.8), the intensity is $I_0 = 0.020 \text{ W/m}^2$. At this same distance, find (a) the intensity in the direction $\theta = 4.0^\circ$; (b) the direction near $\theta = 0$ for which the intensity is $I_0/2$; and (c) the directions in which the intensity is zero.

IDENTIFY and SET UP This problem involves the intensity distribution as a function of angle. Because the 700 m distance from the antennas to the point at which the intensity is measured is much greater than the distance d = 10.0 m between the antennas, the amplitudes of the waves from the two antennas are very nearly equal. Hence we can use Eq. (35.14) to relate intensity *I* and angle θ . **IDENTIFY and SET UP** This problem involves the intensity distribution as a function of angle. Because the 700 m distance from the antennas to the point at which the intensity is measured is much greater than the distance d = 10.0 m between the antennas, the amplitudes of the waves from the two antennas are very nearly equal. Hence we can use Eq. (35.14) to relate intensity *I* and angle θ .

EXECUTE The wavelength is $\lambda = c/f = 5.00$ m. The spacing d = 10.0 m between the antennas is just twice the wavelength (as was the case in Example 35.2), so $d/\lambda = 2.00$ and Eq. (35.14) becomes

$$I = I_0 \cos^2 \left(\frac{\pi d}{\lambda} \sin \theta\right) = I_0 \cos^2 \left[(2.00\pi \text{ rad}) \sin \theta \right]$$

(a) When $\theta = 4.0^{\circ}$,

$$I = I_0 \cos^2[(2.00\pi \text{ rad})\sin 4.0^\circ] = 0.82I_0$$

= (0.82)(0.020 W/m²) = 0.016 W/m²

(b) The intensity *I* equals $I_0/2$ when the cosine in Eq. (35.14) has the value $\pm 1/\sqrt{2}$. The smallest angles at which this occurs correspond to $2.00\pi \sin\theta = \pm \pi/4$ rad, so $\sin\theta = \pm (1/8.00) = \pm 0.125$ and $\theta = \pm 7.2^{\circ}$.

(c) The intensity is zero when $\cos[(2.00\pi \text{ rad})\sin\theta] = 0$. This occurs for $2.00\pi \sin\theta = \pm \pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$, or $\sin\theta = \pm 0.250, \pm 0.750, \pm 1.25, \ldots$. Values of $\sin\theta$ greater than 1 have no meaning, so the answers are

$$\theta = \pm 14.5^{\circ}, \pm 48.6^{\circ}$$

EVALUATE The condition in part (b) that $I = I_0/2$, so that $(2.00\pi \text{ rad})\sin\theta = \pm \pi/4$ rad, is also satisfied when $\sin\theta = \pm 0.375$, ± 0.625 , or ± 0.875 so that $\theta = \pm 22.0^\circ$, $\pm 38.7^\circ$, or $\pm 61.0^\circ$. (Can you verify this?) It would be incorrect to include these angles in the solution, however, because the problem asked for the angle *near* $\theta = 0$ at which $I = I_0/2$. These additional values of θ aren't the ones we're looking for.

Interference in Thin Films (1 of 2)



Light reflected from the upper and lower surfaces of the film comes together in the eye at *P* and undergoes interference.

Some colors interfere constructively and others destructively, creating the color bands we see. aAir bFilm e Index n t



Phase Shifts During Reflection

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Interference in Thin Films (2 of 2)

 For light of normal incidence on a thin film with wavelength λ in the film, in which neither or both of the reflected waves have a half-cycle phase shift:



• When only one of the reflected waves has a half-cycle phase shift:

Constructive reflection:	$2t = (m + \frac{1}{2})\lambda_{k}$ $(m = 0, 1, 2,)$
(From thin film, half-cycle phase shift)	Thickness of film Wavelength
Destructive reflection:	$2t = m\lambda$ $(m = 0, 1, 2, \dots)$

Video Tutor Solution: Example 35.5

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$$E_{\rm r} = \frac{n_a - n_b}{n_a + n_b} E_{\rm i} \qquad (\text{normal incidence}) \tag{35.16}$$

This result shows that the incident and reflected amplitudes have the same sign when n_a is larger than n_b and opposite signs when n_b is larger than n_a . Because amplitudes must always be positive or zero, a *negative* value means that the wave actually undergoes a half-cycle (180°) phase shift. Figure 35.13 shows three possibilities:



Figure 35.13 Upper figures: electromagnetic waves striking an interface between optical materials at normal incidence (shown as a small angle for clarity). Lower figures: mechanical wave pulses on ropes.

Waves travel slower on heavy ropes than on light ropes.

Figure 35.14 (a) Light reflecting from a thin film produces a steady interference pattern, but (b) light reflecting from a thick film does not.

(a) Light reflecting from a thin film



(b) Light reflecting from a thick film



Nonreflective Coatings

Destructive interference occurs when

- the film is about $\frac{1}{4}\lambda$ thick and
- the light undergoes a phase change at both reflecting surfaces,

so that the two reflected waves emerge from the film about $\frac{1}{2}$ cycle out of phase.



Video Tutor Solution: Example 35.7

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EXAMPLE 35.4 Thin-film interference I

Suppose the two glass plates in Fig. 35.12 are two microscope slides 10.0 cm long. At one end they are in contact; at the other end they are separated by a piece of paper 0.0200 mm thick. What is the spacing of the interference fringes seen by reflection? Is the fringe at the line of contact bright or dark? Assume monochromatic light with a wavelength in air of $\lambda = \lambda_0 = 500$ nm.

Figure 35.15 Our sketch for this problem.



EXECUTE Since only one of the reflected waves undergoes a phase shift, the condition for *destructive* interference (a dark fringe) is Eq. (35.18b):

$$2t = m\lambda_0$$
 (*m* = 0, 1, 2, ...)

From similar triangles in Fig. 35.15 the thickness *t* of the air wedge at each point is proportional to the distance *x* from the line of contact:

$$\frac{t}{x} = \frac{h}{l}$$

Combining this with Eq. (35.18b), we find

$$\frac{2xh}{l} = m\lambda_0$$

$$x = m\frac{l\lambda_0}{2h} = m\frac{(0.100 \text{ m})(500 \times 10^{-9} \text{ m})}{(2)(0.0200 \times 10^{-3} \text{ m})} = m(1.25 \text{ mm})$$

Successive dark fringes, corresponding to m = 1, 2, 3, ..., are spaced 1.25 mm apart. Substituting m = 0 into this equation gives x = 0, which is where the two slides touch (at the left-hand side of Fig. 35.15). Hence there is a dark fringe at the line of contact.



Newton's Rings

Figure 35.16a shows the convex surface of a lens in contact with a plane glass plate. A thin film of air is formed between the two surfaces. When you view the setup with monochromatic light, you see circular interference fringes (Fig. 35.16b). These were studied by Newton and are called Newton's rings.

Figure 35.16 (a) Air film between a convex lens and a plane surface. The thickness of the film t increases from zero as we move out from the center, giving (b) a series of alternating dark and bright rings for monochromatic light.

(a) A convex lens in contact with a glass plane

(b) Newton's rings: circular interference fringes





Figure 35.18 A nonreflective coating has an index of refraction intermediate between those of glass and air.

Destructive interference occurs when



so that the two reflected waves emerge from the film about $\frac{1}{2}$ cycle out of phase.



Michelson Interferometer



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Michelson Interferometer

Suppose the angle between mirror M_2 and the virtual image of M_1 is just large enough that five or six vertical fringes are present in the field of view. If we now move the mirror M_2 slowly either backward or forward a distance $\lambda/2$, the difference in path length between rays 1 and 2 changes by λ , and each fringe moves to the left or right a distance equal to the fringe spacing. If we observe the fringe positions through a telescope with a crosshair eyepiece and *m* fringes cross the crosshairs when we move the mirror a distance *y*, then

$$y = m\frac{\lambda}{2}$$
 or $\lambda = \frac{2y}{m}$ (35.19)

